Gaussian Broadcast Channel with Partial Feedback

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Abstract—We present new achievable regions for the two-user Gaussian broadcast channel with noiseless feedback from one of the users. Our results improve on previous achievable regions.

Classification: Information theory, communications.

I. CHANNEL MODEL

We consider a two-receiver broadcast scenario where a single transmitter wishes to send Message $M_1$ to Receiver 1 and an independent message $M_2$ to Receiver 2. The messages $M_1$ and $M_2$ are assumed to be uniformly distributed over the sets $\{1, \ldots, 2^nR_1\}$ and $\{1, \ldots, 2^nR_2\}$, where $n$ denotes the block-length and $R_1$ and $R_2$ the rates of transmission.

The transmission takes place over a memoryless discrete-time Gaussian broadcast channel (BC). The time-$t$ received symbols corresponding to the transmitted symbol $x_t$ are thus

$$Y_{1,t} = x_t + Z_{1,t} \quad \text{and} \quad Y_{2,t} = x_t + Z_{2,t},$$

where $\{Z_{1,t}\}$ and $\{Z_{2,t}\}$ are independent sequences of independent and identically distributed zero-mean Gaussian random variables of variances $\sigma_1^2 > 0$ and $\sigma_2^2 > 0$.

The transmitter is assumed to have feedback from Receiver 2. Thus, it can compute $X_t$ not only as a function of the messages $M_1$ and $M_2$ but also of the previous channel outputs $Y_{2,t-1} = (Y_{2,1}, \ldots, Y_{2,t-1})$:

$$X_t = x_t^{(n)} (M_1, M_2, Y_{2,t-1}).$$

The channel inputs are subject to an average power constraint

$$\frac{1}{n} \mathbb{E} \left[ \sum_{t=1}^n X_t^2 \right] \leq P.$$ 

The two receivers decode their intended message based on the observed output sequences $Y_1^n = (Y_{1,1}, \ldots, Y_{1,n})$ and $Y_2^n = (Y_{2,1}, \ldots, Y_{2,n})$, respectively. In the described setup achievable a pair of nonnegative rates $(R_1, R_2)$ is defined as usual.

II. RESULTS

Theorem 1. A rate pair $(R_1, R_2)$ is achievable whenever

$$R_1 \leq \frac{1}{2\eta} \log \left( \frac{\det (K_{U1} + K_{U2} + BB^T \sigma_2^2 + I_\eta \sigma_1^2)}{\det (K_{U2} + BB^T \sigma_2^2 + I_\eta \sigma_1^2)} \right)$$

$$R_2 \leq \frac{1}{2\eta} \log \left( \frac{\det (K_{U2} + (B + I_\eta)(B + I_\eta)^T \sigma_2^2)}{\det (I_\eta \sigma_2^2)} \right)$$

for some positive integer $\eta$, two positive semi-definite $\eta \times \eta$-matrices $K_{U1}, K_{U2}$, and a strictly lower-triangular $\eta \times \eta$-matrix $B$ such that

$$\text{tr} (K_{U1} + K_{U2} + BB^T \sigma_2^2) \leq \eta P,$$

and where $I_\eta$ denotes the $\eta \times \eta$ identity matrix.

Theorem 2. A rate-pair $(R_1, R_2)$ is achievable whenever

$$R_1 \leq \frac{1}{4\pi} \int_0^{2\pi} \log \left( 1 + \frac{S_{U1}(\omega)}{S_{U2}(\omega) + |H(\omega)|^2 \sigma_2^2 + \sigma_1^2} \right) d\omega$$

$$R_2 \leq \frac{1}{4\pi} \int_0^{2\pi} \log \left( \frac{S_{U2}(\omega)}{\sigma_2^2} + |H(\omega) + I|^2 \right) d\omega$$

for some strictly causal filter with Fourier-Transform $H$ and some power-spectral densities $S_{U1}$ and $S_{U2}$ that satisfy:

$$\frac{1}{2\pi} \int_0^{2\pi} (S_{U1}(\omega) + S_{U2}(\omega) + |H(\omega)|^2 \sigma_2^2) d\omega \leq P.$$

The region in Theorem 2 is a subset of the region in Theorem 1. The reason for also presenting the possibly smaller region in Theorem 2 is that for this second region it is possible to partly determine the optimal parameters (see the following Remark 3), which seems out of reach for the first region.

Remark 3. For a specific choice of the filter $H$ it is possible to derive the optimal power-spectral densities $S_{U1}$ and $S_{U2}$. For brevity the description of these waterfilling-type solutions is omitted.

Specializing the achievable region in Theorem 2 for different values of $\alpha$ to the choice of parameters $H(\omega) = -\frac{e^{-i\omega}}{\sqrt{\alpha^2 \sigma_1^2 + \sigma_2^2}}$, $S_{U2}(\omega) = 0$, and the optimal $S_{U1}^*(\omega)$ recovers the achievable region in [1]. However, in general this choice of parameters is not optimal, and our achievable region in Theorem 2 is larger than the achievable region in [1]. In fact, even the achievable region in Theorem 1 with parameter $\eta = 2$ can be larger than the region in [1]. The same techniques can be applied for the double sided feedback. This direction is under investigation now.

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