

# Mixed Delay Constraints at Maximum Sum-Multiplexing Gain

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**Abstract**—Coding schemes are proposed for Wyner’s *soft-handoff* model and for the sectorized hexagonal model when some of the messages are delay-sensitive and cannot profit from transmitter or receiver cooperation. For the soft-handoff network we also provide a converse. It matches the multiplexing-gain achieved by our scheme when the multiplexing gain of the delay-sensitive messages is low or moderate or when the cooperation links have high capacities. In these cases, the sum-multiplexing gain is the same as if only delay-tolerant messages (which can profit from cooperation) were sent. A similar conclusion holds for the sectorized hexagonal model, when the capacities of the cooperation links are large.

## I. INTRODUCTION

One of the major challenges of today’s wireless communication networks is to design coding schemes for transmission of heterogeneous traffic types. Such a design, for example, has to account for the fact that delay-tolerant applications can profit from cooperation between transmitting or receiving terminals, but delay-sensitive applications cannot. Such mixed delay constraints in wireless networks have recently been studied in [1]–[5]. In this paper, we study the capacity of two cellular network models (Wyner’s soft-handoff model [6], [7] and the hexagonal sectorized model) under mixed delay constraints. In a previous work, we have studied the capacity region of Wyner’s soft-handoff network under mixed delay constraints when only the transmitters or only the receivers can cooperate [5]. We determined the multiplexing gain region in function of the capacities of the cooperation links. The results show that when delay-sensitive applications are of low rates, the total sum-rate is not decreased compared to a scenario where communication is only for delay-tolerant applications. In contrast, for high-rate delay-sensitive applications, this is not the case, and 1 bit of delay-sensitive rate comes at the expense of 2 bits delay-tolerant rate. Moreover, no positive delay-tolerant multiplexing gain is possible when one insists on sending at maximum delay-sensitive multiplexing gain.

As we show in this work, these (rather pessimistic) conclusions do not hold when both the transmitters *and* the receivers can cooperate. In fact, when the cooperation rates are sufficiently large, then by using sophisticated coding schemes it is possible to accommodate the largest possible multiplexing gain for delay-sensitive applications without decreasing the maximum *sum*-multiplexing gain. The stringent delay constraints thus do not harm the overall performance. For smaller

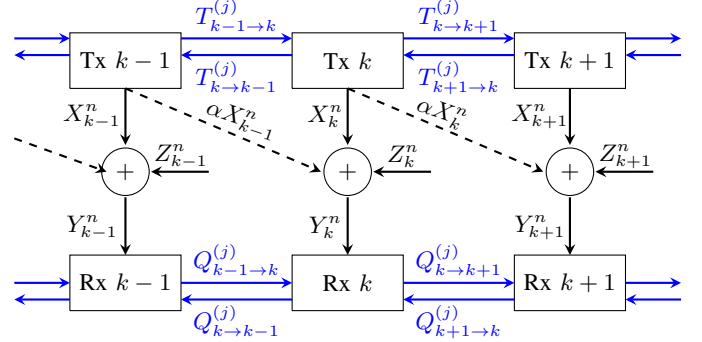


Fig. 1. System model

cooperation rates, this performance is achievable only when the delay-sensitive multiplexing gain is small. In this case, the behaviour is similar to when only transmitters or only receivers can cooperate.

## II. WYNER’S *Soft Hand-off* NETWORK

Consider the communication system as in Fig. 1 with  $K$  interfering transmitter (Tx) and receiver (Rx) pairs  $1, \dots, K$ . Transmitters and receivers are each equipped with a single antenna, and channel inputs and outputs are real valued. Interference is short-range so that the signal sent by Tx  $k$  is observed only by Rx  $k$  and  $k + 1$ . As a result, the time- $t$  channel output at Rx  $k$  is

$$Y_{k,t} = X_{k,t} + \alpha X_{k-1,t} + Z_{k,t}, \quad (1)$$

where  $X_{k,t}$  and  $X_{k-1,t}$  are the symbols sent by Tx  $k$  and  $k - 1$  at time  $t$ , respectively;  $\{Z_{k,t}\}$  are independent and identically distributed (i.i.d.) standard Gaussians for all  $k$  and  $t$ ;  $\alpha \neq 0$  is a fixed real number smaller than 1; and  $X_{0,t} = 0$  for all  $t$ .

Each Tx  $k$  wishes to send a pair of independent source messages  $(M_k^{(F)}, M_k^{(S)})$  to Rx  $k$ . The “fast” source message  $M_k^{(F)}$  is uniformly distributed over  $\mathcal{M}_k^{(F)} := \{1, \dots, \lfloor 2^{nR_k^{(F)}} \rfloor\}$  and the “slow” source message  $M_k^{(S)}$  over  $\mathcal{M}_k^{(S)} := \{1, \dots, \lfloor 2^{nR_k^{(S)}} \rfloor\}$ . Here,  $n$ ,  $R_k^{(F)}$ , and  $R_k^{(S)}$  denote the block-length and rates of transmission. Transmitters can cooperate with their immediate neighbours to exchange information about “slow” but not “fast” messages. In each cooperation round  $j \in \{1, \dots, D_t\}$ , Tx  $k$  produces the two cooperation messages  $T_{k→k-1}^{(j)}$  and  $T_{k→k+1}^{(j)}$ , where

$$T_{k→\bar{k}}^{(j)} = \xi_{k→\bar{k}}^{(n)}(M_k^{(S)}, T_{k-1→k}^{(1)}, \dots, T_{k-1→k}^{(j-1)}),$$



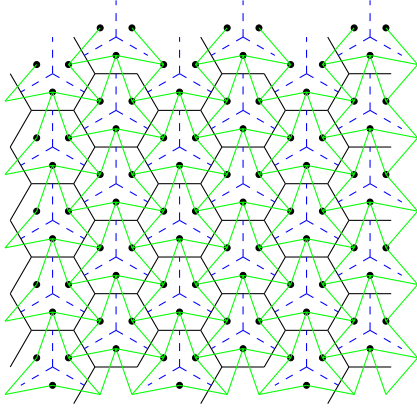


Fig. 3. Illustration of the sectorized hexagonal network. Black hexagonal regions depict the cells and the dashed blue lines determine their sectorization. Interfering sectors are connected by green lines.

If

$$S^{(F)} \leq \frac{1}{2} - 2\mu \frac{D-1}{D}, \quad (17)$$

then  $(S^{(F)}, S^{(S)}) \in \mathcal{S}^*(\mu, D)$  if, and only if, it satisfies (15b).

From Fig. 2 (and Corollary 1) we also see that when  $\mu$  exceeds a threshold, then for any  $S^{(F)}$  the same maximum sum-multiplexing gain of  $\min\{\frac{1}{2} + 2\mu, \frac{2D+1}{2D+2}\}$  is achievable as when only slow messages are transmitted. There is thus no penalty in sum-multiplexing gain by sending also “fast” messages. This is in contrast to the setup with only transmitter or receiver cooperation where a moderate or large multiplexing gain for “fast” messages causes a decrease in sum-multiplexing gain [5].

### III. SECTORIZED HEXAGONAL MODEL

In this model, the  $N$  cells are hexagons and consist of three sectors. The BS of each cell is equipped with  $3M$  directional antennas, with  $M$  antennas pointing to each sector. This allows avoiding interference between communications from different sectors in the same cell. For simplicity, and because the focus is on the multiplexing gain, we assume a single mobile user in each sector. The models with the cells, sectors, and the interference pattern, is depicted in Fig. 3. The small circles indicate mobile users, and the solid green lines indicate that the communication in two given sectors interfere.

For a given sector  $k \in \{1, \dots, 3N\}$ , let the set  $\mathcal{I}_k$  contain the indices of the neighbouring sectors whose signals interfere with sector  $k$ . The time- $t$  signal received at the  $M$  receive antennas directing to sector  $k$  can then be written as:

$$Y_{k,t} = H_{k,k} X_{k,t} + \sum_{\hat{k} \in \mathcal{I}_k} H_{k,\hat{k}} X_{\hat{k},t} + Z_{k,t}, \quad (18)$$

where  $\{H_{k,\hat{k}}\}$  denote the  $M$ -by- $M$  channel matrices.

As before, “fast” and “slow” messages are sent simultaneously over the network. Mobile transmitters in neighbouring sectors can cooperate “slow” messages during  $D_t$  rounds of cooperation. Receiving BSs decode their “fast” messages based solely on their own outputs and their “slow” messages based on these outputs and the cooperation signals that they exchanged with neighbouring BSs over  $D_r$  cooperation rounds.

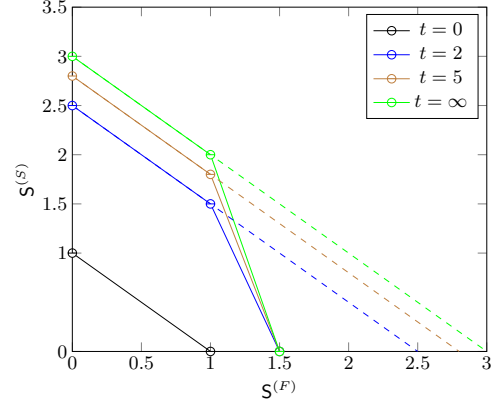


Fig. 4. Inner bound on multiplexing gain region for  $M = 3$  and different values of  $t$ .

So, BS  $i$  produces as its guesses of the “fast” and “slow” messages:

$$\hat{M}_i^{(F)} = g_i^{(n)}(\mathbb{Y}_i) \quad (19)$$

and

$$\hat{M}_i^{(S)} = b_i^{(n)}(\mathbb{Y}_i, \mathbb{V}_{to i}^j), \quad (20)$$

where  $\mathbb{Y}_i := \{(Y_{k,1}, \dots, Y_{k,n}) : \text{sector } k \text{ is in cell } i\}$ ,  $M_i^{(F)} := \{M_k^{(F)} : \text{sector } k \text{ is in cell } i\}$ ,  $M_i^{(S)} := \{M_k^{(S)} : \text{sector } k \text{ is in cell } i\}$  and  $\mathbb{V}_{to i}^j$  denote all conferencing messages received at BS  $i$ . The multiplexing gain region is defined analogously to the previous section, but for simplicity we assume that  $\mu = 1$ .

*Theorem 2:* The multiplexing gain region includes all non-negative pairs  $(S^{(F)}, S^{(S)})$  that satisfy

$$S^{(F)} + S^{(S)} \leq \frac{M(3t-1)}{3t} \quad (21)$$

$$S^{(F)} + c S^{(S)} \leq \frac{M}{2}. \quad (22)$$

where  $c := \frac{t}{2(2t-1)} < 1$  and  $t \in \{1, \dots, D_r/2\}$ .

*Proof:* See Section V. ■

This inner bound on the multiplexing gain region is illustrated in Figure 4. We notice that for small or moderate values of  $S^{(F)}$ , the sum-multiplexing gain achieved by our scheme is the same as when only “slow” messages are sent.

### IV. SCHEMES ACHIEVING THEOREM 1

To achieve the performance in Theorem 1, four schemes need to be time-shared depending on the operating point and on the available cooperation prelog  $\mu$ .

#### A. Scheme 1: Transmitting only “fast” messages

In this scheme, every second transmitter is switched off, which decomposes the network into  $\lceil K/2 \rceil$  non-interfering point-to-point links. A “fast” message of multiplexing gain 1 is transmitted over each of these point-to-point links. The scheme thus achieves multiplexing gain pair  $(S^{(F)} = 1/2, S^{(S)} = 0)$  without any requirement on the cooperation rate.

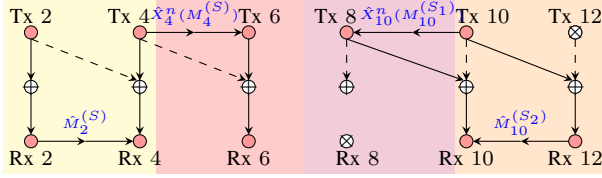


Fig. 5. Communicating “slow” messages in different groups of Scheme 4.

### B. Scheme 2: Transmitting only “slow” messages

Use the scheme [8] to send only “slow” messages. It achieves multiplexing gain pair  $(S^{(F)} = 0, S^{(S)} = \frac{2D+1}{2D+2})$  with cooperation rate  $\mu = \frac{D}{4(D+1)}$ .

### C. Scheme3: Opportunistically sending “fast” messages

Our third scheme is closely related to the second scheme. We notice that [8] splits the network into subnetworks by periodically silencing every  $2D + 2$ -th transmitter. Also, the message of the left-most and the  $D + 3$ -rd transmitters in each subnetwork do not participate in the transmitter cooperation and their decoding can be performed directly based on the corresponding channel output sequences. These messages can thus be sent as “fast” messages, whereas all other transmitted messages are “slow” messages. This scheme achieves multiplexing gain pair  $(S^{(F)} = \frac{2}{2D+2}, S^{(S)} = \frac{2D-1}{2D+2})$  and requires cooperation rate  $\mu = \frac{D}{4(D+1)}$ , see [8].

### D. Scheme 4: Alternating “fast” and “slow” messages

We again silence periodically every  $2D + 2$ -th transmitter, so as to split the network into smaller subnetworks. In a given subnetwork, all odd transmitters  $1, 3, 5, \dots, 2D + 1$  send a “fast” message with multiplexing gain 1 using a simple Gaussian codebook. They further precancel (up to noise level) the interference from their left-neighbours. To facilitate this, each even transmitter (which only sends a slow message as we explain shortly) quantizes and describes its input signal to its right neighbour over the cooperation link. All odd receivers  $1, 3, 5, \dots, 2D + 1$  decode their desired “fast” message based on their own channel outputs using a simple point-to-point channel decoder. They also describe their decoded messages to their right-neighbours which then cancel the influence of the transmitted Gaussian codewords.

In the following, we describe the communication of “slow” messages, which is from even transmitters. Recall that all odd transmitters have precanceled a quantized version of their left-neighbours transmit signal. This implies that the signal sent by a given even Tx  $k$  interferes at Rx  $k + 2$ . Moreover, even receivers immediately cancel the interference of “fast” messages stemming from their left neighbours. The interference graph for “slow” messages thus has the form in Figure 5, where odd receivers are ignored. To describe communication of “slow” messages in more detail, we partition Tx/Rx pairs of each subnetwork into four groups.

Transmitters in  $\mathcal{G}_1 := \{1, \dots, D_r + 1\}$  use Gaussian point-to-point codes of power  $P$  to transmit their “slow” messages over the channel. Rx  $k$  uses the cooperation messages received

from its left neighbours Rx  $k - 2$  (over two hops) and from Rx  $k - 1$  to delete the interference

$$\hat{Y}_k^n = Y_k^n - \alpha X_{k-1}^n(\hat{M}_{k-2}^{(S)}, \hat{M}_{k-1}^{(F)}), \quad (23)$$

and it decodes the source message  $M_k^{(S)}$  based on this difference. Then it describes its decoded message to Rx  $k + 2$ .

Transmitters in  $\mathcal{G}_2 := \{D_r + 2, \dots, D_r + D_t + 1\}$  use dirty-paper coding to mitigate interference from the left. To this end, after receiving and reconstructing the quantization message  $\hat{X}_{k-2}^n(M_{k-2}^{(S)})$  sent by Tx  $k - 2$  (over two conferencing hops), Tx  $k$  encodes its source message  $M_k^{(S)}$  using a power  $P$  dirty-paper code that eliminates the interference  $\alpha \hat{X}_{k-2}^n(M_{k-2}^{(S)})$ . Then it sends the decoded message over the channel. Moreover, Tx  $k$  quantizes its produced input  $X_k^n(M_k^{(S)})$  using a rate  $(1/2) \log(1 + P)$  quantizer and sends the resulting message to Tx  $k + 1$  and Tx  $k + 2$ . Rx  $k$  receives the decoded message  $\hat{M}_{k-1}^{(F)}$  through the conferencing link to its left. Then it uses the received message to reconstruct  $\alpha X_{k-1}^n(\hat{M}_{k-1}^{(F)})$  and forms

$$\hat{Y}_k^n = Y_k^n - \alpha X_{k-1}^n(\hat{M}_{k-1}^{(F)}). \quad (24)$$

It finally decodes message  $M_k^{(S)}$  based on this difference.

Transmitters in  $\mathcal{G}_3 := \{D_r + D_t + 2, \dots, D_r + 2D_t + 2\}$  use dirty-paper coding to mitigate interference from the right. The desired communication path of a message  $M_k^{(S)}$  is Tx  $k \rightarrow$  Tx  $k - 2 \rightarrow$  Rx  $k$ . More specifically, Tx  $k$  encodes  $M_k^{(S)}$  and sends a quantized version  $\hat{X}_k^n(M_k^{(S)})$  to Tx  $k - 2$  which then transmits  $X_{k-2}^n = \hat{X}_k^n(M_k^{(S)})$  over the channel. Encoding is performed using dirty-paper coding where Tx  $k$  precancels the interference of the signal  $X_k^n$  it is sending over the channel. Receivers in  $\mathcal{G}_3$  decode using standard dirty-paper decoding.

Transmitters in  $\mathcal{G}_4 := \{D_r + 2D_t + 3, \dots, 2D_r + 2D_t + 1\}$ , use a Gaussian point-to-point code of power  $P$  to transmit their source messages  $M_k^{(S)}$  over the channel through the path Tx  $k \rightarrow$  Rx  $k + 2 \rightarrow$  Rx  $k$ . Rx  $k$  uses the cooperation messages received from its right neighbours Rx  $k + 2$  (over two hops) and from Rx  $k + 1$  to decode  $M_k^{(S)}$ .

In what we described, in some cases the last transmitter in  $\mathcal{G}_3$  and the first transmitter in  $\mathcal{G}_4$  are supposed to send the same message. In this case, we split this message into two independent parts,  $M_k^{(S_1)}$  and  $M_k^{(S_2)}$ , and let each of the two transmitters send a different part. In the example in Fig. 5, Tx 8 sends  $M_{10}^{(S_1)}$  and Tx 10 sends  $M_{10}^{(S_2)}$ .

The described scheme achieves multiplexing gain pair  $(S^{(F)} = \frac{1}{2}, S^{(S)} = \frac{D}{2D+2})$ , and requires cooperation rate  $\mu_{\max} = \frac{3D}{4(2D+2)}$ .

## V. SCHEMES ACHIEVING THEOREM 2

To achieve the performance in Theorem 2, three schemes need to be time-shared depending on the operating point.

### A. Scheme 1: Transmitting only “fast” messages

Transmission is based on *interference alignment* [10], and cooperation links are completely ignored. This scheme achieves multiplexing gain pair  $(S^{(F)} = \frac{M}{2}, S^{(S)} = 0)$ .

### B. Scheme 2: Transmitting only “slow” messages

Use the scheme in [9] to send only “slow” messages. It achieves multiplexing gain pair  $(S^{(F)} = 0, S^{(S)} = M^{\frac{3t-1}{3t}})$ .

### C. Scheme 3: Alternating “fast” and “slow” messages

The scheme is closely related to the second scheme. We notice that [9] defines *master cells* so that they build a regular grid of equilateral triangles where the three master cells forming each of the triangles lay  $3t$  cell-hops apart from each other for  $t \in \{1, \dots, D_r/2\}$ . Layer-  $1, 2, \dots, t$  cells are cells that are  $1, 2, \dots, t$  hops away from master cells. The scheme in [9] deactivates users in layer-“ $t$ ” cells so as to split the network into non-interfering clusters of sectors, see Fig. 6. In our scheme here, some users send “slow” messages and others send “fast” messages. Users sending “slow” messages use simple Gaussian codebooks to send their messages. They quantize their inputs and send the quantization information to all their neighbours that send “fast” messages. The transmitters sending “fast” messages precancel the interference from “slow” messages. As we will see shortly, communication of “fast” messages is interfered only by “slow” messages, and since this interference is precanceled, receivers can decode their “fast” messages based on almost interference free signals. Decoded “fast” messages are sent to neighbouring receivers, which cancel the corresponding interference. In our scheme, all the three users of a master cell send “fast” messages. Users of layer-“1” cells that interfere users in the master cell send “slow” messages and the remaining users in this layer send “fast” message. Similarly, users of layer-“2” cells that interfere layer-“1” users send “slow” messages and the remaining layer-“2” users send “fast” messages. A similar procedure is applied for layer-“3”, layer-“4”, ..., layer-“ $t-1$ ” cells. In layer-“ $t$ ” cells, all the active users send “slow” messages. Figure 6 shows the deactivated sectors in white, the sectors with “fast” messages in blue, and the sectors with “slow” messages in red.

This scheme achieves multiplexing gain pair  $(S^{(F)} = \frac{M}{3}, S^{(S)} = M^{\frac{2t-1}{3t}})$ .

## VI. CONCLUSION

We considered two cellular models and characterized the multiplexing gain region with transmitter and receiver cooperation when some source messages are subject to stringent delay constraints. In our previous work [5], increasing the rate of delay-sensitive messages by  $\Delta$  required to decrease the rate of delay-tolerant messages approximately by  $2\Delta$ . This penalty does not arise for the setup with transmitter *and* receiver cooperation that is considered for Wyner’s soft-handoff model and the hexagonal sectorized model. In fact, when delay-sensitive messages have moderate or small multiplexing gains, then the *sum*-multiplexing gain is not decreased compared to when only delay-tolerant messages are transmitted. For Wyner’s soft-handoff model, this conclusion even holds for large multiplexing gains of “fast” messages when the cooperation rates are sufficiently large. The obtained results on transmitter and receiver cooperation may have interesting

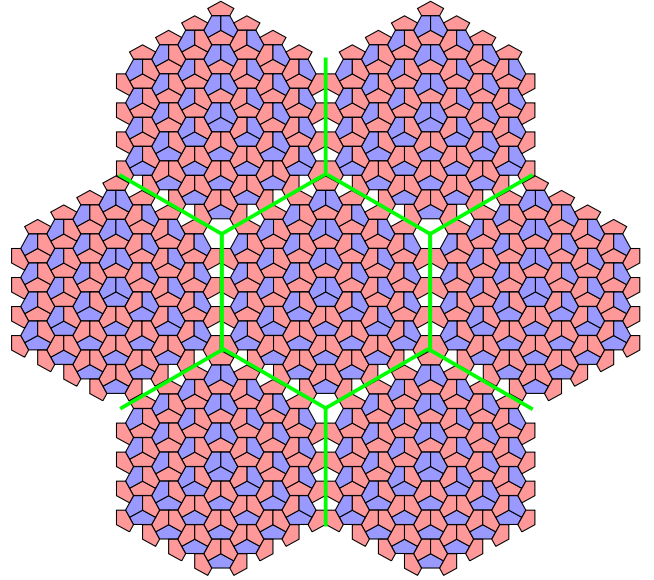


Fig. 6. Illustration of sector allocation for  $t = 4$ . Users in blue sectors send “fast” messages and users in red send “slow” messages. White sectors indicate deactivated users, and thick lines indicate non-interfering clusters.

practical application due to the increase in different latency requirements.

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