Precoded Transmit Path Diversity in FS-OFDM

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Abstract

In Impulse-Radio or DS/SS UWB communications, a prerake scheme has been proposed to achieve diversity while reducing computational complexity in a mobile terminal. On the other hand, in an OFDM system, fractional sampling (FS) has been proposed to achieve path diversity with a single antenna. However, in the FS-OFDM system, it is necessary to oversample a received signal that leads to larger power consumption in the terminal side. In this paper, a precoded transmit path diversity scheme in an OFDM system has been proposed. The proposed scheme can achieve path diversity without oversampling the received signal.

I. Introduction

In DS/SS or Impulse radio (IR) UWB communications, rake receiver has been proposed to resolve multipath components and achieve path diversity [1]–[3]. In this scheme, a receiver needs to perform channel estimation and diversity combining, etc. These signal processing increases computational complexity in a mobile terminal. Prerake has been proposed to shift these signal processing from the mobile terminal to a base station (BS) [4]–[7]. In this system, the BS transmits signal generated by convolution between transmit symbols and the FIR filter whose coefficients are generated from reverse and conjugate operations of the impulse response of a channel known at the transmitter side. The same as the rake scheme, prerake can achieve path diversity without combining at the receiver side. In [5], a prerake combining scheme is proposed when a pulse interval is smaller than a path interval and the coefficients of the impulse response of the channel are independent and Rayleigh distributed in an IR-UWB system. [6], [7] have proposed precombining schemes which tackle an inter-pulse interference (IPI) problem when a pulse interval is longer than a path interval. On the other hand, in an OFDM system, prerake has not been proposed while Fractional Sampling (FS) has been proposed to achieve path diversity with a single antenna [8], [9]. However, in the FS-OFDM system, it is necessary to oversample a received signal and it leads to large power consumption in a small mobile terminal. In this paper, a precoded transmit path diversity scheme in an OFDM system has been proposed. The proposed scheme makes use of the impulse response of the channel with the resolution of the FS interval and can achieve path diversity without oversampling the received signal.

II. System Model

A. OFDM System

Suppose the information symbol on the kth subcarrier is $S[k](k=0,\ldots,N-1)$, the OFDM symbol is then given as

$$x[n]=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}S[k]e^{j\frac{2\pi nk}{N}} \tag{1}$$

where $n(n=0,\ldots N-1)$ is the time index. A guard interval (GI) is appended before transmission. N_{GI} is the length of the GI.

The baseband signal at the output of the filter is given by

$$x(t)=\sum_{n=-N_{\mathrm{GI}}}^{N-1}x[n]p(t-nT_s) \tag{2}$$

where $p(t)$ is the impulse response of the baseband filter and T_s is the symbol duration. This signal is upconverted and transmitted through a multipath channel with the impulse response of a physical channel $c(t)$. The received signal after the down conversion is given as

$$y(t)=\sum_{n=-N_{\mathrm{GI}}}^{N-1}x[n]h(t-nT_s)+v(t) \tag{3}$$

where $h(t)$ is the impulse response of the composite channel and is given by $h(t):=p(t)\otimes c(t)\otimes p(-t)$ and $v(t)$ is the additive Gaussian noise generated at the receiver. For the multipath channel, $c(t)$ and $h(t)$ can be expressed as follows.

$$c(t)=\sum_{l=0}^{L-1}\alpha_l\delta(t-\tau_l), \tag{4}$$

$$h(t)=\sum_{l=0}^{L-1}\alpha_l p_2(t-\tau_l) \tag{5}$$

where $p_2(t) := p(t) \otimes p(-t)$ is the deterministic correlation of $p(t)$ and normalized as $p_2(0) = 1$, L is the number of multipath, and α_l and τ_l are amplitude and delay of l th path, respectively.

If $y(t)$ is sampled at the rate of T_s, its polyphase components can be expressed as.

$$y[n] = \sum_{m=-N_{\mathrm{GI}}}^{N-1} x[m]h[n-m] + v[n]$$

where $y[n], h[n], v[n]$ are the sampled versions of $y(t), h(t), v(t)$ and expressed as follows, respectively.

$$y[n] = y(nT_s), \tag{6}$$

$$h[n] = h(nT_s), \tag{7}$$

$$v[n] = v(nT_s) \tag{8}$$

After removing the GI and taking DFT at each branch, the received symbol is given by

$$Y[k] = H[k]S[k] + V[k] \tag{9}$$

where $Y[k], H[k], V[k]$ are the DFT outputs of $y[n], h[n], v[n]$, respectively.

B. Transmit path diversity

1) conventional prerake model

In this subsection, the conventional prerake scheme is applied to the OFDM system. The transmitted prerake signal, $x_c(t)$, is given as

$$\begin{aligned} x_c(t) &= \sqrt{\frac{1}{\nu_c}} c^*(-t) \otimes x(t) \\ &= \sqrt{\frac{1}{\nu_c}} \left(\sum_{m=0}^{L-1} \alpha_m^* \delta(t + \tau_m) \right) \otimes x(t) \\ &= \sqrt{\frac{1}{\nu_c}} \sum_{m=0}^{L-1} \alpha_m^* x(t + \tau_m) \end{aligned} \tag{10}$$

Where ν_c is a power normalization factor. Therefore, the total impulse response of the channel including the prerake combining is expressed as

$$\begin{aligned} h_c(t) &= \sqrt{\frac{1}{\nu_c}} c^*(-t) \otimes p(t) \otimes c(t) \otimes p(-t) \\ &= \sqrt{\frac{1}{\nu_c}} \sum_{m=0}^{L-1} \sum_{l=0}^{L-1} \alpha_m^* \alpha_l p_2(t + \tau_m - \tau_l) \end{aligned} \tag{11}$$

With the sampled version of $h_c[n] := h_c(nT_s)$, the frequency response of the channel, $H_c[k]$, is given as

$$\begin{aligned} H_c[k] &= \mathrm{DFT}[H_c[k]] \\ &= \sqrt{\frac{1}{\nu_c}} \sum_{n=0}^{N-1} \sum_{m=0}^{L-1} \sum_{l=0}^{L-1} \alpha_m^* \alpha_l p_2(t + \tau_m - \tau_l) e^{-j\frac{2\pi kn}{N}}. \end{aligned} \tag{12}$$

By using multipath coefficient vector, $\boldsymbol{\alpha} = [\alpha_0 \cdots \alpha_{L-1}]^T$, Eq. (12) is expressed as

$$\sqrt{\frac{1}{\nu_c}} \boldsymbol{\alpha}^H \mathbf{R}[k] \boldsymbol{\alpha} \tag{13}$$

where $\mathbf{R}[k]$ is the path correlation matrix of size $L \times L$ and it's (m,l)th component is expressed as

$$[\mathbf{R}[k]]_{m,l} = \sum_{n=0}^{N-1} p_2(nT_s + \tau_m - \tau_l) e^{-j\frac{2\pi nk}{N}}. \tag{14}$$

The argument of the received signal, $\arg\{\boldsymbol{\alpha}^H \mathbf{R}[k] \boldsymbol{\alpha}\}$, is estimated at the receiver. However $|\boldsymbol{\alpha}^H \mathbf{R}[k] \boldsymbol{\alpha}|$ is degraded at certain subcarriers. Different from DS/SS of IR-UWB systems, the IPI expressed in Eq. (14) is generated. Therefore, path diversity gain of the conventional prerake combining scheme is limited.

2) proposed scheme

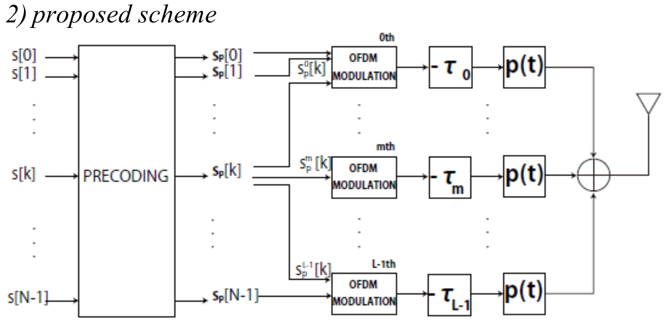

Fig. 1. Proposed scheme.

The block diagram of the proposed scheme is shown in Fig. 1. The proposed scheme precodes the information symbols on the k th subcarrier that is expressed as $S[k](k = 0, \ldots, N-1)$. The precoded information symbol vector of size $L \times 1$ is given as the following equation

$$\mathbf{S}_p[k] = \begin{bmatrix} S_p^0[k] \\ S_p^1[k] \\ \vdots \\ S_p^{L-1}[k] \end{bmatrix} = \begin{bmatrix} w_0[k] \\ w_1[k] \\ \vdots \\ w_{L-1}[k] \end{bmatrix} S[k] \tag{15}$$

where $\mathbf{w}[k] = [w_0[k] \cdots w_{L-1}[k]]^T$ is a weightening vector of size $L \times 1$ given as

$$\mathbf{w}[k] = \mathbf{R}^{-1}[k] \boldsymbol{\alpha}^*. \tag{16}$$

The precoded information symbols $\mathbf{S}_p[k]$ are input to the OFDM demodulators in parallel. The $m(m = 0, \ldots, L-1)$th

OFDM symbol, $x_p^m[n]$, and the baseband signal, $x_p^m(t)$, are expressed as following equations, respectively.

$$
\begin{aligned}
x_p^m[n] &= \sqrt{\frac{1}{N}} \sum_{k=0}^{N-1} S_p^m[k] e^{j\frac{2\pi nk}{N}} \\
&= \sqrt{\frac{1}{N}} \sum_{k=0}^{N-1} w_m[k] S[k] e^{j\frac{2\pi nk}{N}},
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
x_p^m(t) &= \sum_{n=-N_{\mathrm{GI}}}^{N-1} x_p^m[n] p(t - nT_s + \tau_m) \\
&= \sqrt{\frac{1}{N}} \sum_{n=-N_{\mathrm{GI}}}^{N-1} \sum_{k=0}^{N-1} w_m[k] S[k] e^{j\frac{2\pi nk}{N}} p(t - nT_s)
\end{aligned} \tag{18}
$$

The transmitted signal is generated by

$$
\begin{aligned}
& x_p(t) \\
&= \sqrt{\frac{1}{\nu_p}} \sum_{m=0}^{L-1} x_p^m(t + \tau_m) \\
&= \sqrt{\frac{1}{\nu_p N}} \sum_{m=0}^{L-1} \sum_{n=-N_{\mathrm{GI}}}^{N-1} \sum_{k=0}^{N-1} w_m[k] S[k] e^{j\frac{2\pi nk}{N}} p(t - nT_s + \tau_m)
\end{aligned} \tag{19}
$$

where ν_p is the power normalization factor. Suppose $g(t)$ is obtained by convolution between the impulse response of the channel, $c(t)$, and the impulse response of the filter at the receiver side, $p(-t)$, $g(t)$ is expressed as

$$
\begin{aligned}
g(t) &= c(t) \otimes p(-t) \\
&= \sum_{l=0}^{L-1} \alpha_l p(-t - \tau_l)
\end{aligned} \tag{20}
$$

The received signal is obtained by the following equations.

$$
\begin{aligned}
& y_p(t) \\
&= x_p(t) \otimes g(t) + v(t) \\
&= \int x_p(t - \xi) g(\xi) d\xi + v(t) \\
&= \sqrt{\frac{1}{\nu_p N}} \int \sum_{m=0}^{L-1} \sum_{l=0}^{L-1} \sum_{n=-N_{\mathrm{GI}}}^{N-1} \sum_{k=0}^{N-1} w_m[k] \alpha_l S[k] e^{j\frac{2\pi nk}{N}} \\
&\qquad p(t - \xi - nT_s + \tau_m) p(\xi - \tau_l) d\xi + v(t) \\
&= \sqrt{\frac{1}{\nu_p N}} \sum_{m=0}^{L-1} \sum_{l=0}^{L-1} \sum_{n=-N_{\mathrm{GI}}}^{N-1} \sum_{k=0}^{N-1} w_m[k] \alpha_l S[k] e^{j\frac{2\pi nk}{N}} \\
&\qquad p_2(t - \xi - nT_s + \tau_m - \tau_l) + v(t)
\end{aligned} \tag{21}
$$

As shown in Eq. (13), the frequency response of the total precoded channel is expressed as

$$
\begin{aligned}
\mathbf{w}^T[k] \mathbf{R}[k] \boldsymbol{\alpha} &= \left(\mathbf{R}^{-1}[k] \boldsymbol{\alpha}^*\right)^T \mathbf{R}[k] \boldsymbol{\alpha} \\
&= \boldsymbol{\alpha}^H \mathbf{R}^{-1}[k] \mathbf{R}[k] \boldsymbol{\alpha} \\
&= \boldsymbol{\alpha}^H \boldsymbol{\alpha} \\
&= \sum_{l=0}^{L-1} |\alpha_l|^2 .
\end{aligned} \tag{22}
$$

By precoding the information symbols to remove IPI, path diversity can be achieved in the proposed scheme.

III. Simulation Conditions

TABLE I
SIMULATION CONDITIONS

Modulation	1st: QPSK 2nd: OFDM
Channel	2path Rayleigh Fading
Channel Estimation	Ideal
Points of FFT	128
Number of subcarriers	128
Number of packets	100000

Simulation conditions are presented in Table 1. The impulse response of a channel is assumed to be constant during one OFDM packet and known at the transmitter side. The total response of the transmitter and receiver filters is assumed to be truncated sinc pulse with the duration of T_s. Channel models are 2 path Rayleigh fading with uniform delay profile. Regarding the 2 path Rayleigh fading model, BER performance is evaluated with changing delay between the paths, $\tau_d = |\tau_0 - \tau_1|$.

IV. Simulation Results

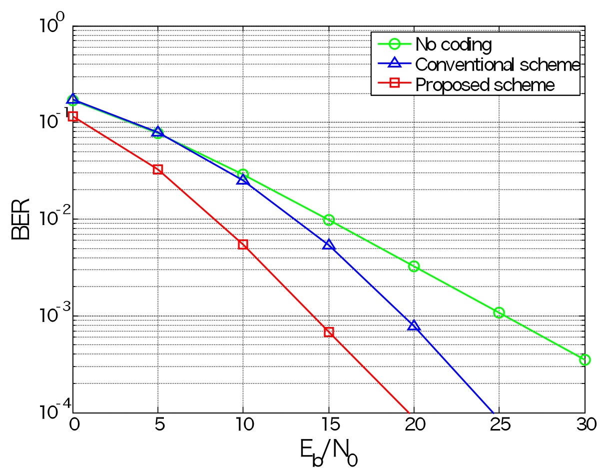

Fig. 2. BER vs. $E_b/N_0 \left(\tau_d = \frac{1}{4} T_s\right)$.

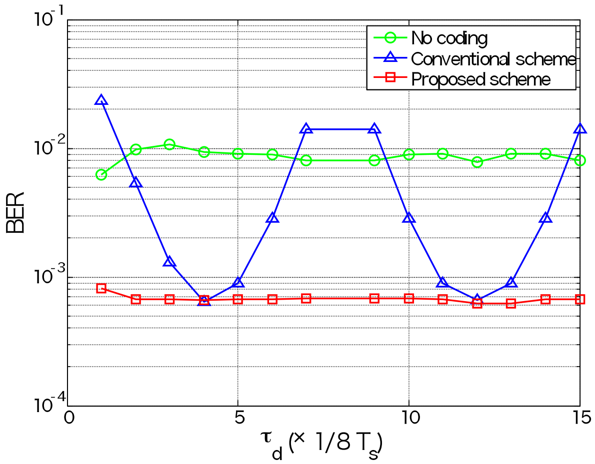

Fig. 3. BER vs. $\tau_d \left(E_b/N_0 = 15[\mathrm{dB}]\right)$.

The BER performances with $\tau_d = T_s/4$ are shown in Fig. 2. The proposed scheme achieves the second order diversity in this figure. This is expected in Eq. (22). On the other hand, the conventional scheme can also achieve transmit path diversity. However, its BER performance is inferior to that of the proposed scheme by about 5[dB] at $\mathrm{BER} = 10^{-4}$. It is because the IPI within one OFDM symbol limits the BER performance of the conventional scheme.

Figure 3 shows the BER vs. τ_d with $E_b/N_0 = 15[\mathrm{dB}]$. From this figure, the BER performance of the proposed scheme is independent of τ_d. It shows that the proposed scheme can remove IPI completely for any τ_d. On the other hand, that of the conventional scheme is dependent on τ_d. On this channel model, the normalization factors, ν_c, ν_d, are given as following equations.

Suppose $\mathbf{R}[k]$ is expressed as

$$\mathbf{R}[k] = \begin{bmatrix} 1 & \psi[k] \\ \psi^*[k] & 1 \end{bmatrix}.$$

The frequency response of the channel expressed in Eq. (13) is given as follows.

$$\begin{aligned} & H_c[k] \\ &= \sqrt{\frac{1}{\nu_c}}\boldsymbol{\alpha}^H \mathbf{R}[k]\boldsymbol{\alpha} \\ &= \sqrt{\frac{1}{\nu_c}}\begin{bmatrix}\alpha_0^* & \alpha_1^*\end{bmatrix}\begin{bmatrix} 1 & \psi[k] \\ \psi^*[k] & 1 \end{bmatrix}\begin{bmatrix}\alpha_0 \\ \alpha_1\end{bmatrix} \\ &= \sqrt{\frac{1}{\nu_c}}\Bigg\{\underbrace{|\alpha_0|^2 + |\alpha_1|^2}_{\text{disired signal}} + \underbrace{\psi^*[k]\alpha_0\alpha_1^* + \psi[k]\alpha_0^*\alpha_1}_{\text{IPI within one OFDM symbol}}\Bigg\} \end{aligned}$$

$$\nu_c = |\alpha_0|^2 + |\alpha_1|^2 + |\psi^*[k]\alpha_0\alpha_1^*| + |\psi[k]\alpha_0^*\alpha_1|. \quad (23)$$

By Eq. (22), ν_p is obtained as

$$\nu_p = |\alpha_0|^2 + |\alpha_1|^2. \quad (24)$$

Therefore, the signal to noise ratio (SNR) of the conventional scheme and the proposed scheme, γ_c, γ_p, is

$$\begin{aligned} & \gamma_c[k] \\ &= \frac{|\boldsymbol{\alpha}^H \mathbf{R}[k]\boldsymbol{\alpha}|^2}{\nu_c}\cdot\frac{E_b}{N_0} \quad (25) \\ &= \frac{\left||\alpha_0|^2 + |\alpha_1|^2 + \psi^*[k]\alpha_0\alpha_1^* + \psi[k]\alpha_0^*\alpha_1\right|^2}{|\alpha_0|^2 + |\alpha_1|^2 + |\psi^*[k]\alpha_0\alpha_1^*| + |\psi[k]\alpha_0^*\alpha_1|}\cdot\frac{E_b}{N_0} \end{aligned}$$

$$\begin{aligned} & \gamma_p \\ &= \frac{|\mathbf{w}^T \mathbf{R}[k]\boldsymbol{\alpha}|^2}{\nu_p}\cdot\frac{E_b}{N_0} \quad (26) \\ &= \left(|\alpha_0|^2 + |\alpha_1|^2\right)\frac{E_b}{N_0} \end{aligned}$$

From Eq. (26), it can be shown that the performance of the conventional scheme is dependent on τ_d. On the other hand, Eq. (26) shows that the proposed scheme can remove the IPI within one OFDM symbol completely and achieve path diversity. This also explains why the performance of the proposed scheme is independent of τ_d. Comparing Eq. (26) with Eq. (26), the following inequation can be induced

$$\gamma_c[k] \leq \gamma_p[k] \quad (27)$$

The equality holds when $\tau_d = \frac{T_s}{2} + nT_s \quad (n = 0, 1, \ldots)$.

Therefore, the performance of the proposed scheme is the best of the three on this channel model.

V. Conclusions

In this paper, the precoded transmit path diversity scheme in the OFDM system has been proposed. In the proposed scheme, the information symbols are precoded to remove the IPI by using the correlation matrix $\mathbf{R}[k]$ and can achieve path diversity without oversampling the received signal. On the 2 path Rayleigh fading model, it is clarified that the proposed scheme can achieve path diversity and it's BER performance is independent of the path delay.

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