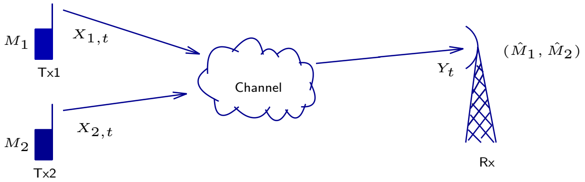
# Cooperation on the Multiple-Access Channel

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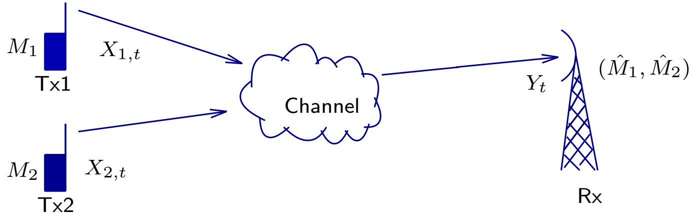
Based on Collaborations with Amos Lapidoth and Shraga I. Bross

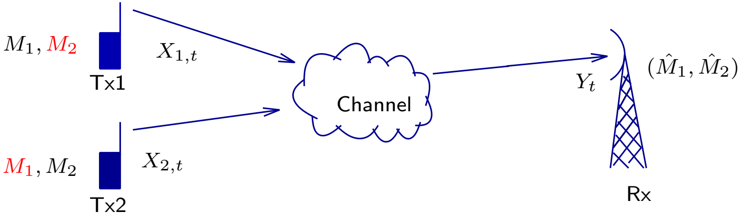
# Multiple-Access Channel (MAC): Many-to-One




No transmitter cooperation (classical MAC)

- Each transmitter knows only its own message
- Joint code design
- Time-sharing

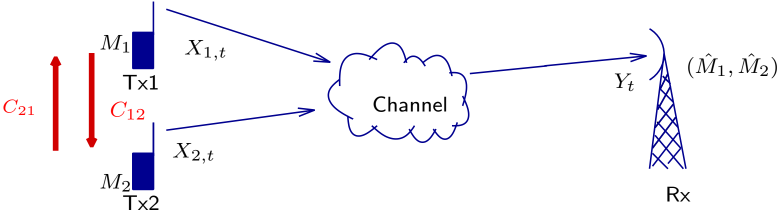
# Multiple-Access Channel (MAC): Many-to-One




Full transmitter cooperation

- Both transmitters know both messages
- Joint encodings of messages

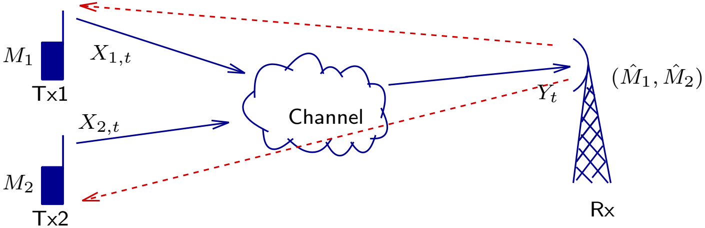
# Multiple-Access Channel (MAC): Many-to-One




In this talk: partial transmitter cooperation

1. MAC with conferencing encoders (e.g. closely located transmitters)
   - Before transmission, Txs communicate over rate-limited bit-pipes
   - Txs can communicate parts of their messages to other Tx
   - $\Rightarrow$ Txs can cooperate based on pipe-outputs

# Multiple-Access Channel (MAC): Many-to-One




In this talk: partial transmitter cooperation

2. MAC with imperfect feedback (e.g. uplink/downlink scenarios)
   - Txs observe (noisy) feedback from channel outputs
   - Cooperation? Past channel outputs give information about both messages
   - $\Rightarrow$ Txs can cooperate in future transmissions

# Capacity Region for Two-User MAC

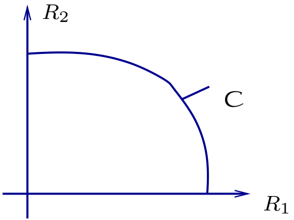
- $M_1 \sim \mathcal{U}\{1, \ldots, \lfloor 2^{nR_1} \rfloor\}, \qquad M_2 \sim \mathcal{U}\{1, \ldots, \lfloor 2^{nR_2} \rfloor\}$

- $n$: block-length of transmission

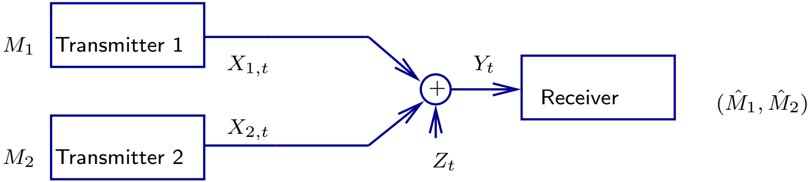
- $R_1, R_2$: rates of transmission

- Capacity $C$: closure of set of pairs $(R_1, R_2)$ for which $\exists$ block-length $n$ schemes such that

$$p(\text{error}) \to 0 \qquad n \to \infty.$$

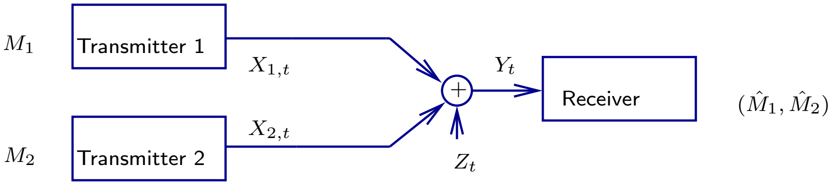


# Additive White Gaussian Noise (AWGN) MAC



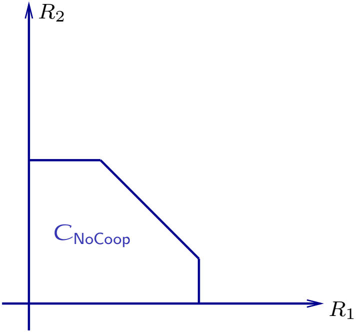

- $Y_t = X_{1,t} + X_{2,t} + Z_t, \qquad t \in \{1, \dots, n\}$
- $\{Z_t\} \sim$ IID $\mathcal{N}(0, N)$
- Power constraints: $\frac{1}{n} \sum_{t=1}^{n} \mathsf{E}\left[X_{\nu,t}^2\right] \leq P_\nu, \quad \nu \in \{1, 2\}$

# Additive White Gaussian Noise (AWGN) MAC

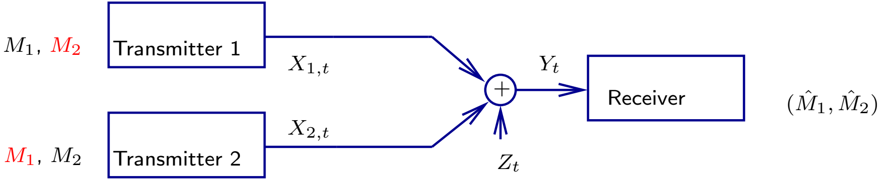



No transmitter cooperation (classical MAC)

$$C_{\text{NoCoop}} = \left\{ \begin{array}{ccc} (R_1, R_2) : & & \\ R_1 & \leq & \frac{1}{2}\log\left(1 + \frac{P_1}{N}\right) \\ R_2 & \leq & \frac{1}{2}\log\left(1 + \frac{P_2}{N}\right) \\ R_1 + R_2 & \leq & \frac{1}{2}\log\left(1 + \frac{P_1+P_2}{N}\right) \end{array} \right\}$$

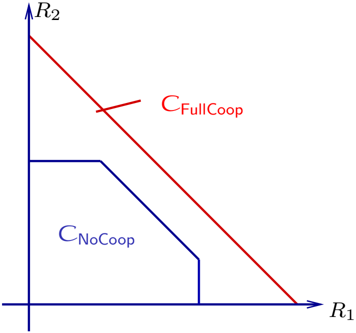


# Additive White Gaussian Noise (AWGN) MAC




Full transmitter cooperation

$$C_{\mathsf{FullCoop}} = \Bigg\{ (R_1, R_2) : R_1 + R_2 \le \frac{1}{2} \log \left( 1 + \frac{P_1 + P_2 + 2\sqrt{P_1 P_2}}{N} \right) \Bigg\}$$

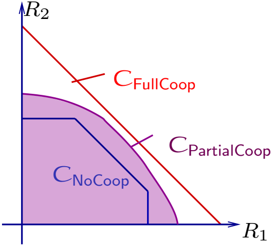


# What about Partial Transmitter Cooperation?

Goal: Capacity of settings with partial transmitter cooperation!

- $C_{\text{NoCoop}} \subseteq C_{\text{PartialCoop}} \subseteq C_{\text{FullCoop}}$
- Are inclusions strict?
- Does partial coop. help at all?
- Is partial coop. $\approx$ full coop.?

$\Rightarrow$ Important design issues

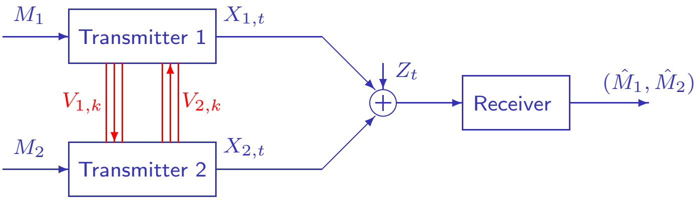



- We consider two scenarios:
  - AWGN MAC with Conferencing Encoders
  - AWGN MAC with Feedback

# Part 1:

# AWGN MAC with Conferencing Encoders

# AWGN MAC with Conferencing Encoders

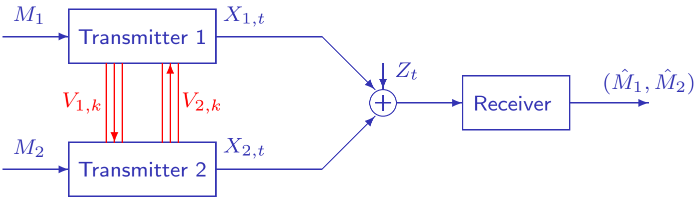



1. phase: Conference (Willems'83):

- $\kappa$ sequential uses of the perfect bit-pipes
- $V_{1,k} = f_{1,k}^{(\kappa)}\left(M_1, V_2^{k-1}\right) \qquad V_{2,k} = f_{2,k}^{(\kappa)}\left(M_2, V_1^{k-1}\right)$
- Rate-limitations:

$$\sum_{k=1}^{\kappa} \log|\mathcal{V}_{1,k}| \leq nC_{12} \qquad \text{and} \qquad \sum_{k=1}^{\kappa} \log|\mathcal{V}_{2,k}| \leq nC_{21}$$

# AWGN MAC with Conferencing Encoders




2. phase: Transmission over channel

$$X_{1,t} = \varphi_{1,t}^{(n)}\left(M_1, V_2^\kappa\right) \qquad X_{2,t} = \varphi_{2,t}^{(n)}\left(M_2, V_1^\kappa\right)$$

# Main Result for Conferencing Encoders

**Theorem 1: Capacity region of AWGN MAC with Conferencing Encoders**

$$C_{\mathsf{Conf}} = \bigcup_{\substack{\rho_1,\rho_2 \\ \in[0,1]}} \left\{ (R_1, R_2) : \begin{array}{rcl} R_1 & \le & \frac{1}{2}\log\left(1 + \frac{P_1\left(1-\rho_1^2\right)}{\sigma^2}\right) + C_{12} \\ R_2 & \le & \frac{1}{2}\log\left(1 + \frac{P_2\left(1-\rho_2^2\right)}{N}\right) + C_{21} \\ R_1 + R_2 & \le & \frac{1}{2}\log\left(1 + \frac{P_1\left(1-\rho_1^2\right)+P_2\left(1-\rho_2^2\right)}{N}\right) + C_{12} + C_{21} \\ R_1 + R_2 & \le & \frac{1}{2}\log\left(1 + \frac{P_1+P_2+2\rho_1\rho_2\sqrt{P_1P_2}}{N}\right) \end{array} \right\}$$

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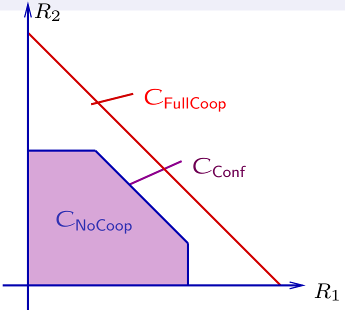
If and only if $C_{12}=C_{21}=0$



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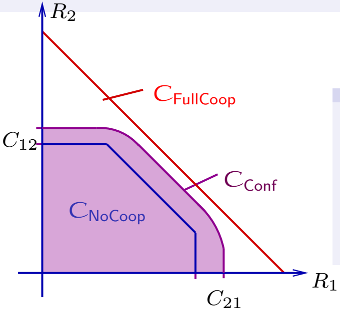
If $C_{12}, C_{21} \neq 0$,
but "small"




If $C_{12} > 0$ or $C_{21} > 0$

$\mathcal{C}_{\mathsf{NoCoop}} \subset \mathcal{C}_{\mathsf{Conf}}$

strictly!

# Main Result for Conferencing Encoders

**Theorem 1: Capacity region of AWGN MAC with Conferencing Encoders**

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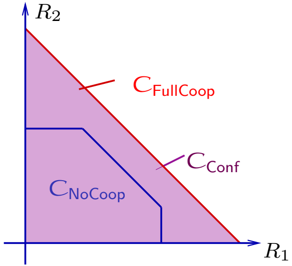
If and only if

$C_{12}, C_{21} \geq \frac{1}{2}\log\left(1 + \frac{P_1+P_2+2\sqrt{P_1P_2}}{N}\right)$



# Achievability (Inner Bound)

- Transmitters split messages: $M_1 = (M_{1,c}, M_{1,p})$ and $M_2 = (M_{2,c}, M_{2,p})$
- Conference: Transmitters exchange $M_{1,c}$ and $M_{2,c}$ over bit-pipes
- Rate of $M_{1,c} < C_{12}$ and rate of $M_{2,c} < C_{21}$
- Transmitters use Gaussian codebooks and add up codewords for transmission over AWGN MAC
- Successive decoding at the receiver

No superposition encoding and joint decoding necessary!
$\Rightarrow$ easier than Willems's scheme

# Converse (Outer Bound)

- Converse as in Willems'83, but accounting for power constraints:

$$C_{\mathsf{Conf}} \subseteq \bigcup_{\substack{X_1 -\circ- U -\circ- X_2 \\ \mathsf{E}[X_1^2]\leq P_1,\ \mathsf{E}[X_2^2]\leq P_2}} \mathcal{R}_{X_1,U,X_2}$$

where

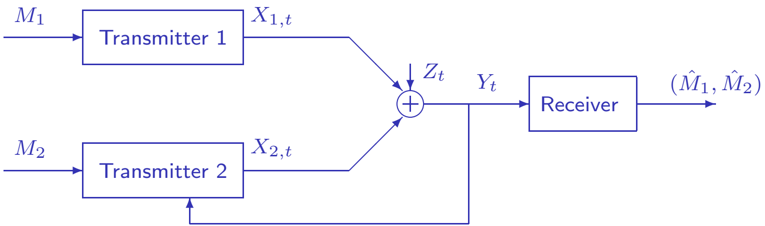
$$\mathcal{R}_{X_1,U,X_2} \triangleq \left\{ (R_1,R_2) : \begin{array}{ccll} R_1 & \leq & I(X_1;Y|X_2U) & +C_{12} \\ R_2 & \leq & I(X_2;Y|X_1U) & +C_{21} \\ R_1+R_2 & \leq & I(X_1X_2;Y|U) & +C_{12}+C_{21} \\ R_1+R_2 & \leq & I(X_1X_2;Y) & \end{array} \right\}$$

**Propose technique to prove:**

Suffices to take union over Gaussian Markov Triples $X_1^{\mathcal{G}} -\circ- U^{\mathcal{G}} -\circ- X_2^{\mathcal{G}}$

Traditional Max-Entropy techniques fail because of Markov condition!

# Technique also Applies to Cover-Leung Region

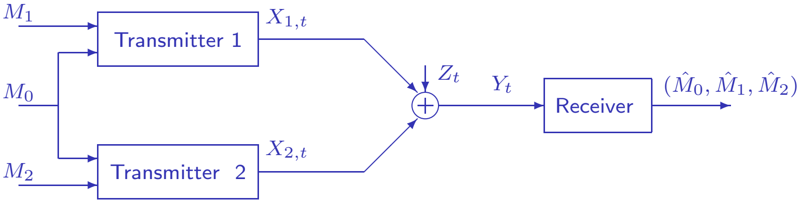



**Achievable region for AWGN MAC with perfect partial or perfect feedback**

$$\mathcal{R}_{\mathsf{CL}} \triangleq \bigcup_{\substack{X_1 -\circ- U -\circ- X_2 \\ \mathsf{E}[X_1^2] \le P_1, \\ \mathsf{E}[X_2^2] \le P_2}} \left\{ (R_1, R_2) : \begin{array}{ccc} R_1 & \le & I(X_1; Y | X_2 U) \\ R_2 & \le & I(X_2; Y | X_1 U) \\ R_1 + R_2 & \le & I(X_1 X_2; Y) \end{array} \right\}$$

Suffices to take union over Gaussian Markov triples $X_1^{\mathcal{G}} -\circ- U^{\mathcal{G}} -\circ- X_2^{\mathcal{G}}$!

# Technique also Applies to Slepian-Wolf Region

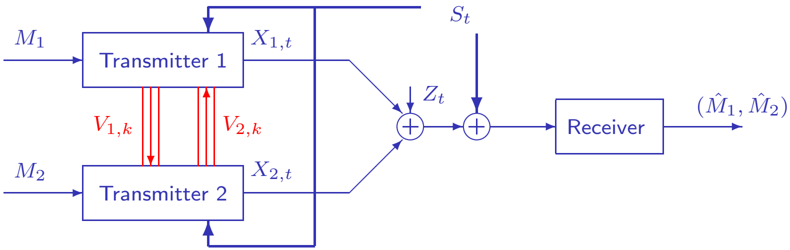



**Capacity region for AWGN MAC with common message**

$$\mathcal{R}_{\mathsf{SW}} \triangleq \bigcup_{\substack{X_1 -\!\circ\!- U -\!\circ\!- X_2 \\ \mathsf{E}[X_1^2] \le P_1, \\ \mathsf{E}[X_2^2] \le P_2}} \left\{ (R_1, R_2) : \begin{array}{rcl} R_1 & \le & I(X_1; Y | X_2 U) \\ R_2 & \le & I(X_2; Y | X_1 U) \\ R_1 + R_2 & \le & I(X_1 X_2; Y | U) \\ R_0 + R_1 + R_2 & \le & I(X_1 X_2; Y) \end{array} \right\}$$

Suffices to take union over Gaussian Markov triples $X_1^{\mathcal{G}} -\!\circ\!- U^{\mathcal{G}} -\!\circ\!- X_2^{\mathcal{G}}$!

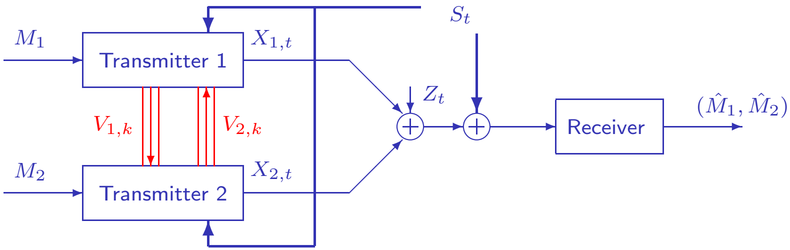
# Dirty-Paper MAC with Conferencing Encoders




- $\{S_t\} \sim$ IID $\mathcal{N}(0, Q)$
- Transmitters know interference $S^n \triangleq (S_1, \ldots, S_n)$ non-causally
- Inputs $X_\nu^n \triangleq (X_{\nu,1}, \ldots, X_{\nu,n})$ at Transmitter $\nu$:

$$X_1^n = \varphi_1^{(n)}\left(M_1, V_2^\kappa, S^n\right)$$
$$X_2^n = \varphi_2^{(n)}\left(M_2, V_1^\kappa, S^n\right)$$

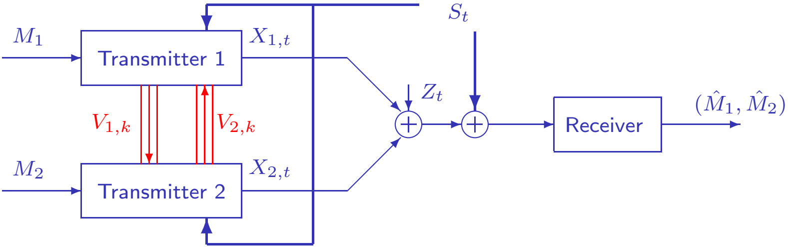
# Dirty-Paper MAC with Conferencing Encoders




2 Settings:

- Transmitters learn $S^n$ before the conference
  - $V_{1,k} = f_{1,k}^{(\kappa)}(M_1, V_2^{k-1}, S^n)$ and $V_{2,k} = f_{1,k}^{(\kappa)}(M_2, V_1^{k-1}, S^n)$
- Transmitters learn $S^n$ after the conference

# Interference acausally known at Txs can perfectly be canceled




**Theorem 2**

*For two-user Gaussian MAC with conferencing encoders:*

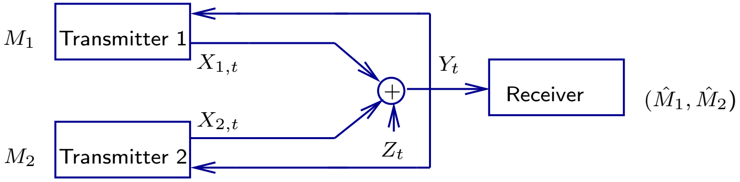
$$C_{\text{Int,before}} = C_{\text{Int,after}} = C_{\text{Conf}}, \qquad \forall Q \geq 0,$$

*if interference known non-causally at both encoders.*

# Part 2:

# AWGN MAC with Feedback

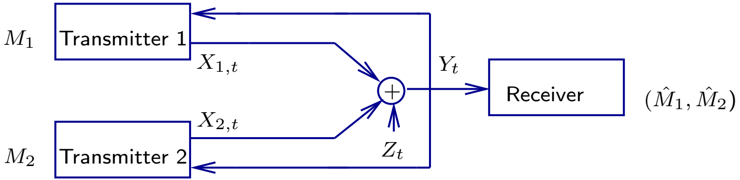
# AWGN MAC with Perfect Feedback




- Transmitters observe perfect causal output feedback:

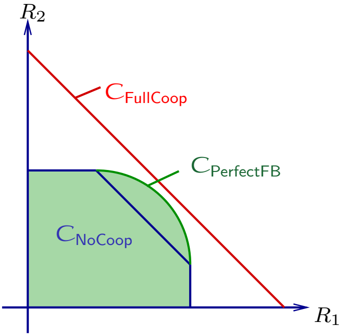
$$X_{\nu,t} = \varphi_{\nu,t}^{(n)}(M_\nu, Y_1, \ldots, Y_{t-1}), \quad \nu \in \{1, 2\}.$$

# AWGN MAC with Perfect Feedback

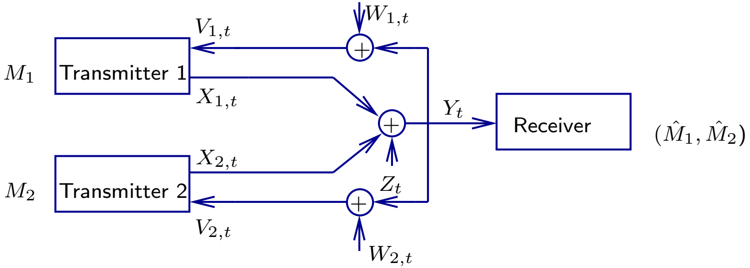



Ozarow'84:

$$C_{\text{PerfectFB}} = \bigcup_{\rho \in [0,1]} \left\{ \begin{array}{lll} (R_1, R_2): & & \\ R_1 & \leq & \frac{1}{2}\log\left(1 + \frac{P_1(1-\rho^2)}{N}\right) \\ R_2 & \leq & \frac{1}{2}\log\left(1 + \frac{P_2(1-\rho^2)}{N}\right) \\ R_1 + R_2 & \leq & \frac{1}{2}\log\left(1 + \frac{P_1 + P_2 + 2\sqrt{P_1 P_2}\rho}{N}\right) \end{array} \right\}$$



# AWGN MAC with Noisy Feedback



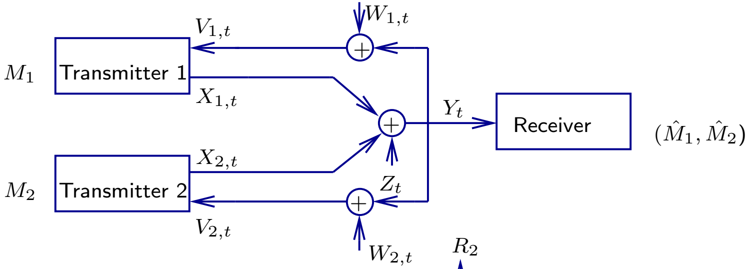

- Noisy feedback:

$$V_{\nu,t} = Y_t + W_{\nu,t}, \qquad \{(W_{1,t}, W_{2,t})\} \sim \text{IID } \mathcal{N}(\mathbf{0}, \mathsf{K}_{W_1 W_2}), \qquad \nu \in \{1,2\}$$

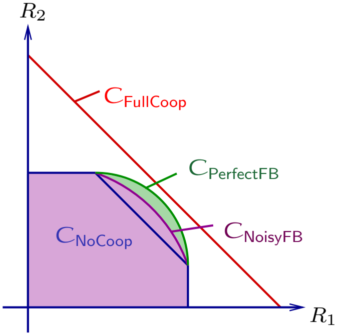
- Transmitters observe noisy feedback:

$$X_{\nu,t} = \varphi_{\nu,t}^{(n)}(M_\nu, V_{\nu,1}, \ldots, V_{\nu,t-1}), \quad \nu \in \{1,2\}.$$

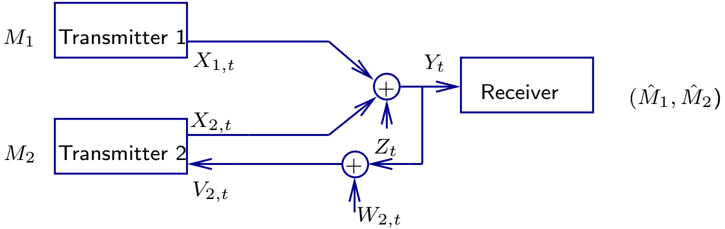
# AWGN MAC with Noisy Feedback




- $\mathcal{C}_{\text{NoisyFB}} = $ ?
- $\mathcal{C}_{\text{NoCoop}} \subseteq \mathcal{C}_{\text{NoisyFB}} \subseteq \mathcal{C}_{\text{PerfectFB}}$
- Are inclusions strict?
- Previous schemes collapse to $\mathcal{C}_{\text{NoCoop}}$ if fb-noise variances too large



# Noisy or Perfect Partial Feedback to Tx 2

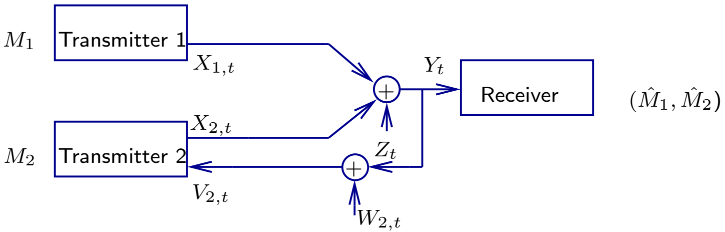



- Noisy partial feedback: $V_{2,t} = Y_t + W_{2,t}, \quad \{W_{2,t}\} \sim$ IID $\mathcal{N}\big(0, \sigma_2^2\big)$
- Transmitter 1 has no feedback: $X_{1,t} = \varphi_{1,t}^{(n)}(M_1)$
- Transmitter 2 observes noisy feedback:

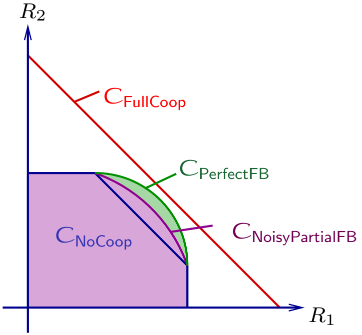
$$X_{2,t} = \varphi_{2,t}^{(n)}(M_2, V_{2,t}, \ldots, V_{2,t-1}),$$

- $\sigma_2^2 = 0$: perfect partial feedback

# Noisy or Perfect Partial Feedback to Tx 2




- $C_{\text{NoisyPartialFB}} =$ ?
- $C_{\text{NoCoop}} \subseteq C_{\text{NoisyPartialFB}} \subseteq C_{\text{PerfectFB}}$
- Are inclusions strict?



# Main Results for Noisy Feedback

**Theorem 3a: Noisy feedback is always beneficial!**

- *Noisy feedback always increases capacity region*
- *Fixed* $P_1, P_2, N > 0$*:* $\mathcal{C}_{\mathsf{NoCoop}} \subset \mathcal{C}_{\mathsf{NoisyFB}}, \qquad \forall\, \mathsf{K}_{W_1 W_2} \succeq 0.$

  *Inclusion is strict!*

**Theorem 4: Almost-perfect feedback ≈ perfect feedback!**

- *Capacity with noisy feedback converges to perfect-feedback capacity*
- *Fixed* $P_1, P_2, N > 0$ *:*

$$\mathsf{cl}\Bigg( \bigcup_{\sigma^2 \geq 0} \bigcap_{\mathsf{K}:\ \mathsf{tr}(\mathsf{K}) \leq \sigma^2} \mathcal{C}_{\mathsf{NoisyFB}}(P_1, P_2, N, \mathsf{K}) \Bigg) = \mathcal{C}_{\mathsf{PerfectFB}}(P_1, P_2, N)$$

# Main Results for Partial Feedback

**Theorem 3b: Noisy partial feedback is always beneficial!**

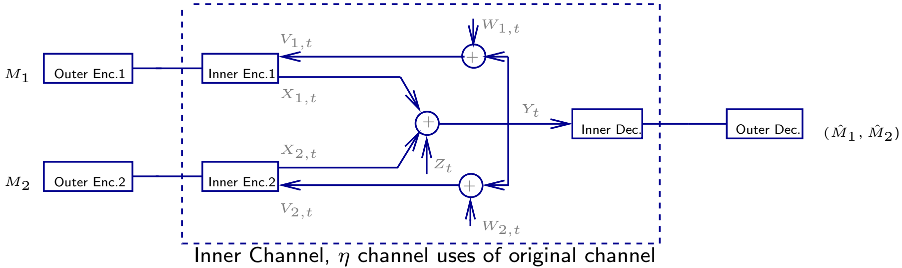
- *Noisy partial feedback always increases capacity region*
- *Fixed $P_1, P_2, N > 0$:* $$\mathcal{C}_{\mathsf{NoCoop}} \subset \mathcal{C}_{\mathsf{NoisyPartialFB}}, \qquad \forall\, \sigma_2^2 \geq 0.$$

  *Inclusion is strict!*

**Theorem 5: Perfect partial-feedback capacity $\neq$ Cover-Leung region! (Answer to van der Meulen)**

- *Perfect partial-feedback capacity > Cover-Leung region*

# Robust Noisy-Feedback Scheme: Concatenated Structure




- Inner Encoders/Decoder: (Exploit Feedback)
  - Use feedback
  - Use original channel $\eta$ times per fed symbol
  - Generalize Ozarow's perfect-feedback scheme; linear encodings/decodings
- Outer Encoders/Decoder: (Robustify Inner Scheme)
  - Ignore feedback
  - Use inner channel once every $\eta$ channel uses of original channel
  - Code to achieve capacity of inner channel

# Summary

- AWGN MAC with conferencing encoders
  - Determined capacity region
  - Conference always increases capacity
  - New technique for proving opt. of Gaussians under a Markovity constraint
  - Acausally known interference at both txs can perfectly be canceled
- AWGN MAC with imperfect feedback
  - Robust noisy-feedback scheme
  - Feedback always increases capacity region, even if very noisy or only partial
  - Almost-perfect feedback $\approx$ perfect feedback
  - Cover-Leung region $\neq$ perfect partial-feedback capacity (answer to v.d. Meulen)