Opt Quant Electron (2015) 47:3775–3782
DOI 10.1007/s11082-015-0245-8

Intrinsic error vector magnitude on pulsed local oscillator coherent optical receiver measurements

P. Gallion[1] · C. Gosset[1] · X. You[1] · J. Zhou[2]

Received: 13 January 2015 / Accepted: 3 August 2015 / Published online: 14 August 2015

Abstract Using a quantum optical noise description, we derive the fundamental noise limitation in the simultaneous measurements of the two quadrature of a signal performed by a double balanced pulsed local oscillator coherent receiver. The best achievable quantum limited signal-to-noise ratio and error vector measurement are derived. The general results are discussed more in detail for the equivalent time under sampling of optical communication signal with different modulation constellations. Results may be extended to real time signal sampling.

Keywords Homodyning · Optical communications · Coherent communications · Quantum communications · Quantum cryptography

1 Introduction

Coherent optical detection is based on the mixing of the received signal with a continuous wave (CW) local oscillator (LO) and is now widely used for long span and high bit rate communication systems. It takes benefit of the noise free mixing gain provided by the strong level LO and of the preservation of the phase and of the signal spectrum in the down conversion of the signal from the optical range to the radiofrequency one. Post detection electrical sampling is usually achieved afterward by using analog to digital conversion. The digital signal processing techniques, already developed for communications in the radiofrequency, are used afterward to cancel electronically the phase rotation induced by

✉ P. Gallion
philippe.gallion@telecom-paristech.fr

1 CNRS LTCI, Telecom ParisTech, Ecole Nationale Supérieure des Télécommunications, 46, rue Barrault, 75013 Paris, France

2 Department of Electronics Science and Engineering, Tongji University, Shanghai, People's Republic of China

signal-LO frequency detuning and phase mismatch, to correct the linear impairments of the optical channel, such as dispersion, and to manage the polarization dispersion in terms of a channel diversity.

In coherent detection applications, the signal optical field is usually weak, as compared to the local one, and a strong and noise free mixing gain, overcoming thermal noise, is obtained without optical pre amplification, making the quantum optical noise to be the fundamental limitation. In optical engineering and for major applications, the influence of intrinsic quantum light fluctuations incoming through the signal and LO ports of the optical mixer are not explicitly considered and quantum noise is usually taken into account in terms of LO shot noise. Such an approach involves, in the same time, the wave nature of the light, as the signal is concerned, and the corpuscular one for the shot noise description and is therefore only approximated. Furthermore the shot noise is sometimes thought of as having its fundamental origin only at photo receiver level. However, this approach gives approximately good results since intrinsic quantum power fluctuation corresponding to the shot noise can be interpreted as the cross term between the local field and the vacuum fluctuation, and the later have the same level for coherent states signal and LO at the inputs of the mixer. Furthermore, in long span optical communication the shot noise contribution is far below the in line optical amplification noise accumulation.

A semiclassical analysis of the two photo detectors balanced homodyne detection (BHD) have been proposed (Abbas et al. 1983), demonstrating the property of canceling the LO excess noise, but still interpreting the quantum limit as the result of the LO shot-noise. By using a quantum mechanical treatment (Yuen and Chan 1983; Schumaker 1984) have demonstrated that the quantum limit is governed by quantum fluctuation incoming through the signal port. The optical quantum noise limitation of the single quadrature signal measurement performed with a passive 2-photo detector BHD using CW LO is today perfectly understood and have been experimentally verified (Machida and Yamamoto 1986).

Less attention have been paid up to now to the active LO coherent detection. A general derivation of the pulsed local oscillator (PLO) coherent receiver sensitivity is provided here in terms of signal-to-noise (SNR) ratio and error vector magnitude (EVM). As an example, the results are discussed more in detail for the linear optical undersampling situation but can be obviously extended to real time signal system operation.

2 Pulsed local oscillator (PLO) coherent receivers

The first proposed active LO quantum receiver uses a conditional an amplitude tuning and a phase switching of the LO during the symbol duration of binary phase shift keyed signal to cancel out one of the two antipodal binary signal values (Dolinar 1973).

Another active LO situation, displayed in Fig. 1a, is the signal time gating by a PLO, introduced for communication system operation (Zhang et al. 2009). By using parallel arrangement, this technique allows to overcome the electronics bandwidth limitations of the photo receivers, of the electrical AD converters and of the post detection signal processing. These limitations become more stringent as the transmission rate and as the number of samples, required during the symbol duration, increase by using high order M-ary Quadrature Amplitude Modulation (MQAM) modulation formats.

Thanks to real time systems operation, synchronization is mandatory required to recovery the clock and phase of the data. The optical linear sampling, using PLO, allows

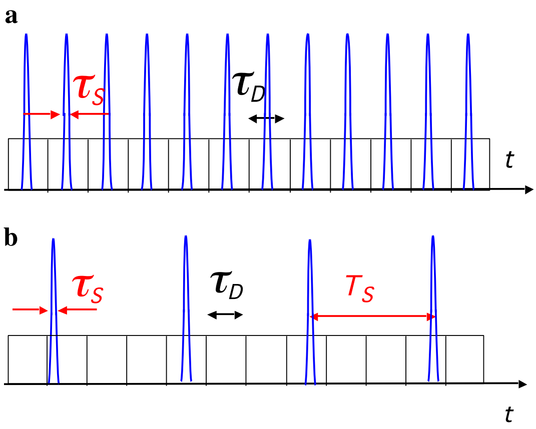

Fig. 1 **a** Real time synchronous sampling. **b** Equivalent time asynchronous undersampling

the real time de-multiplexing of optical signal at Tb/s rate, by using a parallel implementation at coherent receiver and its associated digital signal processing.

A second application of PLO coherent detection receiver is the undersampling technique (Shake et al. 1998; Dorrer 2006), providing low cost, blind and asynchronous characterization of high bit rate optical data flows, free of receiver bandwidth limitation for symbol rate up to the 100 Gbd/s range (Fig. 1b). As far as only the characterization of the optical channel, or the monitoring, of an optical system is concerned, only the access to the intrinsic statistical properties of received signal is required. A typical undersampling configuration is depicted on Fig. 1b. Thanks to the equivalent time sampling operation, high bandwidth receiver and synchronization are no more required and therefore no more impair the measurements, making the intrinsic receiver noise impairment is the major limitation.

A third application of a PLO coherent detection receiver is to reach the standard quantum limit for quantum cryptography application using a quadrature phase shift keying (QPSK) constellation (Gallion et al. 2009).

In optical communications systems, performances are usually simultaneously limited by the optical channel impairments and by the receiver noise and bandwidth limitations, making the intrinsic characterization of channel difficult. As both the optical communication channel and the optical receiver impairments are separately inducing deviations of the measured signal vectors from theirs nominal initial positions, the channel characterization requires the accurate knowledge of the intrinsic noise of the receiver and of the corresponding EVM. The intrinsic noise of a PLO coherent receiver is the fundamental sensitivity limitation for these three applications. As coherent optical detection usually allows to avoid optical pre-amplification, and as the signal and local fields have different time extensions, a specific analysis of the quantum noise limitation of a PLO coherent detection is called for.

3 Optical sample energy

Nowadays, the coherent optical communication receivers perform the simultaneous measurements of the two quadratures components (usually referred as I and Q) of the optical signal vector $e_D(t)$. It has been pointed out earlier (Oliver et al. 1962; Yuen and Shapiro

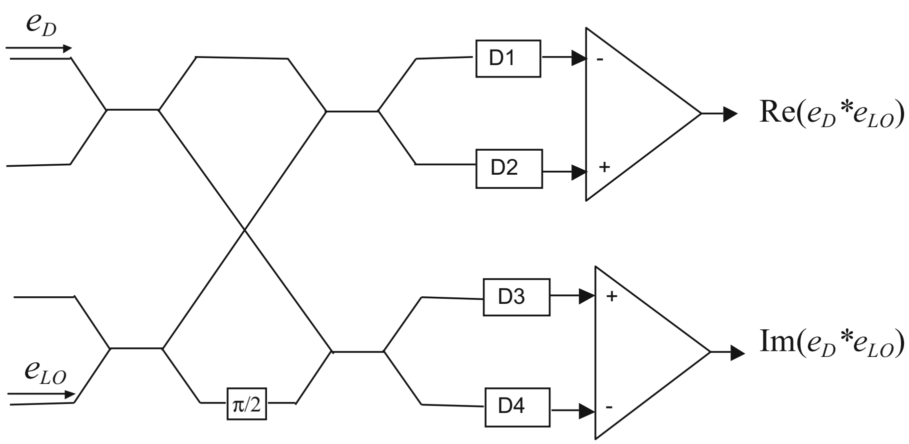

Fig. 2 The double balanced homodyne detection (DBHD) arrangement

1980) that homodyne and heterodyne detections represent respectively the ideal technique for a single quadrature and simultaneous two-quadrature measurements of the signal field.

For a given polarization mode of the signal, the receiver use an optical 4 × 4 optical hybrid and a four photo detectors double balanced homodyne detection (DBHD) depicted on Fig. 2. $e_{LO}(t)$ is the LO field and $e_D^*(t)$ stands for complex conjugate of the signal field.

Let us assume that the normalized electrical field $e_{LO}(t)$ of the PLO consists in a train of optical pulses with a time envelope $E_{LO}(t)$, a with duration τ_S, a central angular frequency ω_{LO} and a repetition period T_S

$$e_{LO}(t) = \left[\sum_n E_{LO}(t - nT_S)\right] \exp j[\omega_{LO}t + \varphi_{LO}(t)] \tag{1}$$

The signal under test is written as

$$e_D(t) = E_D(t) \exp j(\omega_C t + \varphi_C) \tag{2}$$

where $E_D(t)$ is the complex envelope of the signal under test, affected by channel impairment and noise accumulation, and ω_C is its carrier angular frequency.

Discarding square signal and local fields terms, the two quadratures measurements at the output of the mixer are the time integration of the real and the imaginary part of

$$e_D^*(t)e_{LO}(t) = \sum_{\text{pulses}} E_{LO}^*(t)E_S(t - nT_S) \exp(-j(\Delta\omega_C t + \Delta\varphi)) \tag{3}$$

where $\Delta\omega_C = \omega_C - \omega_{LO}$ and $\Delta\varphi(t) = \varphi_C(t) - \varphi_{LO}(t)$ are the angular optical difference and phase difference, between the signal under test and the LO, respectively. The phase shifts of the four-port coupler are omitted for sake of simplification. As we are a in PLO configuration, the integration time for the optical signal is not the electronics integration time τ_E of the photo receivers but is shortened down to the LO pulse duration τ_S. However the former is to be selected in relation with pulse periodicity to avoid sample overlap. Assuming a slow variation of the complex envelop of the signal under test over time τ_S, the I and Q component of the #n sample are the real and the imaginary part of

$$W_n = E_D^*(nT_S)\int_{-\infty}^{+\infty} E_{LO}(t - nT_S)\exp(-j\Delta\omega_C t)dt \tag{4}$$

As a function of the relative values of the beating frequency period and of sampling pulse duration, different situations may occur. For $\Delta\omega_C\tau_S \gg \pi$, the output signal is averaged out by phase rotation thanks to the fast variation of the beating term at the angular frequency $\Delta\omega_C$. On the other hand, for $\Delta\omega_C\tau_S \ll \pi$ it can be approximated as

$$W_n = T_S A_0(\Delta\omega_C)E_D^*(nT_S)\exp(-j\Delta\omega_C nT_S) \tag{5}$$

where $A_0(\omega) = A(\omega + \omega_0)$ is the equivalent baseband of the spectral amplitude envelope $A(\omega)$ of the modes of the LO source, related to the Fourier transform of the pulse envelope by $A(\omega)T_S = \hat{E}_{LO}(\omega)$. The bandwidth is optically determined by the PLO spectrum coherent and the optical detection, operating in an intradyne mode for the LO, can be described as a homodyne arrangement. Operation at the center of the optical bandwidth is assumed in the following for sake of simplicity.

4 Noise sources and error vector magnitude

As already mentioned, the quantum theory of homodyne detection, developed in the 80's, points out that the optical noise originates in fundamental quantum noise contribution induced by the so-called vacuum fluctuation entering through each of the four ports of the 4×4 optical hybrid mixer. The vacuum fluctuation, incoming through the two unused ports of the optical hybrid, stands for the signal splitting noise which is the price to pay for simultaneous quadrature measurements (Oliver 1965; Haus 1995). Using a symmetrized quantum noise energy operator, an additive Gaussian circular white noise (AGWN) with a spectral density $S_N = h\nu/2$ at each optical port is assumed and noise-to-noise cross terms are obviously discarded. The other noise source is the classical electronics thermal noise of the transimpedance amplifiers.

Assuming a nearly balanced and symmetric 4×4 hybrid mixer, the SNR is

$$SNR = \frac{2W_{LO}W_D\frac{\tau_S}{\tau_D}}{h\nu\left(F_D W_{LO} + F_{LO}W_D\frac{\tau_E}{\tau_D}\right) + 2\frac{F_D F_{LO} kT_{EQ}}{R^2 R_{EQ}}\tau_E} \tag{6}$$

where $W_{LO} = P_{LO}T_S$ is the LO sampling pulse energy, $W_D = P_D\tau_D$ the average symbol energy, τ_D the symbol duration, R the photodiode sensitivity and $2kT_{EQ}\tau_E/R^2R_{EQ}$ the equivalent thermal noise contribution reported at the mixer input. F_D and F_{LO} are the noise figure associated to the total loss of signal under test input and sampling signal input ports respectively.

The EVM is now the key parameter to characterize the receiver impairment on the signal under test and is directly related to the accuracy of its two-quadrature measurements (Fig. 3).

Let us consider firstly the simplest case of a constant envelope modulation QPSK. The EVM is expressed in this case as

$$EVM = \sqrt{\frac{\left\langle (I_M - I_R)^2\right\rangle + \left\langle (Q_M - Q_R)^2\right\rangle}{(I_R)^2 + (Q_R)^2}} \tag{7}$$

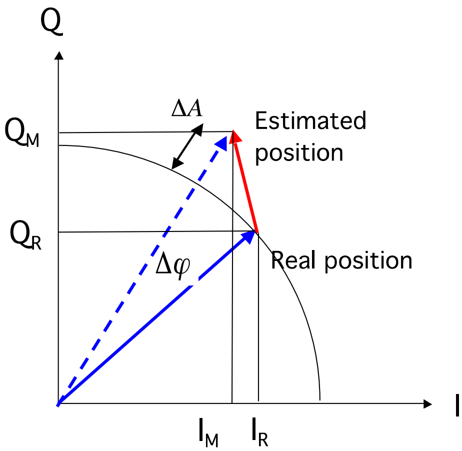

Fig. 3 The error vector magnitude (EVM)

where $(I_M - I_R)$ and $(Q_M - Q_R)$ are the differences between the measured values, denoted with the subscript M and the actual values, denoted with the subscript R, for the two quadratures I and Q of the measured signal vector, respectively.

For a constant envelope modulation and or a Gaussian circular noise, the relation $EVM = (SNR)^{-1/2}$ directly relates the EVM to the signal-to-noise ratio (SNR).

For multi level usual square MQAM modulation, EVM is usually defined by using normalization with the power averaging over the signal alphabet. In this case EVM is obviously enlarged to account for the reduction of average signal power $\langle E \rangle$ as compared to the peak power E_{MAX} of the symbol at the constellation corners. For a square MQAM we have

$$E_{MAX} = 2\left(M^{1/2} - 1\right)^2 \text{ and } \langle E \rangle = \frac{2}{3}(M - 1) \tag{8}$$

Therefore, for high LO power level the EVM enlargement factor ρ given by

$$\rho = \left[3\left(M^{1/2} - 1\right) \Big/ \left(M^{1/2} + 1\right)\right]^{1/2} \tag{9}$$

For instance, ρ equals 1.35 and 1.51 for 16QAM and 64QAM constellations, respectively.

5 Application to under sampling

In a typical undersampling configuration the LO is a train of brief optical pulses with typical duration τ_S in the 1 ps range, with a sampling period T_S far larger than symbol duration τ_D which is typically in the 10 ps range. The pulse repetition rate is typically in the few hundreds of MHz range and allows a low frequency photoreceivers and post detection signal processing.

As a low rate sampling LO is used for the characterization of a high symbol rate data flow, only a few symbols of the data flow are sampled. No frequency and phase relations between the symbol and the sampling rates are required and an unsynchronized random sampling is obtained. Only an appropriate non-integer part δt of the sampling period $T_S = k\tau_D + \delta t$, where k is an integer, is required for a fast acquisition of a statistically representative set of samples. From a statistically representative set of samples, the statistical properties of the signal under test can be recovered in an equivalent time sampling technique. The dedicated software signal processing is required, to perform the blind optical frequency and phase recovery will be not discussed here. This asynchronous characterization method of optical data flows is free of receiver bandwidth limitation, transparent to the bit rate, transparent to the modulation format and requires only low speed and low cost electronics. Because it is free of receiver limited bandwidth impairments, it allows an intrinsic channel impairment characterization.

Let us assume $F_{LO} = F_D = 2$, i.e. set at their minimal values, $R = 1$, a symbol duration $\tau_D = 10^{-11}$ s, a sampling duration $\tau_S = 10^{-12}$ s, a sampling period $T_S = 4 \times 10^{-9}$ s and an electrical integration time $\tau_E = 10^{-9}$ s. Figure 4 shows the signal to noise ratio, expressed by Eq. 6, as a function of the LO power and for various values of the signal

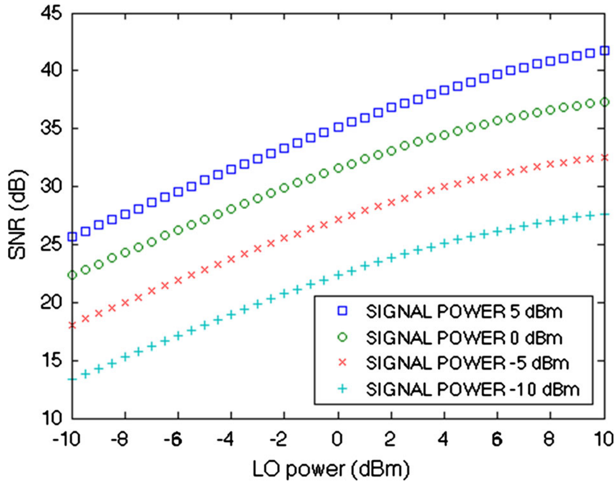

Fig. 4 Signal-to-noise ratio, as a function of the local oscillator power, for various values of the signal under test power

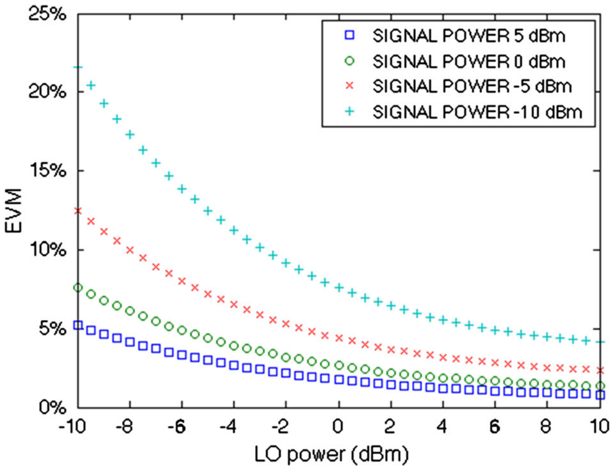

Fig. 5 EVM as a function of the local oscillator power, for various values of the signal power

under test power. For a LO sampling power $P_{LO} = W_{LO}/T_S$ equal to 0 dBm, a SNR ratio of 25 dB is obtained for a signal under test level of −5 dBm.

The so-called "limitation by the shot noise of the LO" is in fact the limitation by the fundamental quantum fluctuations incoming through the signal port in the quantum coherent detection theory. In this ideal situation, the SNR no longer depends on the LO power for which a few to 10 dBm level is required due to the low duty cycle of the pulsed LO. Figure 5 shows the EVM, expressed by Eq. 7, for a constant envelope PSK modulation, as a function of the LO power.

Obviously, for high value of the sampling power and a sampling pulse duration equal to the electronics integration time, Eq. 5 turns to $SNR = P_D/h\nu B_0$, where $B_0 = 1/\tau_S$ is the equivalent optical bandwidth. It is the half of the SNR of the standard CW LO quantum limited homodyne detection, the two factor reduction being the price to pay for simultaneous measurements of the two quadrature components of the field.

6 Conclusion

We derive the fundamental quantum noise limitation for the simultaneous measurements of the two quadratures of a signal, performed by a double balanced PLO coherent receiver. We propose a general expression of the PLO coherent receiver sensitivity. For the particular situation of linear optical undersampling, Eq. 6 gives the SNR ratio. Equation 7 provides a general expression of EVM and points out the LO power level requirement allowing quantum noise limitation. Results can be easily extended to real time signal system operation.

Acknowledgments This work is supported by the "OCELOT" project, from the French National Research Agency (ANR) program.

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