A Schalkwijk-Kailath Type Encoding Scheme for the Gaussian Relay Channel with Receiver-Transmitter Feedback

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***Abstract*— We propose an encoding scheme for the Gaussian relay channel with receiver-transmitter feedback based on the Schalkwijk-Kailath coding strategy for the memoryless Gaussian single-user channel with feedback. The scheme has the advantage over previous schemes for the relay channel of being of very low complexity and, for certain channel parameters, achieving much higher rates.**

I. Introduction

The relay channel was first introduced by van der Meulen in [1]. Cover and El Gamal in [2] then introduced the idea of block-Markov superposition encoding for the relay channel by proposing two encoding schemes based on this idea: the compress-and-forward scheme and the decode-and-forward scheme. Block-Markov superposition encoding is also used in the best schemes [6] known today. Nevertheless, only in a few special cases, i.e., for semi-deterministic relay channels [4] or for physically degraded relay channels [2] a block-Markov encoding strategy was shown to achieve capacity. In general the capacity of the relay channel is not known, not even in the Gaussian case.

In the presence of feedback links either from the receiver to the transmitter, or from the relay to the transmitter, the best schemes [5] proposed so far are as well based on block-Markov strategies, and are known to be capacity achieving only in the same special cases as without feedback. The situation changes only when a feedback link from the receiver to the relay is introduced. For this case it was shown in [2] that a block-Markov strategy achieves capacity.

In this work we consider the Gaussian relay channel where the transmitter has access to feedback from the receiver. We propose an encoding scheme which does not rely on block-Markov encoding, instead we design the transmitter and the relay to be linear. Thus, in this scheme the transmitter, relay and receiver are all of very low complexity.

The encoding scheme is based on the Schalkwijk-Kailath signaling scheme [7] for the additive white Gaussian single-user channel with feedback. Their scheme has been extended to various basic memoryless Gaussian communication settings with feedback: Ozarow extended the scheme for the two-user multiple-access channel [8] and the broadcast channel [10]; Kramer extended it for the multi-user access channel and the interference channel [9], and finally Merhav and Weissman extended the scheme for Costa's Writing on Dirty Paper channel [11]. Note that for the single-user channel, the two-user multiple-access channel and Costa's Writing on Dirty Paper channel the schemes are capacity achieving. Unfortunately, we cannot prove this property for our extension for the relay channel. Nevertheless, for specific channel parameters the scheme outperforms all existing block-Markov encoding schemes by far. Thus, we show that at least in the case of a receiver-transmitter feedback link there are low-complexity alternatives which outperform the best known block-Markov encoding schemes.

II. Setting and Results

The Gaussian relay channel consists of three terminals, a transmitter, a receiver, and a relay. The transmitter wishes to transmit a message W to the receiver, and the relay helps in this transmission. The message W is a random variable which is uniformly distributed over the set $\mathcal{W} = \{1, \ldots, \lfloor e^{nR} \rfloor\}$ where n denotes the blocklength and R denotes the transmission rate.

Independent of the message W let $\{(Z_{1,k}, Z_{2,k})\}$ be a sequence of independent identically distributed (i.i.d.) pairs of independent Gaussian random variables of zero mean and variances N_1 and N_2. The sequence $\{Z_{1,k}\}$ models the noise on the link from the transmitter to the relay and the sequence $\{Z_{2,k}\}$ models the noise on the multiple-access link from the transmitter plus relay to the receiver. Thus, the time-k channel outputs at the relay and at the receiver for given channel inputs $x_{1,k}$ at the transmitter and $x_{2,k}$ at the relay are

$$\begin{aligned} Y_{1,k} &= x_{1,k} + Z_{1,k}, \\ Y_k &= x_{1,k} + dx_{2,k} + Z_{2,k}. \end{aligned}$$

Here d is the gain coefficient of the relay-to-receiver link. The gain coefficients of the other links can be set to one without loss of generality.

The transmitter has access to causal and noise-free feedback from the receiver's output and thus forms its time-k channel input $x_{1,k}$ as a function of the given message $w \in \mathcal{W}$

and the sequence of previous channel outputs at the receiver $y_1, \ldots, y_{k-1}$. The relay does not have any feedback and forms the time-k channel input $x_{2,k}$ as a function of the sequence of previous channel outputs at the relay $y_{1,1}, \ldots, y_{1,k-1}$. Thus, the time-$k$ encoding functions are of the form

$$f_{1,k}^{(n)}: \quad \mathcal{W} \times \mathbb{R}^{k-1} \longrightarrow \mathbb{R}, \tag{1}$$

$$f_{2,k}^{(n)}: \quad \mathbb{R}^{k-1} \longrightarrow \mathbb{R} \tag{2}$$

for $k = 1, \ldots, n$ and the time-k channel inputs are given by

$$x_{1,k} = f_{1,k}^{(n)}(w, y^{k-1}),$$
$$x_{2,k} = f_{2,k}^{(n)}(y_1^{k-1})$$

where v^{k-1} denotes the vector $(v_1, \ldots, v_{k-1})$.

The encoding functions are restricted to fulfill average block power constraints

$$\frac{1}{n}\sum_{k=1}^{n} \mathbb{E}\left[\left(f_{1,k}^{(n)}(W, Y^{k-1})\right)^2\right] \leq P_1 \tag{3}$$

and

$$\frac{1}{n}\sum_{k=1}^{n} \mathbb{E}\left[\left(f_{2,k}^{(n)}(Y_1^{k-1})\right)^2\right] \leq P_2, \tag{4}$$

where $\mathbb{E}$ denotes the expectation operator.

A rate R is said to be achievable if for every blocklength n there exists a sequence of pairs of encoding functions $\{f_{1,k}^{(n)}, f_{2,k}^{(n)}\}_{k=1}^{n}$ as in (1) and (2) fulfilling the constraints (3) and (4), and a decoding function

$$\phi^{(n)}: \quad \mathbb{R}^n \longrightarrow \mathcal{W}$$

such that the probability of a decoding error

$$P_e \triangleq \Pr\left(\phi^{(n)}\left(Y^n\right) \neq W\right)$$

tends to 0 when $n \to \infty$.

Now we are ready to state the main result of this work.

Theorem 1: Consider the Gaussian relay channel with causal noiseless feedback from receiver to transmitter, power constraints P_1 and P_2, noise variances N_1 and N_2, and a gain d on the link from the relay to the receiver. A rate R is achievable for this channel if

$$R \leq \max_{0 \leq \tilde{P}_2 \leq P_2} \frac{1}{2}\ln\left(1 + \frac{P_1\left(1 + d\sqrt{\frac{\tilde{P}_2}{P_1+N_1}}\rho^*\right)^2}{d^2\frac{\tilde{P}_2}{P_1+N_1}N_1 + N_2}\right) \tag{5}$$

where the correlation coefficient ρ^* is given by the unique solution in $[0, 1]$ of the following quartic equation in ρ

$$\rho^2\left(\left(\sqrt{P_1} + d\sqrt{\frac{P_1}{P_1+N_1}}\sqrt{\tilde{P}_2}\rho\right)^2 + d^2\frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2\right)$$
$$= d^2\frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2. \tag{6}$$

From [12] we report the highest rate which is known to be achievable with a block-Markov strategy in a receiver-transmitter feedback setting. The encoding scheme achieving this rate combines the ideas of restricted decoding [13] and the form of backward decoding introduced in [6] for the relay channel without feedback.

Theorem 2 ([12], Theorem 4 and Corollary 2): A rate R is achievable for the Gaussian relay channel with causal noiseless feedback from receiver to transmitter, power constraints P_1 and P_2, noise variances N_1 and N_2, and link gain d if

$$R \leq \sup\min\left\{\frac{1}{2}\ln\left(1 + \frac{P_1 + d^2P_2 + 2d\sqrt{P_1P_2}\sqrt{\bar{\alpha}_1\bar{\alpha}_2}\rho}{N_2}\right)\right.$$
$$-\frac{1}{2}\ln\left(1 + \frac{N_1}{N'}\right),\ \frac{1}{2}\ln\left(1 + \frac{\alpha_1P_1}{N_2} + \frac{\alpha_1P_1}{N_1+N'}\right)$$
$$\left.+\frac{1}{2}\ln\left(1 + \frac{\bar{\alpha}_1P_1\left(1-\rho^2\right)}{\alpha_1P_1 + N_1}\right)\right\} \tag{7}$$

where the supremum is over the parameters $\alpha_1, \alpha_2, \rho, N'$ fulfilling

$$0 \leq \alpha_1, \alpha_2, \rho \leq 1 \quad \text{and} \quad N' \geq \frac{N_1N_2}{d^2\alpha_2P_2}. \tag{8}$$

III. Encoding Scheme

In this section we describe the encoding scheme which achieves the rates in Theorem 1.

Prior to transmission the encoder maps the message W into a real number on the unit interval $[-1/2, 1/2]$ with the following one-to-one mapping

$$\theta: \quad w \mapsto \frac{w-1}{\lfloor e^{nR}\rfloor - 1} - \frac{1}{2}.$$

Consequently, the random variable $\theta(W)$ is distributed uniformly over $\lfloor e^{nR}\rfloor$ equally spaced values within $\left[-\frac{1}{2}, \frac{1}{2}\right]$. We will denote $\theta(W)$ as the message point and simply write θ for it.

The transmitter wishes to convey this message point to the receiver and uses the following scheme: in the first transmission step the encoder transmits a scaled version of the message point θ; based on the noisy channel output the receiver then produces an estimate of θ; thanks to the feedback link the encoder can observe the channel output as well and thus can compute the receiver's estimate; in the next transmission step the encoder sends a scaled version of the receiver's estimation error of θ; based on the noisy channel output the receiver corrects its estimate of θ; again thanks to the feedback link the encoder can compute the decoder's new estimate, and in the next step it sends a scaled version of the new estimation error; and so on and so forth. Thus, the strategy is to send maximally informative updates at the transmitter in order to successively refine the receiver's estimate of the message point, a strategy first described by Schalkwijk-Kailath in [7].

The relay helps in this transmission by simply amplifying and forwarding its previous observation.

After the reception of the n channel outputs the receiver guesses the message W based on its estimate of the message point θ.

In the remaining of this section we describe the transmission steps and the decoding in detail, followed by an analysis of the performance.

First Transmission Step: $k = 1$

In the first transmission step the encoder transmits a scaled version of power P_1 of the message point θ, i.e., $X_{1,1} = \sqrt{\frac{P_1}{\text{Var}(\theta)}}\theta$. Here $\text{Var}(\theta)$ denotes the variance of θ.

The relay stays quiet and the decoder observes $Y_1 = \sqrt{\frac{P_1}{\text{Var}(\theta)}}\theta + Z_{2,1}$ and estimates θ as follows

$$\hat{\theta}_1 = \sqrt{\frac{\text{Var}(\theta)}{P_1}}Y_1 = \theta + \sqrt{\frac{\text{Var}(\theta)}{P_1}}Z_{2,1}.$$

As a result, the decoder's estimation error $\epsilon_1 \triangleq \hat{\theta}_1 - \theta = \sqrt{\frac{\text{Var}(\theta)}{P_1}}Z_{2,1}$ is zero-mean Gaussian and of variance

$$\alpha_1 \triangleq \text{Var}(\epsilon_1) = \frac{\text{Var}(\theta)\, N_2}{P_1}.$$

In the subsequent transmissions the encoder sends resolution information in order to successively refine the decoder's estimate of ϵ_1 and equivalently of θ.

Second Transmission Step: $k = 2$

Before the second transmission step the encoder observes the channel output Y_1 via the instantaneous feedback link and thus can compute the receiver's estimate $\hat{\theta}_1$. Additionally, the encoder of course also knows θ and can compute the estimation error ϵ_1.

In the second transmission step the encoder transmits a scaled version of power P_1 of the estimation error ϵ_1, this is, $X_{1,2} = \sqrt{\frac{P_1}{\alpha_1}}\epsilon_1$.

The relay again stays quiet and the receiver observes the channel output $Y_2 = \sqrt{\frac{P_1}{\alpha_1}}\epsilon_1 + Z_{2,2}$. Based on Y_2 the receiver computes the linear minimum mean square error (LMMSE) estimate of ϵ_1, this is,

$$\hat{\epsilon}_1 = \frac{\sqrt{\alpha_1 P_1}}{P_1 + N_2}Y_2$$

and updates its estimate of the message point θ as $\hat{\theta}_2 = \hat{\theta}_1 - \hat{\epsilon}_1$. The new estimation error is then $\epsilon_2 \triangleq \hat{\theta}_2 - \theta = \epsilon_1 - \hat{\epsilon}_1$ and is of variance

$$\alpha_2 \triangleq \text{Var}(\epsilon_2) = \alpha_1 \frac{N_2}{P_1 + N_2}.$$

In this second transmission step the relay observes $Y_{1,2} = \sqrt{\frac{P_1}{\alpha_1}}\epsilon_1 + Z_{1,2}$.

Further Transmission Steps: $k = 3, \ldots, n$

Prior to transmission step k the encoder observes the feedback outputs $Y_1, \ldots, Y_{k-1}$. Based on these observations and the message point θ it computes ϵ_{k-1}, the error of the decoder's time-$(k-1)$ estimate $\hat{\theta}_{k-1}$ of the message point θ, i.e., $\epsilon_{k-1} \triangleq \hat{\theta}_{k-1} - \theta$. Note that in this scheme ϵ_{k-1} is also the decoder's LMMSE-estimation error when estimating ϵ_1 based on $Y_2, \ldots, Y_{k-1}$. The computation of ϵ_{k-1} can be performed recursively as $\epsilon_{k-1} = \epsilon_{k-2} - \hat{\epsilon}_{k-2}$, where ϵ_{k-2} is the LMMSE-estimation error when estimating ϵ_1 given $Y_2, \ldots, Y_{k-2}$ and $\hat{\epsilon}_{k-2}$ is the LMMSE-estimate of ϵ_{k-2} given $Y_2, \ldots, Y_{k-1}$.

In transmission step k the encoder sends a scaled version of power P_1 of the estimation error ϵ_{k-1}

$$X_{1,k} = \sqrt{\frac{P_1}{\alpha_{k-1}}}\epsilon_{k-1}$$

where $\alpha_{k-1} \triangleq \text{Var}(\epsilon_{k-1})$.

The relay applies an amplify-and-forward strategy, that is, it transmits a scaled version of $Y_{1,k-1}$, its observation in the previous step. The scaling factor is chosen as $\sqrt{\frac{\tilde{P}_2}{P_1+N_1}}$ for some $\tilde{P}_2 \in [0, P_2]$ and hence the expected power of the input symbol $X_{2,k}$ equals $\tilde{P}_2$

$$\begin{aligned} X_{2,k} &= \sqrt{\frac{\tilde{P}_2}{P_1 + N_1}}Y_{1,k-1} \\ &= \sqrt{\frac{\tilde{P}_2}{P_1 + N_1}}\left(\sqrt{\frac{P_1}{\alpha_{k-2}}}\epsilon_{k-2} + Z_{1,k-1}\right). \end{aligned}$$

Thus, the time-k channel output at the receiver is given by

$$\begin{aligned} Y_k &= X_{1,k} + dX_{2,k} + Z_{2,k} \\ &= \sqrt{\frac{P_1}{\alpha_{k-1}}}\epsilon_{k-1} + d\sqrt{\frac{\tilde{P}_2}{P_1 + N_1}}\sqrt{\frac{P_1}{\alpha_{k-2}}}\epsilon_{k-2} \\ &\quad + d\sqrt{\frac{\tilde{P}_2}{P_1 + N_1}}Z_{1,k-1} + Z_{2,k}. \end{aligned}$$

In the above expression only ϵ_{k-2} depends on the previous channel outputs $(Y_2, \ldots, Y_{k-1})$. Therefore, the best predictor of Y_k based on $(Y_2, \ldots, Y_{k-1})$ is a scaled version of the estimate $\hat{\epsilon}_{k-2}$, this is,

$$\hat{Y}_k = d\sqrt{\frac{\tilde{P}_2}{P_1 + N_1}}\sqrt{\frac{P_1}{\alpha_{k-2}}}\hat{\epsilon}_{k-2}.$$

With this predictor the receiver can form the following innovation

$$\begin{aligned} I_k &\triangleq Y_k - \hat{Y}_k \\ &= \sqrt{\frac{P_1}{\alpha_{k-1}}}\epsilon_{k-1} + d\sqrt{\frac{\tilde{P}_2}{P_1 + N_1}}\sqrt{\frac{P_1}{\alpha_{k-2}}}\epsilon_{k-1} \\ &\quad + d\sqrt{\frac{\tilde{P}_2}{P_1 + N_1}}Z_{1,k-1} + Z_{2,k} \end{aligned} \tag{9}$$

which is independent of the previous channel outputs $(Y_2, \ldots, Y_{k-1})$. Based on I_k the receiver can further compute $\hat{\epsilon}_{k-1}$, the LMMSE-estimate of ϵ_{k-1} when observing

$(Y_2, \ldots, Y_k)$, i.e.,

$$\hat{\epsilon}_{k-1} = \frac{\text{Cov}\,(\epsilon_{k-1}, I_k)}{\text{Var}\,(I_k)} I_k.$$

The receiver then uses this term to update its estimate of θ

$$\hat{\theta}_k = \hat{\theta}_{k-1} - \hat{\epsilon}_{k-1}$$

and hence the new estimation error $\epsilon_k \triangleq \hat{\theta}_k - \theta$ is

$$\begin{aligned} \epsilon_k &= \epsilon_{k-1} - \hat{\epsilon}_{k-1} \\ &= \epsilon_{k-1} - \frac{\text{Cov}\,(\epsilon_{k-1}, I_k)}{\text{Var}\,(I_k)} I_k. \end{aligned} \tag{10}$$

Note that ϵ_k is also the LMMSE-estimation error when estimating ϵ_1 based on $(Y_2, \ldots, Y_k)$.

With expressions (9) and (10) the variance of the estimation error ϵ_k can be computed

$$\begin{aligned} \alpha_k &= \alpha_{k-1} - \frac{\text{Cov}\,(\epsilon_{k-1}, I_k)^2}{\text{Var}\,(I_k)} \\ &= \frac{\alpha_{k-1}\left(d^2 \frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2\right)}{\left(\sqrt{P_1} + d\sqrt{\frac{P_1}{P_1+N_1}}\sqrt{\frac{\alpha_{k-1}}{\alpha_{k-2}}}\sqrt{\tilde{P}_2}\right)^2 + d^2\frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2}. \end{aligned}$$

Then, introducing $\rho_{k-1} \triangleq \sqrt{\frac{\alpha_{k-1}}{\alpha_{k-2}}}$ in the above recursion we obtain

$$\alpha_k = \frac{\alpha_{k-1}\left(d^2 \frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2\right)}{\left(\sqrt{P_1} + d\sqrt{\frac{P_1}{P_1+N_1}}\rho_{k-1}\sqrt{\tilde{P}_2}\right)^2 + d^2\frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2} \tag{11}$$

where

$$\rho_2 = \sqrt{\frac{N_2}{P_1 + N_2}},$$

and recursively for $k = 3, \ldots, n-1$

$$\rho_k = \sqrt{\frac{d^2\frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2}{P_1\left(1 + d\sqrt{\frac{\tilde{P}_2}{P_1+N_1}}\rho_{k-1}\right)^2 + d^2\frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2}}. \tag{12}$$

Note that ρ_k equals the correlation coefficient of ϵ_{k-1} and ϵ_k, and thus is proportional to the correlation of the time-$(k+1)$ signal from the encoder to the receiver and the time-$(k+1)$ signal from the relay to the receiver.

Decoding of the Message after Step n

After the n-th transmission step the decoder's estimate of the message point θ is given by $\hat{\theta}_n = \theta + \epsilon_n$. The decoder then guesses that the message $\hat{W} = \hat{w}$ was sent if $\theta(\hat{w})$ is the message point closest to $\hat{\theta}_n$, i.e.,

$$\hat{w} = \arg\min_{w \in \mathcal{W}} |\hat{\theta}_n - \theta(w)|.$$

Performance analysis

Without loss of generality we can assume that $W = w$. Then an error in the decoding occurs only if there is a $w' \neq w$ such that the message point $\theta(w')$ is closer to $\hat{\theta}_n$ than the message point $\theta(w)$. The probability of this event is upper bounded by the probability that the magnitude of ϵ_n is greater than half the distance between adjacent message points which in its turn can be upper bounded as follows

$$P_e \leq \Pr\left[|\epsilon_n| > \frac{1}{2(e^{nR}-1)}\right] < 2Q\left(\frac{1}{2e^{nR}\sqrt{\alpha_n}}\right),$$

where $Q(x) \triangleq \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \mathrm{d}t$ is the tail of the standard Gaussian distribution evaluated at x. In the above term the variance α_n can be expressed by iteratively applying (11)

$$\alpha_n = \alpha_2 \cdot \prod_{k=3}^{n}\left(\frac{d^2\frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2}{\left(\sqrt{P_1} + d\sqrt{\frac{P_1}{P_1+N_1}}\rho_{k-1}\sqrt{\tilde{P}_2}\right)^2 + d^2\frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2}\right). \tag{13}$$

Then we obtain the upper bound on the probability of a decoding error in (12) (shown on top of the next page) and we see that the probability of error tends to 0 when $n \to \infty$ if

$$R < \lim_{n\to\infty} \frac{1}{2n} \sum_{k=3}^{n} \ln\left(1 + \frac{\left(\sqrt{P_1} + d\sqrt{\frac{P_1}{P_1+N_1}}\rho_{k-1}\sqrt{\tilde{P}_2}\right)^2}{d^2\frac{N_1}{P_1+N_1}\tilde{P}_2 + N_2}\right). \tag{13}$$

The convergence of the right hand side of (13) to the bound for R given in Theorem 1 follows by showing that the sequence of correlation coefficients $\{\rho_k\}$ converges to ρ^*, the solution of (6), and then applying Cesáro's Mean Theorem [3, Theorem 4.2.3].

In order to prove the convergence of the sequence $\{\rho_k\}$ to ρ^* we need the following lemma.

Lemma 1: Consider the function $f : x \mapsto \sqrt{\frac{a}{a+p(1+bx)^2}}$ defined on the closed interval $[0,1]$ when $a, b, p \geq 0$. The function $f(\cdot)$ has exactly one fixed point x^* in $[0,1]$ and the infinite sequence $x_0, x_1 = f(x_0), x_2 = f(x_1), \ldots$ converges to this fixed point for any starting point $x_0 \in [0,1]$.

Applying Lemma 1 to the sequence of correlation coefficients $\{\rho_k\}$ it follows that the sequence converges to the unique fixed point in $[0,1]$ of Recursion (12) which is equivalent to the unique solution in the interval $[0,1]$ of (6).

IV. Discussion

In the expression for the achievable rate of Theorem 1 the parameter $\tilde{P}_2$ reflects the effective power used at the relay. Note that the $\tilde{P}_2$ which maximizes this expression is not necessarily equal to P_2 which implies that in the scheme described in Section III it is not necessarily optimal for the relay to transmit with all the available power. This phenomenon arises

$$P_e \leq 2Q\left(\frac{1}{2\sqrt{\alpha_2}}\cdot\exp\left(\sum_{k=3}^{n}\frac{1}{2}\ln\frac{\left(\sqrt{P_1}+d\sqrt{\frac{P_1}{P_1+N_1}}\rho_{k-1}\sqrt{\tilde{P}_2}\right)^2+d^2\frac{N_1}{P_1+N_1}\tilde{P}_2+N_2}{d^2\frac{N_1}{P_1+N_1}\tilde{P}_2+N_2}-nR\right)\right) \quad (12)$$

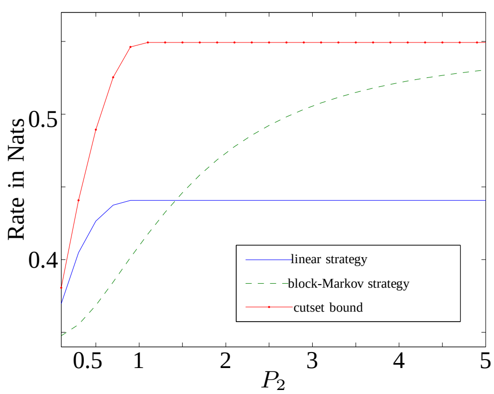

Fig. 1. Bounds on the capacity of the Gaussian relay channel with receiver-transmitter feedback

due to the sub-optimal amplify-and-forward strategy at the relay. With this strategy the relay not only amplifies the signal-part of the previous observation but also the noise-part. Thus, the optimal choice of the effective transmit power is a tradeoff between boosting the signal and enhancing the noise.

Figure 1 compares the performance of the achievable rates in Theorem 1 and in Theorem 2 with the cut-set upper bound on the capacity [3]. The comparison is for the choice of $P_1 = N_1 = N_2 = 1$, $d = 1$, and is a function of the power constraint P_2 at the relay. Note that with the choice $N_1 = N_2$ we only consider settings where at the relay a compress-and-forward strategy or an amplify-and-forward strategy are favorable over a decode-and-forward strategy.

For low power constraint P_2 the rates achieved with the linear scheme increase with P_2, since the relay is exploiting all the available transmit power, and the linear scheme outperforms the block-Markov superposition type scheme. However, for high power constraint P_2 the rates achieved with the linear scheme do not increase with P_2, since only a part of the power is used at the relay, and the linear scheme performs worse than the block-Markov scheme.

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