Source Coding with Side-Information at the Receivers and an Application

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***Abstract*—We review results on the source-coding problem in Figure 1, i.e., on the simple network proposed by Gray and Wyner where additionally the receivers have side-information about the source. We also describe how this problem is relevant for coding over broadcast channels (BC) with feedback.**

I. The Source Coding Problem

We consider the source-coding problem in Figure 1, where a transmitter describes the source sequence $X^n = (X_1, \ldots, X_n)$ to Receivers 1 and 2, who observe the side-information $Y_1^n = (Y_{1,1}, \ldots, Y_{1,n})$ and $Y_2^n = (Y_{2,1}, \ldots, Y_{2,n})$, respectively. Here, $\{(X_t, Y_{1,t}, Y_{2,t})\}_{t=1}^n$ is a sequence of independent and identically distributed (IID) triples of joint law $P_{XY_1Y_2}$.

The transmitter can send a common message M_0 of rate R_0 to both receivers and private messages M_1 and M_2 of rates R_1 and R_2 to Receivers 1 and 2, respectively. The goal is that Receiver i, for $i \in \{1,2\}$, produces a reconstruction sequence $\hat{X}_i^n = (\hat{X}_{i,1}, \ldots, \hat{X}_{i,n})$ that satisfies $\frac{1}{n}\mathsf{E}\left[\sum_{t=1}^n d_i(X_t, \hat{X}_{i,t})\right] \leq D_i$, where d_i is the symbol-wise distortion function and $D_i \geq 0$ the distortion constraint. The described setup includes the case where $X^n = (X_1^n, X_2^n)$ and Receiver i only wishes to reconstruct a lossy version of X_i^n.

Theorem 1 ([5]): A triple (R_0, R_1, R_2) is achievable if

$$\begin{aligned} R_0 + R_i &\geq I(X; V_i, V_0|Y_i), \qquad i \in \{1,2\}, \\ R_0 + R_1 + R_2 &\geq I(X; V_1|Y_1, V_0) + I(X; V_2|Y_2, V_0) \\ &\quad + \max_{i\in\{1,2\}} I(X; V_0|Y_i) \end{aligned}$$

for some V_0, V_1, V_2 and f_1, f_2 s.t $V_0, V_1, V_2 - \circ - X - \circ - Y_1, Y_2$ and $\mathsf{E}[d_i(X, f_i(V_0, V_i, Y_i))] \leq D_i$, for $i \in \{1,2\}$.

Though the region is not known to be optimal in general, it is optimal for all special cases where the rates-distortions region is known. In the following we review these special cases. Gray and Wyner characterized the rates-distortions region when there is no side-information [1]. Heegard and Berger [2] proposed inner and outer bounds on the rate-distortions region when the transmitter can send only a common message, but no private messages, i.e., when $R_1 = R_2 = 0$. The bounds coincide when $X - \circ - Y_1 - \circ - Y_2$ or $X - \circ - Y_2 - \circ - Y_2$. Steinberg and Merhav [3], and Tian and Diggavi [4] considered the case when the transmitter can send a common message and a private message to Receiver 2 but not to Receiver 1, i.e., when $R_1 = 0$. More specifically, [3] characterizes the rates-distortions region for the *successive Wyner-Ziv refinement problem* where Receiver 2 (who receives the additional private message) has the better side-information, i.e., $X - \circ - Y_2 - \circ - Y_1$, and [4] proposes inner and outer bounds on the rates-distortions region for the *scalable source-coding problem* where Receiver 1 has the better side-information, i.e., $X - \circ - Y_1 - \circ - Y_2$. The bounds in [4] are tight when either $D_1 = 0$ or $D_2 = 0$ or when the two distortion measures d_1 and d_2 are deterministic and degraded.

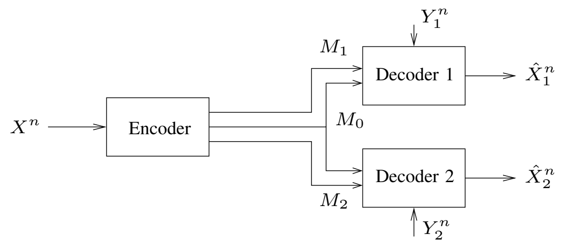

Fig. 1. Gray-Wyner setup with different side-information at the receivers.

II. Application to the BC with Feedback

Our lossy source-coding problem has application to coding over BCs with feedback, see especially [5] but also [6], [7]. The core idea of these schemes is the following. In a first stage the transmitter sends the desired messages at rates that do not allow for decoding at the receivers. Then, using the feedback, it identifies resolution information which it sends during the second stage. To this end, it first compresses the resolution information using a good lossy source-code for the setup in Figure 1, where the side-information at the receivers is given by their channel outputs in the first stage. It then sends the compression indices over the BC so that the receivers can decode them. With the lossy reconstructions of the resolution information learned in the second stage and the outputs from the first stage the receivers then decode the desired messages.

References

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