Analysis of frequency chirping of semiconductor lasers in the presence of optical feedback

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The frequency chirping of a single-mode semiconductor laser in the presence of optical feedback is studied by rate-equation analysis. The model includes the amplitude–phase-coupling and spectral-hole-burning effects. A simple analytical formula is obtained in the small-signal regime that shows that the amplitude–phase-coupling effect also enhances the chirp-reduction ratio and that the maximum reduction of frequency chirping can be achieved at the same time as maximum line narrowing in the in-phase condition.

The strong nonlinear photon and carrier coupling in semiconductor lasers gives rise to simultaneous intensity modulation and frequency modulation or frequency chirping under direct-current modulation.[1] The chirping properties are important for the design of long-distance and high-bit-rate optical communication systems,[2] and the knowledge of chirping provides a useful way to study the interaction mechanism between photons and carriers in a semiconductor laser.[3] An analysis of frequency chirping has been made for the solitary laser[2–4] and for the injection-locked laser.[5] It is clear that the amplitude–phase-coupling effect[6] and the spectral-hole-burning effect[7] are two determinant factors affecting the chirp properties of a solitary semiconductor laser.[2–4] In recent years the great interest in coherent optical communication systems has led to extensive study of external-cavity semiconductor lasers because of their narrow spectral linewidth, good frequency stability, and wavelength tunability.[8–13] The suppression[9–13] of modulation-induced oscillation frequency chirping has also been extensively investigated for this kind of laser configuration. However, it seems that less attention has been paid to the weak-feedback case, including the two effects mentioned above. In this case the multiple reflections in the external cavity can be neglected, so we may consider only the laser cavity itself. Our purpose in this Letter is to give a complete analysis of frequency chirping of a single-mode semiconductor laser in the presence of weak optical feedback by including these effects and to compare the chirp reduction with the linewidth narrowing.

A convenient measure of frequency chirping due to direct-current modulation is the chirp-to-modulated-power ratio (CPR), which is defined as[2,4,5]

$$\mathrm{CPR} = \frac{\delta\nu(j\omega)}{\delta p(j\omega)}, \qquad (1)$$

where $\delta\nu(j\omega)$ is the frequency deviation, $\delta p(j\omega)$ is the modulated output power, and ω is the angular frequency of modulation.

The starting point of our analysis is the rate equations including the effect of amplitude–phase coupling in the presence of optical feedback. If $P(t)$ and $N(t)$ represent the number of photons and the number of minority carriers, respectively, and $\phi(t)$ represents the optical phase of the lasing field, then their dynamic evolution in the weak-feedback case is governed by the following rate equations[8,14,15]:

$$\dot{P}(t) = (G-\gamma)P(t) + R + 2\kappa[P(t)P(t-\tau)]^{1/2} \times \cos[\omega_0\tau + \phi(t) - \phi(t-\tau)], \qquad (2)$$

$$\dot{N}(t) = \frac{I(t)}{e} - GP(t) - \frac{N(t)}{\tau_e}, \qquad (3)$$

$$\dot{\phi}(t) = \omega_{m0} - \omega_0 + \frac{\alpha}{2}(G-\gamma) - \kappa\left[\frac{P(t-\tau)}{P(t)}\right]^{1/2} \times \sin[\omega_0\tau + \phi(t) - \phi(t-\tau)], \qquad (4)$$

where G is the gain or net rate of stimulated emission, R is the rate of spontaneous emission, γ is the inverse of the photon decay time, τ_e is the electron lifetime, α is the linewidth-enhancement factor, κ is the feedback coefficient, τ is the round-trip time in the external cavity, $I(t)$ is the injection current, ω_{m0} is the free angular oscillation frequency, and ω_0 is the lasing angular frequency in the presence of optical feedback. In our rate-equation model, the noise terms arising from spontaneous emission are neglected.

We limit our analysis to the small-modulation-signal regime, in which we have

$$I(t) = I_b + I_m(t), \qquad (5)$$

$$m = \frac{I_m}{I_b - I_{\mathrm{th}}} \ll 1, \qquad (6)$$

where I_m is the modulation current, I_b is the bias current, I_{th} is the threshold current, and m is the modulation depth.

The standard procedure used to solve Eqs. (2)–(4) is to linearize P, N, and ϕ in terms of small deviations around their steady values[14,15]:

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$$P(t) = \overline{P} + \delta P(t), \qquad N(t) = \overline{N} + \delta N(t),$$
$$\phi(t) = \overline{\phi} + \delta\phi. \tag{7}$$

Also, we assume that the optical phase ϕ varies slowly during the external-cavity round-trip time, i.e.,

$$|\delta\phi(t) - \delta\phi(t-\tau)| \ll \pi/2. \tag{8}$$

In the small sinusoidal modulation regime, the optical phase is of the form $\phi = \mathrm{Re}[\beta \exp(j\omega t)]$, so the assumed condition limits our analysis to the short-external-cavity case.

Under the above assumptions, the rate equations can be written as

$$\delta\dot{P} = \left(G_p\overline{P} - \frac{R}{\overline{P}}\right)\delta P(t) - \kappa \cos\omega_0\tau[\delta P(t) - \delta P(t-\tau)]$$
$$+ G_N\overline{P}\delta N - 2\kappa\overline{P}\sin\omega_0\tau[\delta\phi(t) - \delta(t-\tau)], \tag{9}$$

$$\delta\dot{N} = (\overline{G} + G_p\overline{P})\delta P(t) - \left(G_N\overline{P} + \frac{1}{\tau_e}\right)\delta N + \frac{I_m(t)}{e}, \tag{10}$$

$$\delta\dot{\phi} = \frac{\kappa}{2\overline{P}}\sin\omega_0\tau[\delta P(t) - \delta P(t-\tau)]$$
$$+ \frac{\alpha}{2}G_N\delta N - \kappa\cos\omega_0\tau[\delta\phi(t) - \delta\phi(t-\tau)], \tag{11}$$

where $G_N = \partial G/\partial N$ and $G_p = \partial G/\partial P$.

Note that the spectral-hole-burning effect[7] expressed by G_P is included. By using the linear approximation, G is expressed as

$$G = \Gamma A(N - N_0)(1 - \epsilon P), \tag{12}$$

where Γ is the optical confinement factor, A is the differential gain, and ϵ is the gain saturation factor that specifies the spectral-hole-burning effect. Following the standard linear treatment,[14,15] neglecting the term G_p only in Eq. (10), we obtain the following equations after Fourier transformation:

$$(j\omega I - \mathrm{M})\begin{bmatrix}\delta P(j\omega)\\ \delta N(j\omega)\\ \delta\phi(j\omega)\end{bmatrix} = \begin{bmatrix}0\\ \dfrac{I_\mathrm{m}(j\omega)}{e}\\ 0\end{bmatrix}, \tag{13}$$

where

$$M = \begin{bmatrix} -\left[\epsilon\Gamma A(\overline{N} - N_0)\overline{P} + \dfrac{R}{\overline{P}}\right] - \kappa\cos\omega_0\tau[1 - \exp(-j\omega\tau)], & \Gamma A\overline{P}, & -2\kappa\overline{P}\sin\omega_0\tau[1 - \exp(-j\omega\tau)] \\ -\Gamma A(\overline{N} - N_0)(1 - \epsilon P), & -\dfrac{2}{\tau_R}, & 0 \\ \dfrac{\kappa}{2\overline{P}}\sin\omega_0\tau[1 - \exp(-j\omega\tau)], & \dfrac{\alpha}{2}\Gamma A, & -\kappa\cos\omega_0\tau[1 - \exp(-j\omega\tau)] \end{bmatrix}$$

and τ_R is the damping time associated with the relaxation oscillation. By solving Eq. (13) and using the approximation $1 - \exp(-j\omega\tau) = j\omega\tau$, which is valid for a relatively short external cavity, then the CPR in the presence of optical feedback is

$$\mathrm{CPR} = \frac{1}{4\pi\overline{P}}\,\frac{j\omega[\alpha + \kappa\tau(1+\alpha^2)^{1/2}\sin(\omega_0\tau + \tan^{-1}\alpha)] + \alpha(\epsilon G\overline{P} + R/\overline{P})}{1 + \kappa\tau(1+\alpha^2)^{1/2}\cos(\omega_0\tau + \tan^{-1}\alpha)}. \tag{14}$$

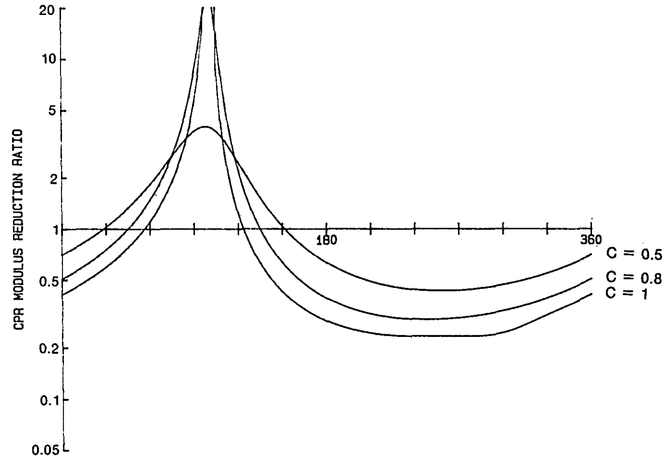

Fig. 1. CPR modulus-reduction ratio $|\mathrm{CPR}/\mathrm{CPR}_0|$ as a function of normalized oscillation phase without optical feedback, i.e., $\omega_{m0}\tau$, for different feedback strengths $C = 0.5$, 0.8, 1.0.

It is evident that when $\kappa = 0$ in Eq. (14), the CPR changes to

$$(\mathrm{CPR})_0 = \frac{\alpha}{4\pi\overline{P}}(j\omega + \epsilon G\overline{P} + R/\overline{P}). \tag{15}$$

This is the well-known CPR expression of a semiconductor laser without optical feedback.[2-5,7] Figure 1 shows the CPR modulus-reduction ratio as a function of normalized oscillation phase without optical feedback, i.e., $\omega_{m0}\tau$, for different feedback strengths that are described by the characteristic parameter C (Ref. 16):

$$C = \kappa\tau(1+\alpha^2)^{1/2}. \tag{16}$$

Other parameters used are the following: linewidth-enhancement factor $\alpha = 5.4$, optical gain $G = 5.0 \times 10^{11}$ sec^{-1}, photon number $\overline{P} = 1.4 \times 10^5$, spontaneous emission rate $R = 1.3 \times 10^{12}$ sec^{-1}, gain saturation factor $\epsilon = 1.0 \times 10^{-8}$, and modulation frequency $f_m (= \omega/2\pi) = 3$ GHz. In Fig. 1 we have used the normalized free-oscillation phase (modulo 2π) because the lasing frequency ω_0 in the presence of optical feedback depends on the feedback strength.[14] From the figure we can see that both chirp narrowing and chirp broadening may occur in the presence of optical feedback, according to the optical phase, and that the condition for narrowing and broadening is approximately the same for linewidth narrowing and broadening. In the in-phase condition, $\omega_0\tau + \tan^{-1}\alpha = 2m\pi$, the maximum reduction of frequency chirping is achieved, and the reduction ratio η is

$$\eta = \frac{1}{1 + \kappa\tau(1 + \alpha^2)^{1/2}}, \tag{17}$$

which is just the square root of the linewidth-reduction ratio.[14] This result is not surprising since the linewidth arises from fluctuations of the optical phase. Also, in the out-of-phase condition chirp increases at the same time as linewidth broadening. For weak feedback, $C \leq 1$, the influence of optical feedback on the CPR phase is negligible. It is worth noting that the amplitude–phase-coupling effect also enhances the chirp-reduction ratio and that it changes the in-phase condition by adding $\tan^{-1}\alpha$. When this coupling effect is neglected, the result reduces to the form obtained by Saito *et al.*[9]

We have investigated the effect of optical feedback on the frequency-chirping properties of a single-mode semiconductor laser. We have found that the optical phase plays an important role in determining chirp narrowing or chirp broadening. We have pointed out that chirp narrowing is enhanced by amplitude–phase coupling in semiconductor lasers. By comparing the chirp reduction with the linewidth narrowing we have shown that the maximum chirp reduction is achieved in the in-phase condition, which is the same for linewidth narrowing, and that this maximum ratio η is just the square root of the linewidth-reduction ratio. A similar discussion about chirp reduction in the presence of optical feedback was proposed recently by Henry and Kazarinov[17] in a different manner. Further experimental verification of the CPR in the presence of optical feedback is in progress.

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