Time–bandwidth product of chirped sech^2 pulses: application to phase–amplitude-coupling factor measurement

P. Lazaridis, G. Debarge, and P. Gallion

Département Communications, École Nationale Supérieure des Télécommunications, 46, rue Barrault, 75634 Paris Cedex 13, France

Received January 27, 1995

An exact analytical expression for the time–bandwidth product $\Delta t \Delta f$ of chirped sech^2 pulses is derived. The relation can be expressed by $\Delta t \Delta f = 0.1786$ arcosh$(\cosh \pi\alpha + 2)$ as a function of the laser's phase–amplitude coupling factor α. An experimental measurement of the α factor that relies on this formula is discussed.

It is well known that gain-switched semiconductor laser pulses suffer from power spectrum broadening as a result of chirp included within the pulse.[1] This spectral broadening is due to the fact that the phase–amplitude-coupling factor α of semiconductor lasers is different from zero and usually lies in the region $\alpha = 2$–8.[2] A simple analytical expression for a Gaussian pulse shape was shown to be[3] $\Delta t \Delta f = 0.44\sqrt{1 + \alpha^2}$, where $\Delta t \Delta f$ is the time–bandwidth product (TBP) and Δt and Δf are the FWHM of the optical pulse intensity and of its power spectrum, respectively. Because of its simplicity this has been the only formula used for estimation of the α factor from $\Delta t \Delta f$ measurements of gain-switched laser pulses. However, it gave systematically higher values for α than did other measurement methods.[1,2] Moreover, experimental evidence suggests that pulses from gain-switched semiconductor lasers have sech^2 rather than Gaussian shapes.[4,5] In this Letter a new formula is derived that, unlike other expressions proposed,[6] is exact for the chirped sech^2 pulse and gives better estimates of the laser's α factor in the large-signal regime than the Gaussian approximation gives.

The Gaussian and sech^2 pulse shapes look much alike, as can be seen from Fig. 1, apart from their decay rate (which is faster for the Gaussian pulses). However small this difference may seem, it results in a different TBP. In the case of unchirped pulses this product is 0.441 for the Gaussian and 0.315 for the sech^2 pulse. When the pulses are chirped, as is the case with gain-switching pulses, the instantaneous frequency changes with time, resulting in a broadened spectrum. The phase equation for the laser pulse is given by[3]

$$\frac{\mathrm{d}\varphi(t)}{\mathrm{d}t} = \frac{\alpha}{2}\,\frac{1}{P}\,\frac{\mathrm{d}P}{\mathrm{d}t}\,, \tag{1}$$

where $P(t)$ is the pulse intensity. In Fig. 2 the variation of instantaneous frequency as a function of time is shown, assuming an α factor of 5. It can be seen that the chirped Gaussian pulse presents a linear frequency deviation with decreasing frequency as a function of time, which is referred to as downchirp. On the other hand, the chirped sech^2 pulse presents a nonlinear frequency deviation, of a hyperbolic tangent variation, which is only approximately linear near the origin and saturates at the tails of the pulse. This nonlinearity results in an even broader spectrum for usual values of α, as will be demonstrated. An outline of the derivation is given below. The pulse shape in the time domain is

$$P(t) = |E(t)|^2 = \mathrm{sech}^2(t/\tau)\,, \tag{2}$$

where $P(t)$ is the output photon density, $E(t)$ is the normalized output electrical field, and τ is a time-scaling parameter. From the above formula it is straightforward to calculate the FWHM Δt, which is given by

$$\Delta t = 2\,\mathrm{arcosh}(\sqrt{2})\tau = 1.763\tau\,, \tag{3}$$

where $\mathrm{arcosh}(x) = \ln(x + \sqrt{x^2 - 1})$ is the inverse hyperbolic cosine function. The spectrum of the pulse is derived by use of the Fourier transform (FT) of sech^p, where $p = 1 + j\alpha$ is a complex exponent[3] representing the chirped nature of the pulse. The spectrum is obtained in terms of gamma functions of a complex argument as[7]

$$\tilde{E}(\omega) = \mathrm{FT}[\mathrm{sech}^p(t/\tau)] = \frac{2^{p-1}\tau}{\Gamma(p)}\Gamma\left(\frac{p + j\omega\tau}{2}\right)\Gamma\left(\frac{p - j\omega\tau}{2}\right)\cdot \tag{4}$$

Then, using the well-known gamma function properties, we obtain the remarkably simple expression for the normalized power spectrum of the chirped pulse in terms of the product of two hyperbolic secants as

$$\frac{|\tilde{E}(\omega)|^2}{|\tilde{E}(0)|^2} = \frac{\mathrm{sech}\left[\frac{\pi}{2}(\omega\tau + \alpha)\right]\mathrm{sech}\left[\frac{\pi}{2}(\omega\tau - \alpha)\right]}{\mathrm{sech}^2(\pi\alpha/2)}\,. \tag{5}$$

When the pulse is unchirped, i.e., $\alpha = 0$, we obtain the familiar sech^2 power spectrum. In Fig. 3 one can notice the difference between the spectra of the chirped Gaussian and sech^2 pulses. We traced both curves by assuming an α factor of 5. Formula (5) is used for the

0146-9592/95/101160-03$6.00/0

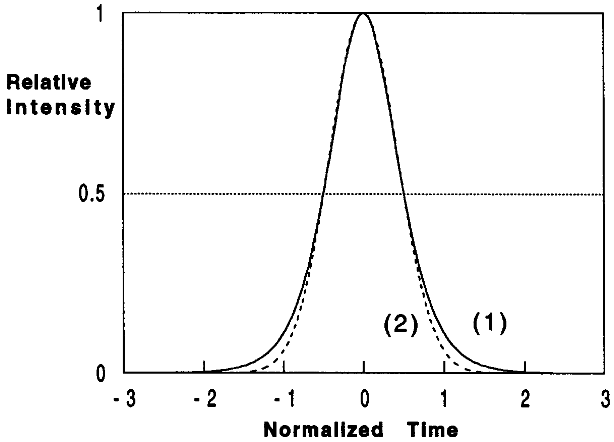

Fig. 1. Normalized intensity of (1) a sech^2 and (2) a Gaussian pulse having the same FWHM time duration. Time is normalized relative to the FWHM.

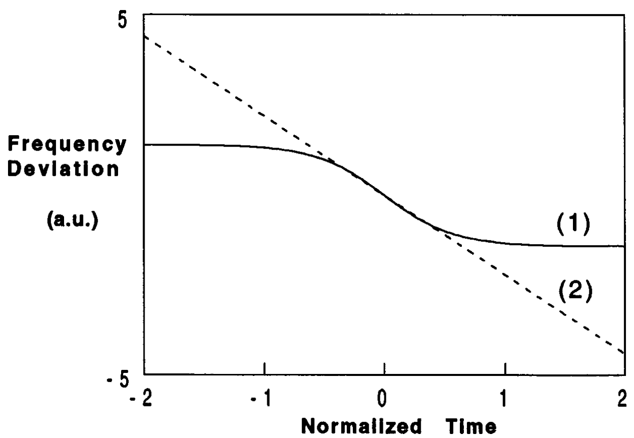

Fig. 2. Instantaneous frequency deviation for the chirped (1) sech^2 and (2) Gaussian pulses versus normalized time.

chirped sech^2 spectrum, the corresponding formula for the chirped Gaussian pulse is

$$\frac{|\tilde{E}(\omega)|^2}{|\tilde{E}(0)|^2} = \exp\left(-\frac{\omega^2 {\tau_1}^2}{1+\alpha^2}\right), \qquad (6)$$

and $\tau_1 = 1.06\tau$, so the sech^2 and the Gaussian $\{P(t) = \exp[-(t/\tau_1)^2]\}$ pulses have the same FWHM time duration.

It is interesting to note that the spectrum is itself Gaussian even for a chirped pulse. Although neither of the curves presents rabbit ears and the curves are perfectly symmetric, the sech^2 curve seems to be more realistic, fitting experimental and numerical results better in that it falls off more rapidly than the Gaussian curve and that it presents an extended plateau region. From formula (5) the FWHM Δf is calculated and multiplied by Eq. (3) to yield finally

$$\Delta t \Delta f = \left[\frac{2\,\mathrm{arcosh}(\sqrt{2})}{\pi^2}\right] \mathrm{arcosh}(\cosh \pi\alpha + 2), \qquad (7)$$

which is our main result. In Fig. 4 the TBP of chirped Gaussian and sech^2 pulses is plotted as a function of the laser's α factor. An interesting feature of this graph is that, although the sech^2 pulse starts with an initially lower TBP value of 0.315 instead of 0.441 for the Gaussian, it soon ends up with higher $\Delta t \Delta f$ values for the usual α factors, i.e., $\alpha \geq 2$. This behavior was actually predicted in Ref. 6 by numerical simulation, and then the authors of Ref. 6 tried a curve fit to explain the calculated results. Formula (7) gives the correct TBP value for transform-limited (unchirped) pulses, i.e., 0.315. On the other hand, neglecting 2 in comparison to $\cosh \pi\alpha$ yields the excellent asymptotic approximation $\Delta t \Delta f = 0.56\alpha$, which is very accurate for $\alpha \geq 2$. This is a useful result, as α varies between 2 and 4 for strained quantum-well lasers and between 5 and 8 for bulk lasers, meaning that this approximate formula is always valid. The corresponding asymptotic result for the chirped Gaussian pulse is easily seen to be 0.44α, so the value of α is overestimated when the Gaussian approximation is used in cases in which the pulse shape is closer to sech^2. The overestimation error asymptotically reaches 27%. In Table 1 the α factor estimations for the two approximations are shown to be significantly different, and this explains the observation in Ref. 1 that the Gaussian formula overestimates α by approximately 20% compared with

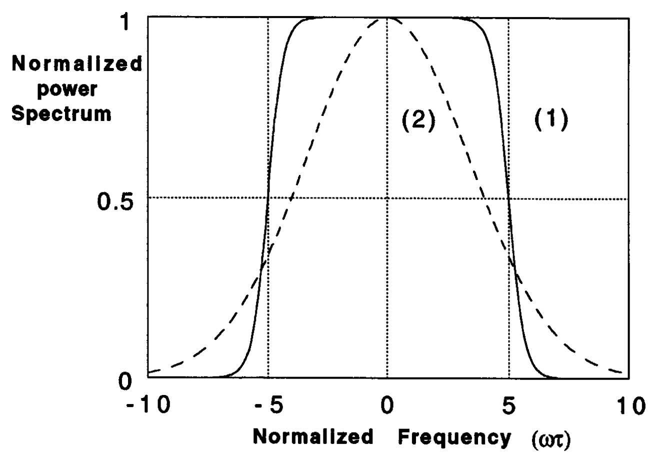

Fig. 3. Normalized power spectrum of the chirped (1) sech^2 and (2) Gaussian pulses as a function of normalized frequency.

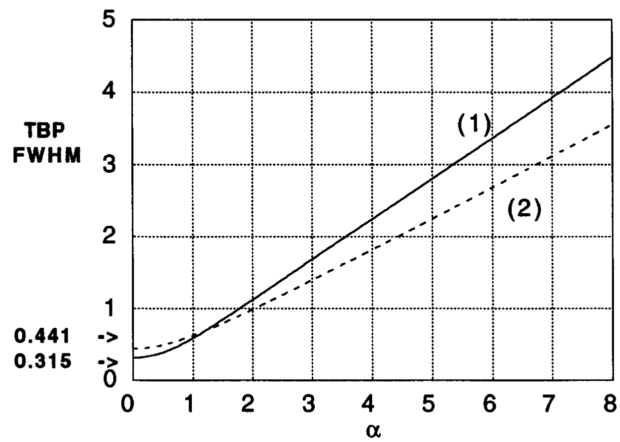

Fig. 4. Time–bandwidth product of the chirped (1) sech^2 and (2) Gaussian pulses as a function of the laser's phase–amplitude-coupling factor.

Table 1. Comparison of Phase–Amplitude-Coupling Factor α Values Estimated by the Gaussian and the Soliton sech^2 Approximations from $\Delta t \Delta f$ Measurement Data

α (G)	α (sech^2)	Difference (%)	$\Delta t \Delta f$
3	2.5	20	1.39
5	4.0	25	2.25
8	6.3	27	3.56

other measurement methods. However, one should always bear in mind that, strictly speaking, the above analytical results hold only in the case of perfectly symmetric Gaussian and sech^2 pulses and therefore symmetric power spectra. So care must be taken in applying these results in the case of very large modulation amplitudes for gain switching, where pulses develop long tails and power spectra become asymmetrical, presenting rabbit ears of different amplitudes. This condition being satisfied, experiment shows that the pulse shape and the power spectrum bear a closer similitude to the sech^2 form than to the Gaussian form. Theoretically, this is explained by the fact that the laser's rate equations support an exponential rise and fall for the pulse rather than the $\exp(-t^2)$ Gaussian evolution. The latter is confirmed by numerical solution of the rate equations.[6] Our numerical experiments with the rate equations gave us excellent agreement between the α factor used in the simulations and the one estimated by $\alpha = \Delta t \Delta f / 0.56$ for $\alpha \geq 2$, as long as the laser's driving conditions were moderate, such as to give rise to almost symmetric pulses.

In conclusion, an exact analytical expression is derived for the TBP of sech^2 chirped pulses as a function of the laser's α factor. It can be used to determine the pulse spectrum broadening and hence the deviation from ideal soliton propagation requirements, i.e., transform-limited pulses, that is due to nonzero laser α factors. On the other hand, the exact as well as the approximate formulas can be used as means to estimate the laser's α by TBP measurements in the gain-switching regime. In the latter case this leads to the simple estimation formula $\alpha = \Delta t \Delta f / 0.56$ that is valid for $\Delta t \Delta f \geq 1$.

This research was partially supported by the Centre National d'Etudes des Télécommunications. The authors thank J.-C. Bouley, C. Kazmierski, and P. Vandamme for helpful discussions during this work.

References

1. M. Osinski, D. F. G. Gallagher, and I. H. White, Electron. Lett. **21,** 981 (1985).
2. M. Osinski and J. Buus, IEEE J. Quantum Electron. **QE-23,** 9 (1987).
3. T. L. Koch and J. E. Bowers, Electron. Lett. **20,** 1038 (1984).
4. H. F. Liu, Y. Ogawa, and S. Oshiba, Appl. Phys. Lett. **59,** 1284 (1991).
5. H. F. Liu, S. Oshiba, Y. Ogawa, and Y. Kawai, Opt. Lett. **17,** 64 (1992).
6. K. Wada and Y. Cho, Opt. Lett. **19,** 1633 (1994).
7. I. S. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products* (Academic, New York, 1980), Eq. (3.985.1), p. 506.