On Cognitive Interference Networks

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Cognitive Interference Networks

Interference networks:

- K Transmitter/Receiver pairs
- Communications interfere with each other
- Non-cooperating transmitters; non-cooperating receivers
- Constant channel, *not time-varying*
- Single-antenna

Cognitive setting:

- Transmitters have side-information: $\rightarrow$messages of other transmitters
- Side-information can arise from local neighborhood

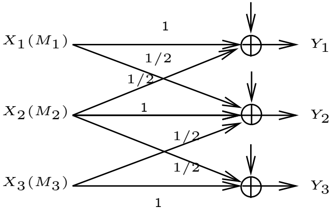
Pre-log η of Cognitive Interference Networks (Intuition)

$$\eta \triangleq \overline{\lim_{P\to\infty}} \frac{C_\Sigma}{C_{\mathsf{AWGN}}}$$

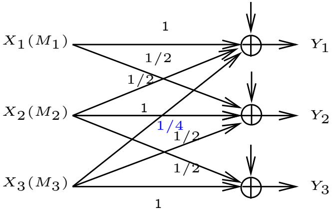
C_Σ: Sum-rate capacity of network;
C_{AWGN}: Single-user AWGN-channel capacity

- Asymptotic gain of sum-rate capacity of network over C_{AWGN}
- High-SNR logarithmic growth of sum-rate capacity of network
- In the real case: should be called "pre-half-log"
- Also called *degrees of freedom* or *multiplexing gain*
- $1 \leq \eta \leq K$ with $K \leftrightarrow$ MIMO

Some Examples

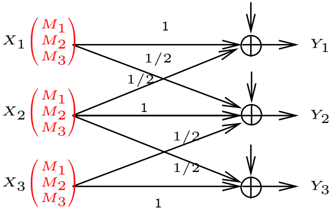


- "No SI": pre-log=2

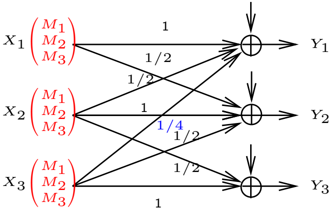


- "No SI": pre-log=1

Some Examples

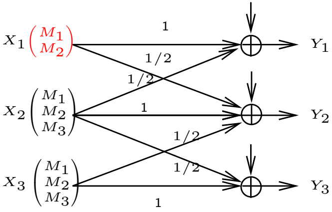


- "No SI": pre-log=2
- "Full SI"/cooperating encoders: pre-log=3

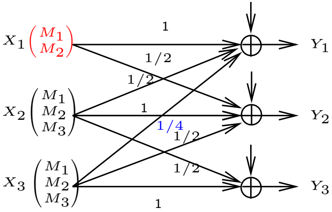


- "No SI": pre-log=1
- "Full SI"/cooperating encoders: pre-log=3

Some Examples

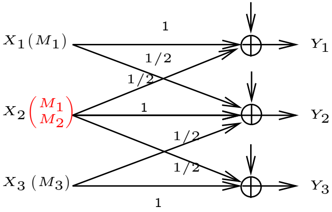


- "No SI": pre-log=2
- "Full SI"/cooperating encoders: pre-log=3
- For all "strictly partial SI": pre-log=2



- "No SI": pre-log=1
- "Full SI"/cooperating encoders: pre-log=3
- "Strictly partial SI", setting a): pre-log=3

Some Examples



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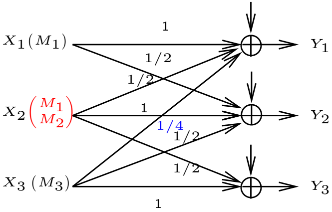
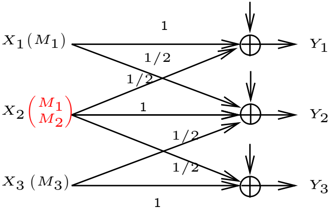


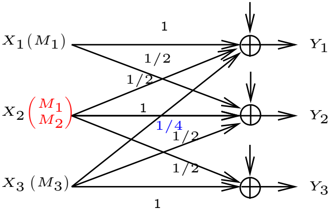
- "No SI": pre-log=1
- "Full SI"/cooperating encoders: pre-log=3
- "Strictly partial SI", setting a): pre-log=3
- "Strictly partial SI", setting b):
  pre-log unknown, $1 \leq$ pre-log $\leq 3/2$

Some Examples



- "No SI": pre-log=2
- "Full SI"/cooperating encoders: pre-log=3
- For all "strictly partial SI": pre-log=2

Similar networks;
very different behavior of pre-log!



- "No SI": pre-log=1
- "Full SI"/cooperating encoders: pre-log=3
- "Strictly partial SI", setting a): pre-log=3
- "Strictly partial SI", setting b):
  pre-log unknown, $1 \leq$ pre-log $\leq 3/2$

Main Open Question

- Pre-log of general cognitive interference networks?
- Open even for general *non-cognitive* networks!

$\longrightarrow$ We have partial results ...

Questions we shall answer

- $\exists$ "strictly partial SI" better than "No SI"?
- $\exists$ "strictly partial SI" such that $\eta = K$?
- Must η be an integer?

Setting

- Transmitters $1, \ldots, K$; Receivers $1, \ldots, K$
- $M_k \sim U\{1, \ldots, \lfloor e^{nR_k} \rfloor\}, \quad k \in \{1, \ldots, K\}$
- Transmitter k's message set $\mathcal{S}_k \subset \{1, \ldots, K\}$:

$$i \in \mathcal{S}_k \quad \Longleftrightarrow \quad \text{Transmitter } k \text{ knows } M_i$$

- Input sequences: $X_{k,t} = f_{k,t}\left(\{M_i\}_{i \in \mathcal{S}_k}\right), \quad t \in \{1, \ldots, n\}$
- Equal power constraints: $\frac{1}{n}\sum_{t=1}^{n} \mathsf{E}\left[X_{k,t}^2\left(\{M_i\}_{i \in \mathcal{S}_k}\right)\right] \leq P$

Setting

- Time-t outputs at Receivers $1, \ldots, K$:

$$\mathbf{Y}(t) = \mathsf{H}\mathbf{X}(t) + \mathbf{Z}(t), \quad t \in \{1, \ldots, n\}$$

- $\mathbf{Y}(t) = (Y_1(t), \ldots, Y_K(t))^\mathsf{T}$, $\quad \mathbf{X}(t) = (X_1(t), \ldots, X_K(t))^\mathsf{T}$
- $\{Z_j(t)\}$ IID $\sim \mathcal{N}\big(0, \sigma^2\big)$
- $\mathsf{H} = (h_{j,k}) \in \mathbb{R}^{K \times K}$ of full rank
- H constant, *not time-varying*
- H models geometry of the setting

Pre-log

- Goal: Receiver j wants to learn M_j
- $(R_1, \ldots, R_K)$ achievable if

$$\mathsf{Pr}[(M_1, \ldots, M_K) \neq (\Phi_1^n(\mathbf{Y}_1), \ldots, \Phi_K^n(\mathbf{Y}_K))] \to 0, \quad n \to \infty$$

- Sum-rate capacity: $C_\Sigma(\mathsf{H}, \{\mathcal{S}_k\}) = \sup \sum_{j=1}^K R_j$

Pre-log:

$$\eta(\mathsf{H}, \{\mathcal{S}_k\}) \triangleq \varlimsup_{P\to\infty} \frac{C_\Sigma(\mathsf{H}, \{\mathcal{S}_k\})}{1/2 \log(1 + P/\sigma^2)}$$

Related Results: “No Side-Information”

Asymptotic:

- Host-Madsen/Nosratinian: Fully connected networks
  - pre-log unknown, $1 \leq \eta \leq K/2$
  - for certain choices of coefficients of *full-rank* K-by-K matrix H, $\eta = 1$

Asymptotic MIMO

- Cadambe/Jafar: 3-by-3 fully connected network, $M > 1$ antennas at each Tx, Rx
  - $\eta = \frac{3}{2}M \Rightarrow$ For MIMO settings η can be non-integer!

Non-asymptotic:

- Etkin/Tse/Wang: 2-by-2 interference network
  - Capacity region to within one bit per user

Related Results: “Full Side-Information”

Setting corresponds to Broadcast channel!

Asymptotic results:

- Caire/Shamai:
  - $\eta = K$

Non-asymptotic results:

- Weingarten/Steinberg/Shamai:
  - Capacity region
  - Achieved with Costa’s writing on dirty paper coding

Related Results: “Partial Side-Information”

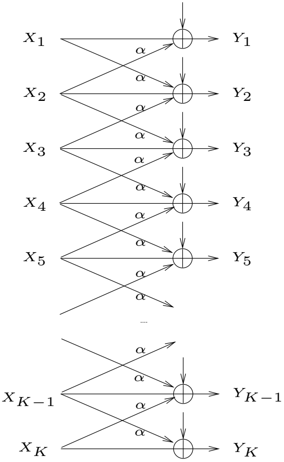
Asymptotic Results:

- Devroye/Sharif: 2-by-2 interference network
  - $\eta = 1$
- Lapidoth/Shamai/Wigger: Wyner’s interference network

Non-asymptotic results:

- 2-by-2 cognitive interference channels:
  - Jovicic/Viswanath
  - Wu/Vishwanath/Arapostatis
  - Devroye/Mitran/Tarokh

Wyner's Linear Model for Cellular Systems



- K Transmitters/Receivers,
- $\alpha > 0$
- "No side-information" $\rightarrow \eta = K - \left\lfloor \frac{K}{2} \right\rfloor$
- "Strictly partial side-information": Next and previous $0 < J < \left\lfloor \frac{K-1}{2} \right\rfloor$ Messages $\rightarrow \eta = K - \left\lfloor \frac{K}{J+2} \right\rfloor$
- Achieved with Costa's writing on dirty paper coding and silencing transmitters (ISIT'07)

Encoding Scheme: (Partial Interference Cancelation, Partial Zero-Forcing)

- Independent Gaussian codebooks

$$\mathcal{C}_i = \left\{\mathbf{u}_i(1), \ldots, \mathbf{u}_i(\lfloor 2^{nR_i} \rfloor)\right\}, \quad i \in \{1, \ldots, K\}$$

- "Linear" encoding: $\mathbf{X}_k \triangleq (X_k(1), \ldots, X_k(n))^\intercal = \sum_{i \in \mathcal{S}_k} d_{i,k} \mathbf{u}_i(M_i)$
- $\{d_{i,k}\}$ s.t. power constraints fulfilled
- Channel outputs: $\mathbf{Y}_j = \sum_{k=1}^{K} h_{j,k} \mathbf{X}_k + \mathbf{Z}_j$
- Decoding: joint typicality decoding

Encoding Scheme: How to Choose $\{d_{i,k}\}$?

- Reordering of channel outputs:

$$\mathbf{Y}_j = \underbrace{\left(\sum_{k:j\in\mathcal{S}_k} h_{j,k}d_{j,k}\right)\mathbf{u}_j(M_j)}_{\text{information}} + \underbrace{\sum_{i\neq j}\left(\sum_{k:i\in\mathcal{S}_k} h_{j,k}d_{i,k}\right)\mathbf{u}_i(M_i)}_{\text{interference}} + \mathbf{Z}_j$$

- Want to cancel as many interferences as possible!

p^*: maximum of *canceled/zero-forced* interferences

$$\Rightarrow \eta(\mathsf{H}, \{\mathcal{S}_k\}) \geq p^*$$

- Lower bound p^* generally not tight

Upper Bound on Pre-log, Lemma

Lemma : (Degraded Networks)

Given $(\mathsf{H}, \{\mathcal{S}_k\})$, *permutation* $\boldsymbol{\pi}$ *on* $\{1, \ldots, K\}$, $1 \le q \le K$.

If for any encoding scheme $\exists$ *functions* $f_{q+1}, \ldots, f_K$ *s.t.*

$$\mathbf{Y}_{\boldsymbol{\pi}(j)} = f_j(\mathbf{Y}_{\boldsymbol{\pi}(1)}, \ldots, \mathbf{Y}_{\boldsymbol{\pi}(j-1)}, M_{\boldsymbol{\pi}(1)}, \ldots, M_{\boldsymbol{\pi}(j-1)}, \text{“noise”}), \quad q < j \le K,$$

then

$$\eta(\mathsf{H}, \{\mathcal{S}_k\}) \le q$$

Upper Bound on Pre-log, Proof of Lemma

Conditions:

$$\mathbf{Y}_{\boldsymbol{\pi}(j)} = f_j(\mathbf{Y}_{\boldsymbol{\pi}(1)}, \ldots, \mathbf{Y}_{\boldsymbol{\pi}(j-1)}, M_{\boldsymbol{\pi}(1)}, \ldots, M_{\boldsymbol{\pi}(j-1)}, \text{“noise”}), \quad q < j \leq K$$

Proof idea:

$$\boxed{\begin{array}{cc} \mathbf{X}_{\boldsymbol{\pi}(1)} & \mathbf{Y}_{\boldsymbol{\pi}(1)} \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \mathbf{X}_{\boldsymbol{\pi}(K)} & \mathbf{Y}_{\boldsymbol{\pi}(K)} \end{array}} \subset_{\text{Cap.}} \boxed{\begin{array}{cc} \mathbf{X}_{\boldsymbol{\pi}(1)} & \mathbf{Y}_{\boldsymbol{\pi}(1)} \\ \cdots & \cdots \\ \mathbf{X}_{\boldsymbol{\pi}(q)} & \mathbf{Y}_{\boldsymbol{\pi}(q)} \\ \mathbf{X}_{\boldsymbol{\pi}(q+1)} & \mathbf{Y}_{\boldsymbol{\pi}(q+1)}, \mathbf{Y}_{\boldsymbol{\pi}(1)}, \ldots, \mathbf{Y}_{\boldsymbol{\pi}(q)}, \text{“noise”} \\ \cdots & \cdots \\ \mathbf{X}_{\boldsymbol{\pi}(K)} & \mathbf{Y}_{\boldsymbol{\pi}(K)}, \mathbf{Y}_{\boldsymbol{\pi}(1)}, \ldots, \mathbf{Y}_{\boldsymbol{\pi}(q)}, \text{“noise”} \end{array}}$$

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Upper Bound on Pre-log, Proof of Lemma

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$$\mathbf{Y}_{\boldsymbol{\pi}(j)} = f_j(\mathbf{Y}_{\boldsymbol{\pi}(1)}, \ldots, \mathbf{Y}_{\boldsymbol{\pi}(j-1)}, M_{\boldsymbol{\pi}(1)}, \ldots, M_{\boldsymbol{\pi}(j-1)}, \text{"noise"}), \quad q < j \leq K$$

Proof idea:

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even with cooperation:
$\rightarrow \eta \leq q$

Upper Bound on Pre-log: When can we apply the Lemma?

Condition:

$$\mathbf{Y}_{\boldsymbol{\pi}(j)} = f_j(\mathbf{Y}_{\boldsymbol{\pi}(1)}, \ldots, \mathbf{Y}_{\boldsymbol{\pi}(j-1)}, M_{\boldsymbol{\pi}(1)}, \ldots, M_{\boldsymbol{\pi}(j-1)}, \text{``noise''}), \quad q < j \leq K$$

Example: Let $q = K - 1$

$$\begin{aligned} \mathbf{Y}_{\boldsymbol{\pi}(K)} &= \sum_{k=1}^{K} h_{\boldsymbol{\pi}(K),k} \mathbf{X}_k + Z_{\boldsymbol{\pi}(K)} = \sum_{k}^{K} \left(\sum_{i=1}^{K-1} c_i h_{\boldsymbol{\pi}(i),k} \right) \mathbf{X}_k + Z_{\boldsymbol{\pi}(K)} - c\mathbf{X}_j \\ &= \sum_{i=1}^{K-1} c_i \mathbf{Y}_{\boldsymbol{\pi}(i)} - c\mathbf{X}_j - \text{``noise''} \end{aligned}$$

- $\boldsymbol{\pi}(K)$-th row of H is linear comb. of other rows, except j-th entry
- $\mathbf{X}_j \perp\!\!\!\perp M_{\boldsymbol{\pi}(K)} \quad \rightarrow$ computable from $M_{\boldsymbol{\pi}(1)}, \ldots, M_{\boldsymbol{\pi}(K-1)}$

Results

Characterization of networks where $\eta = K$ and those where $\eta = K - 1$

Theorem

Given interference network $(\mathsf{H}, \{\mathcal{S}_k\})$*:*

$$\begin{aligned} p^* = K \quad &\Longrightarrow \quad \eta(\mathsf{H}, \{\mathcal{S}_k\}) = K, \\ p^* = K - 1 \quad &\Longrightarrow \quad \eta(\mathsf{H}, \{\mathcal{S}_k\}) = K - 1, \\ p^* \leq K - 2 \quad &\Longrightarrow \quad \eta(\mathsf{H}, \{\mathcal{S}_k\}) < K - 1. \end{aligned}$$

Corollary

Given $(\mathsf{H}, \{\mathcal{S}_k\})$*:*

$$\begin{aligned} \eta(\mathsf{H}, \{\mathcal{S}_k\}) = K \quad &\Longleftrightarrow \quad p^* = K, \\ \eta(\mathsf{H}, \{\mathcal{S}_k\}) = K - 1 \quad &\Longleftrightarrow \quad p^* = K - 1. \end{aligned}$$

Pre-log not in open interval $(K - 1, K)$

Results

Characterization of side-information required for $\eta = K$

Theorem

$$\eta(\mathsf{H}, \{\mathcal{S}_k\}) = K$$
$$\Longleftrightarrow$$
$$\forall j,k \in \mathcal{K}: \quad \left(\operatorname{rank}\left(\mathsf{H}_{(j)}^{(k)}\right) = K-1 \qquad \Longrightarrow \qquad j \in \mathcal{S}_k\right)$$

Corollary

$$\text{“Full side-information”} \quad \text{required for} \quad \eta = K$$
$$\Longleftrightarrow$$
$$\operatorname{rank}\left(\mathsf{H}_{(j)}^{(k)}\right) = K-1 \qquad \text{for all } j \neq k, \quad \text{and } j,k \in \{1,\ldots,K\}$$

$\mathsf{H}_{(j)}^{(k)}$: H without k-th column, j-th row

Results

Theorem

No “strictly partial side-information” can increase pre-log

whenever

$$\mathsf{H} \textit{ is diagonal} \quad \textit{or} \quad \mathsf{H} = \begin{pmatrix} \times & 0 & 0 & \cdots & 0 & \times & 0 & \cdots & 0 & 0 \\ 0 & \times & 0 & \cdots & 0 & \times & 0 & \cdots & 0 & 0 \\ & & & \cdots & & & \cdots & & & \\ & & & \cdots & & & \cdots & & & \\ 0 & 0 & 0 & \cdots & \times & \times & 0 & \cdots & 0 & 0 \\ \times & \times & \times & \cdots & \times & ? & \times & \cdots & \times & \times \\ 0 & 0 & 0 & \cdots & 0 & \times & \times & \cdots & 0 & 0 \\ & & & \cdots & & & \cdots & & & \\ & & & \cdots & & & \cdots & & & \\ 0 & 0 & 0 & \cdots & 0 & \times & 0 & \cdots & \times & 0 \\ 0 & 0 & 0 & \cdots & 0 & \times & 0 & \cdots & 0 & \times \end{pmatrix}$$

x*: non-zero entry,* $?$*: arbitrary entry*

Examples: 2-by-2 interference network

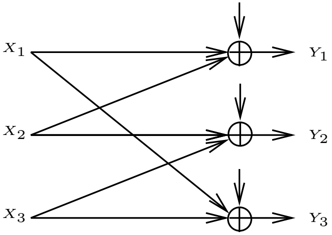
$\longrightarrow$ recovering result by Devroye/Sharif

Extension of Encoding Scheme

- Group μ channel uses into *super-channel* use
- $\rightarrow$ *Super channel* with μ-antenna transmitters/receivers, channel matrix $\mathsf{H} \otimes I_\mu$
- Split messages into μ sub-messages $\rightarrow$ μK cognitive single-antenna transmitters
- Linearly process the μ outputs of each receiver $\rightarrow$ μK single-antenna receivers
- Partial interference cancelation for single-antenna μK-by-μK Tx/Rx network

Inspired by Weingarten/Shamai/Kramer: "On the Compound MIMO BC"

Example of Extended Encoding Scheme



- "No side-information"
- Extend the scheme over 2 channel uses
- $\rightarrow \eta \geq 3/2$

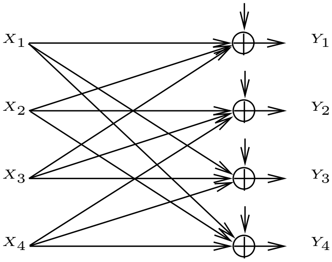
In fact with a modified upper bound:

$$\eta = 3/2 \quad \text{(non-integer!)}$$

$\rightarrow$ We showed: pre-log needs not be an integer!

Example of Extended Encoding Scheme

Construction can be generalized to K-by-K networks:



- Each receiver experiences interference from next $K-2$ transmitters in round robin way
- Extend scheme to $K-1$ channel uses
- $\rightarrow \eta \geq K/(K-1)$

In fact with a modified upper bound:

$$\eta = K/(K-1) \quad \text{(non-integer!)}$$

Summary

- Characterized networks where $\eta = K$ and those where $\eta = K - 1$
- Characterized networks where "full side-information" necessary for $\eta = K$
- Characterized networks where some "strictly partial side-information" *can* increase pre-log
- Pre-log of cognitive (single-antenna) interference networks can be non-integer

Reminder:

$$\eta = \varlimsup_{P \to \infty} \frac{C_{\Sigma}}{C_{\mathsf{AWGN}}}; \qquad \text{K: \# Tansmitters/Receivers}$$