Mixed Delay-Constraints in Wyner's Soft-Handoff Network

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***Abstract*—Wyner's *soft-handoff* network with mixed delay constraints on source messages is considered in which neighbouring receivers are able to cooperate over rate-limited conferencing links. Each source message is a combination of independent "fast" and "slow" bits where the former are subject to a stringent decoding delay. Inner and outer bounds on the capacity region are derived and the per-user multiplexing gain is obtained for only transmitter or only receiver conferencing as a function of the capacity of the conferencing links and the maximum allowed decoding delay of "slow" bits.**

I. Introduction

Wireless communication networks have to accommodate different types of data traffics with different latency constraints. In particular, delay-sensitive video-applications represent an increasing portion of the data traffic. On the other hand, modern networks can increase data rates by means of cooperation between terminals or with helper relays. However, cooperation typically introduces additional communication delays, and is thus not applicable to delay-sensitive applications. In this paper, we analyze the rates of communication that can be attained over an interference network with either transmitter- or receiver- cooperation, and where parts of the messages cannot profit from this cooperation because they are subject to stringent delay constraints. Mixed delay constraints in wireless networks' have previously been studied in [1]–[3]. In particular, [1] proposes a broadcasting approach over a single-antenna fading channel to communicate a stream of "fast" messages, which have to be sent over a single coherence block, and a stream of "slow" messages, which can be sent over multiple blocks. A similar approach was taken in [3] but for a broadcast scenario with K users. Instead of superposing "slow" on "fast" messages, this latter work proposes a scheduling approach to give preference to the communication of "fast" messages.

For simplicity, in this paper, we focus on Wyner's *soft-handoff* model [4]–[6] with K interfering transmitter and receiver pairs that are aligned on a line. Each transmitter sends a pair of independent source messages called "fast" and "slow" messages. Each receiver decodes the fast message immediately and only based on its own channel outputs. Before decoding its fast message, it can communicate with its immediate neighbours over conferencing links during a given maximum number of rounds [7] and subject to a rate-constraint. It then decodes the "slow" message based on its own channel outputs and the cooperation messages received from its neighbours. In the case of only transmitter-conferencing, receivers decode both messages based only on their own channel outputs, but transmitters can hold a conferencing communication that depends only on the "slow" messages but not on the "fast" messages. The "fast" messages here again model data that has to be sent to the receivers without additional delay.

We propose inner and outer bounds on the capacity region of the soft-hand network with receiver-conferencing. We also characterize the per-user multiplexing gain region of the setup with only transmitter-conferencing or only receiver conferencing for given conferencing prelogs and a given maximum decoding delay (number of conferencing rounds) of "slow" messages. The per-user multiplexing gain regions of the two scenarios coincide, and thus show a duality between transmitter- and receiver-conferencing in the high signal-to-noise ratio regime.

Our results also indicate that the sum-rate of "fast" and "slow" messages is approximately constant when "fast" messages are sent at small rate. In this regime, the stringent decoding delay of part of the messages does not cause a loss in overall performance. When "fast" messages have large rates, this is not the case. In this regime, increasing the rate of "fast" messages by Δ, requires that the rate of "slow" messages be reduced by approximately $2 \cdot \Delta$.

II. Problem Setup

Consider a wireless communication system as in Fig. 1 with K interfering transmitter (Tx) and receiver (Rx) pairs $1, \ldots, K$ that are aligned on a line. Transmitters and receivers are each equipped with a single antenna, and channel inputs and outputs are real valued. Interference is short-range so that the signal sent by Tx k is observed only by Rx k and Rx $k+1$. As a result, the time-t channel output at Rx k is

$$Y_{k,t} = X_{k,t} + \alpha X_{k-1,t} + Z_{k,t}, \tag{1}$$

where $X_{k,t}$ and $X_{k-1,t}$ are the symbols sent by Transmitter k and $k-1$ at time t, respectively; $\{Z_{k,t}\}$ are independent and identically distributed (i.i.d.) standard Gaussians for all k and t; $\alpha \neq 0$ is a fixed real number smaller than 1; and $X_{0,t} = 0$ for all t.

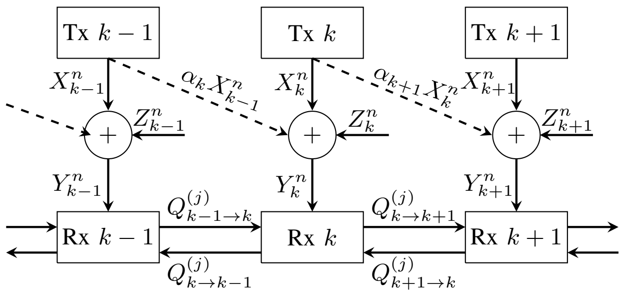

Fig. 1. System model

Each Tx k wishes to send a pair of independent source messages $M_k^{(F)}$ and $M_k^{(S)}$ to Rx k. The "fast" source message $M_k^{(F)}$ is uniformly distributed over the set $\mathcal{M}_k^{(F)} := \{1, \ldots, \lfloor 2^{nR_k^{(F)}} \rfloor\}$ and needs to be decoded subject to a stringent delay constraint, as we explain shortly. The "slow" source message $M_k^{(S)}$ is uniformly distributed over $\mathcal{M}_k^{(S)} := \{1, \ldots, \lfloor 2^{nR_k^{(S)}} \rfloor\}$ and is subject to a less stringent decoding delay constraint. Here, n denotes the blocklength of transmission and $R_k^{(F)}$ and $R_k^{(S)}$ are the rates of transmissions of the "fast" and the "slow" messages. All source messages are independent of each other and of all channel noises.

Tx k computes its channel inputs $X_k^n := (X_{k,1}, \ldots, X_{k,n})$ as a function of the pair $(M_k^{(F)}, M_k^{(S)})$:

$$X_k^n = f_k^{(n)}\big(M_k^{(F)}, M_k^{(S)}\big), \tag{2}$$

for some function $f_k^{(n)}$ on appropriate domains that satisfies the average block-power constraint

$$\frac{1}{n}\sum_{t=1}^{n} X_{k,t}^2 \leq P, \quad \text{a.s.}, \quad \forall\, k \in \{1, \ldots, K\}. \tag{3}$$

Receivers decode in two phases. During the first *fast-decoding phase*, each Rx k has access only to its own sequence of channel outputs $Y_k^n := (Y_{k,1}, \ldots, Y_{k,n})$, and it is required to decode the "fast" source message $M_k^{(F)}$. That means, Rx k produces the guess

$$\hat{M}_k^{(F)} = g_k^{(n)}\big(Y_k^n\big) \tag{4}$$

by means of a decoding function $g_k^{(n)}$ on appropriate domains.

In the subsequent *slow-decoding phase*, the receivers first communicate to each other over orthogonal conferencing links, and then they decode their intended "slow" messages based on their own channel outputs and the conferencing messages received from their neighbours. Only neighbouring receivers can exchange conferencing messages, and conferencing is limited to a maximum number of $\mathsf{D}_{\max}$ rounds and to rate-constraint π Specifically, in conferencing round $j \in \{1, 2, \ldots, \mathsf{D}_{\max}\}$, Rx k sends the conferencing message $Q_{k\to k-1}^{(j)}$ to its left neighbour, Rx $k-1$, and the conferencing message $Q_{k\to k+1}^{(j)}$ to its right neighbour, Rx $k+1$. These conferencing messages are computed based on the outputs Y_k^n and on the conferencing messages Rx k received in the previous $j-1$ rounds. So, for $\tilde{k} \in \{k-1, k+1\}$:

$$Q_{k\to\tilde{k}}^{(j)} := \psi_{k,\tilde{k}}^{(n)}\Big(Y_k^n, Q_{k-1\to k}^{(1)}, Q_{k+1\to k}^{(1)}, \ldots, Q_{k-1\to k}^{(j-1)}, Q_{k+1\to k}^{(j-1)}\Big), \tag{5}$$

for an encoding function $\psi_{k,\tilde{k}}^{(n)}$ on appropriate domains. The $\mathsf{D}_{\max}$ messages sent over a conferencing link in each direction are subject to a rate constraint π. So, for all $k \in \{1, \ldots, K\}$ and $\tilde{k} \in \{k, k+1\}$:

$$\sum_{j=1}^{\mathsf{D}_{\max}} H(Q_{k\to\tilde{k}}^{(j)}) \leq \pi \cdot n. \tag{6}$$

After the last conferencing round $\mathsf{D}_{\max}$, each Rx k decodes its desired "slow" message as

$$\hat{M}_k^{(S)} := b_k^{(n)}\Big(Y_k^n, Q_{k-1\to k}^{(1)}, Q_{k+1\to k}^{(1)}, \ldots, Q_{k-1\to k}^{(\mathsf{D}_{\max})}, Q_{k+1\to k}^{(\mathsf{D}_{\max})}\Big) \tag{7}$$

by means of a decoding function $b_k^{(n)}$ on appropriate domains.

The main interest in this paper is in the achievable sum-rates of "fast" and "slow" messages. Given a maximum conferencing rate π and a maximum allowed power P, the pair of (average) rates $(R^{(F)}, R^{(S)})$ is called *achievable*, if there exists a sequence (in n) of encoding and decoding functions so that

$$\frac{1}{K}\sum_{k=1}^{K} R_k^{(F)} = R^{(F)} \qquad \text{and} \qquad \frac{1}{K}\sum_{k=1}^{K} R_k^{(S)} = R^{(S)}, \tag{8}$$

and the probability of decoding error

$$P_e^{(n)} := \Pr\Bigg[\bigcup_{k\in\{1,\ldots,K\}} \Big\{\hat{M}_k^{(F)} \neq M_k^{(F)} \text{ or } \hat{M}_k^{(S)} \neq M_k^{(S)}\Big\}\Bigg] \tag{9}$$

tends to 0 as $n \to \infty$.

Definition 1: Given power constraint $P > 0$ and maximum conferencing rate π, the *capacity region* $\mathsf{C}(P, \pi)$ is the closure of the set of all rate pairs $(R^{(F)}, R^{(S)})$ that are achievable.

We will particularly be interested in the high signal-to-noise ratio (SNR) behaviour of the capacity-region. Our focus is on the set of achievable *per-user multiplexing gains* when the conferencing capacity also scales logarithmically in the SNR. Given a conferencing prelog $\mu \geq 0$, the *pair of per-user multiplexing gains* $(\mathsf{S}^{(F)}, \mathsf{S}^{(S)})$ is called *achievable*, if for each K there exists a sequence of rates $\{R_K^{(F)}(P), R_K^{(S)}(P)\}_{P>0}$ so that

$$\mathsf{S}^{(F)} := \varlimsup_{K\to\infty} \varlimsup_{P\to\infty} \frac{R_K^{(F)}}{\frac{1}{2}\log(1+P)}, \tag{10}$$

$$\mathsf{S}^{(S)} := \varlimsup_{K\to\infty} \varlimsup_{P\to\infty} \frac{R_K^{(S)}}{\frac{1}{2}\log(1+P)}, \tag{11}$$

and for each K and $P > 0$ the pair $(R_K^{(F)}(P), R_K^{(S)}(P))$ is achievable with conferencing rate at most $\pi = \mu \cdot \frac{1}{2}\log P$.

Definition 2: Given a conferencing-prelog μ, the closure of the set of all achievable per-user multiplexing gains $(\mathsf{S}^{(F)}, \mathsf{S}^{(S)})$ is called *multiplexing gain region* and denoted $\mathcal{S}^\star(\mu)$.

III. Main Results

Our first result is an inner bound on the capacity region. It is based on two schemes, the first for the case $\pi < R^{(F)}$ and the second for the case $\pi > R^{(F)}$. In the first scheme only parts of "fast" source messages are exchanged over the conferencing links. In the second scheme also parts of "slow" source messages are exchanged.

Theorem 1 (Capacity Inner Bound): The capacity region $\mathsf{C}(P,\pi)$ includes all rate-pairs $(R^{(F)}, R^{(S)})$ that satisfy

$$R^{(F)} \leq \min\left\{I(U_2;Y), I(U_2;Y|U_1)+\pi\right\} \tag{12a}$$

and

$$R^{(F)} + R^{(S)} \leq \frac{1}{K}\sum_{k=1}^{K}\Big[I(X;Y,U_1'|U_1) + \min\left\{I(U_2;Y), I(U_2;Y|U_1)+\pi\right\}\Big], \tag{12b}$$

where triples (U_1, U_2, X) and (U_1', U_2', X') are i.i.d. according to some probability distribution $P_{U_1U_2X}$ that satisfying the Markov chain $U_1 \to U_2 \to X$, and where $Y = X + \alpha X' + Z$ with Z standard Gaussian independent of $(U_1, U_2, X, U_1', U_2', X')$.

The capacity region $\mathsf{C}(P,\pi)$ also includes all rate-pairs $(R^{(F)}, R^{(S)})$ that satisfy

$$R^{(F)} \leq I(U;Y) \tag{13a}$$

$$R^{(F)} + R^{(S)} \leq I(U;Y) + I(V_1;Y,U'|U) + \sum_{d=2}^{\mathsf{D}_{\max}-1} I(V_d;Y,V_{d-1}'|V_{d-1}) + I(X;Y,X'|V_{\mathsf{D}_{\max}-1}), \tag{13b}$$

where the tuples $(U, V_1, \ldots, V_{\mathsf{D}_{\max}-1}, X)$ and $(U', V_1', \ldots, V_{\mathsf{D}_{\max}-1}', X')$ are i.i.d. according to some probability distribution $P_{UV_1\ldots V_{\mathsf{D}_{\max}-1}X}$ satisfying the Markov chain $U \to V_1 \to V_2 \to \ldots \to V_{\mathsf{D}_{\max}-1} \to X$ and the rate constraint

$$I(U;Y) + I(V_1;Y,U'|U) + \sum_{d=2}^{\mathsf{D}_{\max}-1} I(V_d;Y,V_{d-1}'|V_{d-1}) \leq \pi, \tag{14}$$

and where $Y = X + \alpha X' + Z$ with Z independent standard Gaussian.

Proof: See Section V. ∎

Theorem 2 (Capacity Outer Bound): Any achievable rate pair $(R^{(F)}, R^{(S)})$ satisfies the following two conditions:

$$R^{(F)} + R^{(S)} \leq \frac{\left(\left\lceil\frac{K-1}{2}\right\rceil+1\right)}{K}\cdot\frac{1}{2}\log\left(1+(1+\alpha^2)P\right) + \frac{\left\lfloor\frac{K-1}{2}\right\rfloor}{K}\cdot\max\{-\log|\alpha|,0\} + \frac{\left\lfloor\frac{K}{2}\right\rfloor}{K}\cdot\frac{1}{2}\log(1+\alpha^2) + \frac{K-1}{K}\cdot\pi, \tag{15}$$

$$2R^{(F)} + R^{(S)} \leq \frac{K-1}{2K}\left(\frac{1}{2}\log\left((1+(1+\alpha^2)P)(1+\alpha^2)\right) + 2\max\{-\log|\alpha|,0\}\right) + \frac{1}{K}\log(1+P). \tag{16}$$

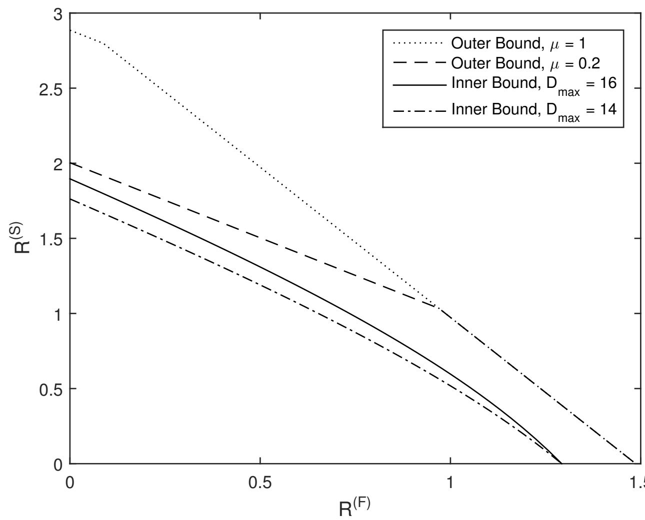

Fig. 2. Capacity outer bound in Theorem 2 and inner bound in Theorem 1 for $\alpha = 0.2$.

Proof: See Section VI. ∎

Figs. 2 illustrates the outer bound on the capacity-region in Theorem 2 and the inner bound in Theorem 1 when this latter is evaluated for jointly Gaussian distributions on the inputs and the auxiliaries.

Theorem 3 (Per-User Multiplexing Gain): Given conferencing-prelog $\mu \geq 0$, the multiplexing gain region $\mathcal{S}^\star(\mu)$ is the set of all nonnegative pairs $(\mathsf{S}^{(F)}, \mathsf{S}^{(S)})$ satisfying

$$2\mathsf{S}^{(F)} + \mathsf{S}^{(S)} \leq 1 \tag{17}$$

$$\mathsf{S}^{(F)} + \mathsf{S}^{(S)} \leq \min\left\{\frac{1}{2}+\mu,\ \frac{2\mathsf{D}_{\max}+1}{2\mathsf{D}_{\max}+2}\right\}. \tag{18}$$

Remark 1: An analogous result can be obtained for the slightly modified setup where each receiver can send conferencing messages only to its left-neighbour or only to its right-neighbour. The same proof techniques apply, and it can be shown that the per-user multiplexing gain region is characterized by (17) and by

$$\mathsf{S}^{(F)} + \mathsf{S}^{(S)} \leq \min\left\{\frac{1}{2}+\frac{\mu}{2},\ \frac{\mathsf{D}_{\max}+1}{\mathsf{D}_{\max}+2}\right\}. \tag{19}$$

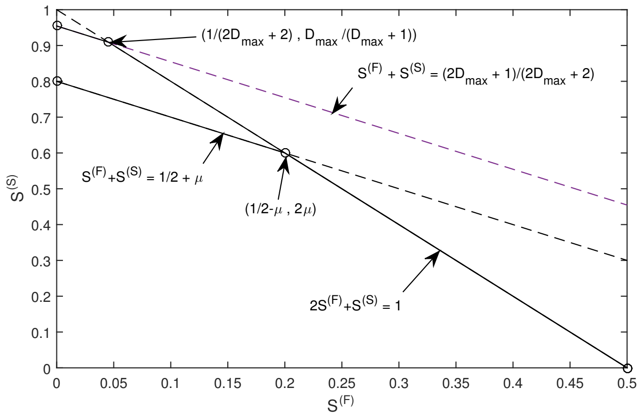

Fig. 3. Multiplexing gain region $\mathcal{S}^\star(\mu)$ for $\mathsf{D}_{\max} = 10$ and $\mu = 0.3$.

Notice that despite the asymmetry of the network, the result is the same for conferencing to left-neighbours and to right-neighbours.

IV. Transmitter-Conferencing

We consider a related setup where the transmitter can send conferencing messages but not the receivers. Transmitter conferencing is limited to $\mathsf{D}_{\max}$ rounds and the exchanged messages can only depend on the "slow" messages but not on the fast messages. This models a setup where the transmitters learn the "slow" messages in advance before they communicate to the receivers, whereas "fast" messages arrive at the transmitters just shortly before this communication.

Specifically, in each round $j \in \{1, \ldots, \mathsf{D}_{\max}\}$, Tx k produces the two conferencing messages $T_{k\to k-1}^{(j)}$ and $T_{k\to k+1}^{(j)}$, where

$$T_{k\to\tilde{k}}^{(j)} = \xi_{k\to\tilde{k}}^{(n)}\big(M_k^{(S)}, T_{k-1\to k}^{(1)}, \ldots, T_{k-1\to k}^{(j-1)}, T_{k+1\to k}^{(1)}, \ldots, T_{k+1\to k}^{(j-1)}\big) \quad (20)$$

for some function $\xi_{k\to\tilde{k}}^{(n)}$ on appropriate domains. It sends these messages over the conferencing links to its left and right neighbours. As before, the conferencing links are rate-limited to rate π. So, for all $k \in \{1, \ldots, K\}$ and $\tilde{k} \in \{k-1, k+1\}$:

$$\sum_{j=1}^{\mathsf{D}_{\max}} H(T_{k\to\tilde{k}}^{(j)}) \leq \pi \cdot n. \quad (21)$$

Each Tx k then computes its channel inputs as

$$X_k^n = \tilde{f}_k^{(n)}\big(M_k^{(F)}, M_k^{(S)}, T_{k-1\to k}^{(1)}, \ldots, T_{k-1\to k}^{\mathsf{D}_{\max}}, T_{k+1\to k}^{(1)}, \ldots, T_{k+1\to k}^{\mathsf{D}_{\max}}\big) \quad (22)$$

subject to the power constraint in (3).

Each Rx k decodes the two messages $M_k^{(F)}, M_k^{(S)}$ only based on its channel outputs Y_k^n:

$$\big(\hat{M}_k^{(F)}, \hat{M}_k^{(S)}\big) = \tilde{g}_k^{(n)}\big(Y_k^n\big). \quad (23)$$

Capacity region and per-user multiplexing gain region $\tilde{\mathcal{S}}^\star(\mu)$ are defined analogously as before.

Theorem 4 (Only Transmitter-Conferencing): Given $\mu \geq 0$, the per-user multiplexing gain region $\tilde{\mathcal{S}}^\star(\mu)$ is the set of all nonnegative pairs $(\mathsf{S}^{(F)}, \mathsf{S}^{(S)})$ that satisfy

$$2\mathsf{S}^{(F)} + \mathsf{S}^{(S)} \leq 1 \quad (24)$$

$$\mathsf{S}^{(F)} + \mathsf{S}^{(S)} \leq \min\left\{\frac{1}{2} + \mu, \ \frac{2\mathsf{D}_{\max}+1}{2\mathsf{D}_{\max}+2}\right\}. \quad (25)$$

Proof: Similar to the proof of Theorem 3. See Section **??**. ■

Remark 2: Our results exhibit a duality between transmitter- and receiver-conferencing. They yield the same per-user multiplexing gain region.

V. Proof of Achievability of Theorem 1

We present two coding schemes. Our first scheme achieves the rate-pairs in (12), and receivers conference only parts of decoded "fast" messages. A single-round of conferencing suffices, and thus also "slow" message are decoded with only two units of delay. The second scheme achieves the rate-pairs in (13), and receivers conference also parts of "slow" messages. In general, here, $\mathsf{D}_{\max}$ conferencing rounds are employed and "slow" messages are decoded with maximum allowed delay $\mathsf{D}_{\max}$.

A. Scheme 1: Conferencing only parts of "fast"-messages

Fix a small number $\epsilon > 0$ and a joint distribution $P_{U_1U_2X}$ that satisfies the Markov chain $U_1 \to U_2 \to X$. Let (U_1', U_2', X') be an independent copy of (U_1, U_2, X) and define

$$Y = X + \alpha X' + Z, \quad (26)$$

where Z is standard Gaussian independent of all other defined random variables.

Split each source message into two parts, $M_k^{(F)} = \big(M_k^{(F_1)}, M_k^{(F_2)}\big)$, of rates $\big(R_k^{(F_1)}, R_k^{(F_2)}\big)$ that sum up to $R_k^{(F)} = R_k^{(F_1)} + R_k^{(F_2)}$ and so that

$$R_k^{(F_1)} < \pi. \quad (27)$$

<u>Codebook construction:</u> For each $k \in \{1, \ldots, K\}$, generate codebooks $\mathcal{C}_{1,k}$, $\{\mathcal{C}_{2,k}(i)\}$, and $\{\mathcal{C}_{x,k}(i,j)\}$ randomly. Codebook

$$\mathcal{C}_{1,k} := \left\{u_{1,k}^n(i)\colon \ \ i = 1, \ldots, \left\lfloor 2^{nR_k^{(F_1)}} \right\rfloor\right\} \quad (28)$$

is generated by picking all entries i.i.d. according to P_{U_1}. For each $i \in \big\{1, \ldots, \lfloor 2^{nR_k^{(F_1)}} \rfloor\big\}$, codebook

$$\mathcal{C}_{2,k}(i) := \left\{u_{2,k}^n(j|i)\colon \ \ j = 1, \ldots, \left\lfloor 2^{nR_k^{(F_2)}} \right\rfloor\right\} \quad (29)$$

is generated by picking the t-th entry of codeword $u_{2,k}^n(j|i)$ independently of all other entries and codewords according to the distribution $P_{U_2|U_1}(\cdot|u_{1,k,t}(i))$. Here, $u_{1,k,t}(i)$ denotes the t-th entry of codeword $u_{1,k}^n(i)$. For each pair (i,j) in $\big\{1, \ldots, \lfloor 2^{nR_k^{(F_1)}} \rfloor\big\} \times \big\{1, \ldots, \lfloor 2^{nR_k^{(F_2)}} \rfloor\big\}$, codebook

$$\mathcal{C}_{x,k}(i,j) := \left\{x_k^n(\ell|i,j)\colon \ \ \ell = 1, \ldots, \left\lfloor 2^{nR_k^{(S)}} \right\rfloor\right\} \quad (30)$$

is generated by picking the t-th entry of codeword $x_k^n(\ell|i,j)$ independently of all other entries according to the distribution $P_{X|U_2}(\cdot|u_{2,k,t}(j|i))$. Here, $u_{2,k,t}(j|i)$ denotes the t-th entry of codeword $u_{2,k}^n(j|i)$.

Reveal all codebooks to all terminals.

Encoding: Tx k sends codeword $x_k^n\big(M_k^{(S)}\big|M_k^{(F_1)}, M_k^{(F_2)}\big)$ over the channel.

Decoding: Each Rx k performs the following steps. Given that it observes $Y_k^n = y_k^n$, it first looks for a unique pair $(\hat{i},\hat{j})$ such that

$$(u_{1,k}^n(\hat{i}), u_{2,k}^n(\hat{j}|\hat{i}), y_k^n) \in \mathcal{T}_\epsilon^{(n)}(P_{U_1U_2Y}). \tag{31}$$

If none or more than one such pair $(\hat{i},\hat{j})$ exists, Rx k declares an error. Otherwise, it declares $\hat{M}_k^{(F)} = (\hat{i},\hat{j})$, and it sends

$$Q_{k\to k+1}^{(1)} = \hat{i}. \tag{32}$$

to its right neighbour, Rx $k+1$.

With the message $Q_{k-1\to k}^{(1)}$, Rx k obtains from its left-neighbour, it decodes also its intended "slow" message. To this end, it looks for an index $\hat{\ell}$ such that

$$\Big(u_{1,k}^n(\hat{i}), u_{2,k}^n(\hat{j}|\hat{i}), x_k^n(\hat{\ell}|\hat{i},\hat{j}),\\ u_{k-1}^n(Q_{k-1\to k}^{(1)}), y_k^n\Big) \in \mathcal{T}_\epsilon^{(n)}(P_{U_1U_2XU_1'Y}), \tag{33}$$

If none or multiple such indices $\hat{\ell}$ exist, an error is declared. Otherwise, Rx k declares $\hat{M}_k^{(S)} = \hat{\ell}$.

Analysis: Decoding in (31) is successful with probability tending to 1 as $n\to\infty$, if

$$R_k^{(F_1)} + R_k^{(F_2)} < I(U_2;Y) \tag{34}$$

$$R_k^{(F_2)} < I(U_2;Y|U_1). \tag{35}$$

Decoding in (33) is successful with probability tending to 1 as $n\to\infty$, if

$$R_k^{(S)} < I(X;Y,U_1'|U_1). \tag{36}$$

The conferencing constraint is satisfied by (27). Apply then Fourier-Motzkin elimination to (27), (34) and (35). Achievability of the pairs (12) follows then by a rate-transfer argument noting that parts of the "slow" messages can also be sent as "fast" messages.

B. Scheme 2: Conferencing also parts of "slow"-messages

to do

VI. Proof of Theorem 2

For convenience of notation, define for any $k\in\{1,\dots,K\}$:

$$M_k := (M_k^{(F)}, M_k^{(S)}). \tag{37}$$

We first prove Inequality (15). By Fano's Inequality and the independence of the messages, we have for any $k \in \{1,\dots,K-1\}$:

$$\begin{aligned}
&R_k^{(F)} + R_k^{(S)} + R_{k+1}^{(F)}\\
&= \frac{1}{n}\Big[H(M_k^{(F)}) + H(M_k^{(S)}) + H(M_{k+1}^{(F)})\Big]\\
&= \frac{1}{n}\Big[H(M_k^{(F)}|M_{k-1})\\
&\qquad +H(M_k^{(S)}|M_1,\dots,M_{k-1},M_k^{(F)},M_{k+1},\dots,M_K)\\
&\qquad\qquad +H(M_{k+1}^{(F)}|M_{k-1},M_{k+1}^{(S)})\Big]\\
&\le \frac{1}{n}\Big[I(M_k^{(F)};Y_k^n|M_{k-1})\\
&\qquad +I(M_k^{(S)};Y_1^n,\dots,Y_K^n|M_1,\dots,M_{k-1},M_k^{(F)}\\
&\qquad\qquad ,M_{k+1},\dots,M_K)\\
&\qquad\qquad +I(M_{k+1}^{(F)};Y_{k+1}^n|M_{k-1},M_{k+1}^{(S)})\Big] + \frac{\epsilon_n}{n}\\
&\overset{(a)}{=} \frac{1}{n}\Big[I(M_k^{(F)};Y_k^n|M_{k-1})\\
&\qquad +I(M_k^{(S)};Y_k^n,Y_{k+1}^n|M_{k-1},M_k^{(F)},M_{k+1})\\
&\qquad\qquad +I(M_{k+1}^{(F)};Y_{k+1}^n|M_{k-1},M_{k+1}^{(S)})\Big] + \frac{\epsilon_n}{n}\\
&\overset{(b)}{=} \frac{1}{n}\Big[I(M_k^{(F)},M_k^{(S)};Y_k^n|M_{k-1})\\
&\qquad +I(M_k^{(S)};Y_{k+1}^n|Y_k^n,M_k^{(F)},M_{k-1},M_{k+1})\\
&\qquad\qquad +I(M_{k+1}^{(F)};Y_{k+1}^n|M_{k-1},M_{k+1}^{(S)})\Big] + \frac{\epsilon_n}{n}\\
&\le \frac{1}{n}\Big[h(X_k^n+Z_k^n) - h(Z_k^n) + h(\alpha X_k^n + Z_{k+1}^n|X_k^n+Z_k^n)\\
&\qquad -h(Z_{k+1}^n) + h(Y_{k+1}^n|M_{k+1}^{(S)}) - h(\alpha X_k^n + Z_{k+1}^n)\Big]\\
&\quad +\frac{\epsilon_n}{n}\\
&\overset{(c)}{\le} \frac{1}{2}\log(1+(1+|\alpha|^2)P) + \frac{1}{2}\log(1+\alpha^2)\\
&\quad +\max\{-\log|\alpha|, \log(\alpha^2-1)\} + \frac{\epsilon_n}{n}.
\end{aligned} \tag{38}$$

Here, (a) follows because given source messages M_{k-1} and M_{k+1}, the triple (M_k, Y_k^n, Y_{k+1}^n) is independent of the rest of the outputs $Y_1^n,\dots,Y_{k-1}^n,Y_{k+2}^n,\dots,Y_K^n$ and source messages $M_1,\dots,M_{k-2},M_{k+2},\dots,M_K$; (b) follows by the chain rule of mutual information and because M_{k+1} is independent of the tuple (M_{k-1},M_k,Y_k^n); (c) is obtained by rearranging terms, and the following bounds (39)–(42). In fact, because conditioning can only reduce entropy, and by the entropy-maximizing property of the Gaussian distribution,

$$\begin{aligned}
h(Y_{k+1}^n|M_{k+1}^{(S)}) &\le h(Y_{k+1}^n)\\
&\le \frac{1}{2}\log((2\pi e)(1+(1+|\alpha|^2)P)).
\end{aligned} \tag{39}$$

Moreover,

$$\begin{aligned}
h(\alpha X_k^n + Z_{k+1}^n|X_k^n+Z_k^n) &= h(Z_{k+1}^n - \alpha Z_k^n|X_k^n+Z_k^n)\\
&\le h(Z_{k+1}^n - \alpha Z_k^n)\\
&= \frac{1}{2}\log((2\pi e)(1+\alpha^2)).
\end{aligned} \tag{40}$$

For the next bound, define T_k^n i.i.d. zero-mean Gaussian independent of all other random variables and with a variance that depends on α. If $\alpha<1$, the variance is $\frac{1}{\alpha^2}-1$. In this case,

$\frac{1}{\alpha}Z_{k+1}^n$ has the same joint distribution with all other random variables as $Z_k^n + T_k^n$ and

$$\begin{aligned} &h(X_k^n + Z_k^n) - h(\alpha X_k^n + Z_{k+1}^n) \\ &\quad = h(X_k^n + Z_k^n) - h(X_k^n + \frac{1}{\alpha}Z_{k+1}^n) - \log|\alpha| \\ &\quad = h(X_k^n + Z_k^n) - h(X_k^n + Z_k^n + T_k^n) - \log|\alpha| \\ &\quad \leq -\log|\alpha|. \end{aligned} \tag{41}$$

If $\alpha \geq 1$, then each symbol of T_k^n has variance $1 - \frac{1}{\alpha^2}$. In this case, Z_k^n has the same joint distribution with all other random variables as $\frac{1}{\alpha}Z_{k+1}^n + T_k^n$. Thus, similarly to before:

$$\begin{aligned} &h(X_k^n + Z_k^n) - h(\alpha X_k^n + Z_{k+1}^n) \\ &= h(X_k^n + \frac{1}{\alpha}Z_{k+1}^n + T_k^n) - h(\alpha X_k^n + Z_{k+1}^n) \\ &= h(X_k^n + \frac{1}{\alpha}Z_{k+1}^n + T_k^n) - h(X_k^n + \frac{1}{\alpha}Z_{k+1}^n | T_k^n) - \log|\alpha| \\ &\leq I(X_k^n + \frac{1}{\alpha}Z_{k+1}^n + T_k^n; T_k^n) - \log|\alpha| \\ &\leq I(\frac{1}{\alpha}Z_{k+1}^n + T_k^n; T_k^n) - \log|\alpha| \\ &= \frac{1}{2}\log\left(\frac{1}{1/\alpha^2}\right) - \log|\alpha| = 0. \end{aligned} \tag{42}$$

Following similar steps, one can also prove that

$$\begin{aligned} R_K^{(F)} + R_K^{(S)} &\leq \frac{1}{n}I(M_k^{(F)}, M_k^{(S)}; Y_k^n | M_{k-1}) + \frac{\epsilon_n}{n} \\ &\leq \frac{1}{2}\log(1+P) + \frac{\epsilon_n}{n}. \end{aligned} \tag{43}$$

We sum up the bound in (38) for all values of $k \in \{1, \ldots, K-1\}$, and combine it with (43). Taking $n \to \infty$, it follows that whenever the probability of error $P_e^{(n)}$ vanishes as $n \to \infty$ (and thus $\frac{\epsilon_n}{n} \to 0$ as $n \to \infty$):

$$\begin{aligned} &\sum_{k=1}^{K}\left(2R_k^{(F)} + R_k^{(S)}\right) \\ &= R_1^{(F)} + \sum_{k=1}^{K-1}\left(R_k^{(F)} + R_k^{(S)} + R_{k+1}^{(F)}\right) + R_K^{(F)} + R_K^{(S)} \\ &\leq (K-1)\frac{1}{2}\log(1+(1+\alpha^2)P) + \log(1+P) \\ &\quad + \frac{K-1}{2}\log(1+\alpha^2) + (K-1)\max\{-\log|\alpha|, 0\}, \end{aligned} \tag{44}$$

We now prove bound (16). We assume K is even. For K odd the bound can be proved in a similar way. Define (recall that $X_0^n = 0$)

$$\begin{aligned} \mathbf{M}_{\text{odd}} &:= \{M_k: \ k \text{ odd}\} \\ \mathbf{M}_{\text{even}} &:= \{M_k: \ k \text{ even}\} \\ \mathbf{X}_{\text{odd}}^n &:= \{X_k^n: k \text{ odd}\} \\ \mathbf{X}_{\text{even}}^n &:= \{X_k^n: \ k \text{ odd}\} \\ \mathbf{Y}_{\text{odd}}^n &:= \{Y_k^n: k \text{ odd}\} \\ \mathbf{Y}_{\text{even}}^n &:= \{Y_k^n: k \text{ even}\} \\ \mathbf{Z}_{\text{odd}}^n &:= \{Z_k^n: k \text{ odd}\} \\ \mathbf{Z}_{\text{even}}^n &:= \{Z_k^n: k \text{ even}\} \end{aligned}$$

and

$$\begin{aligned} \mathbf{Q}_{\text{odd}} &:= \left\{Q_{k\to\tilde{k}}^{(1)}, \ldots, Q_{k\to\tilde{k}}^{(\mathsf{D}_{\max})}: \ k \text{ odd}, \ \tilde{k} \in \{k-1, k+1\}\right\} \\ \mathbf{Q}_{\text{even}} &:= \left\{Q_{k\to\tilde{k}}^{(1)}, \ldots, Q_{k\to\tilde{k}}^{(\mathsf{D}_{\max})}: \ k \text{ even}, \ \tilde{k} \in \{k-1, k+1\}\right\}. \end{aligned}$$

By Fano's inequality, there must exist a sequence $\{\epsilon_n\}_{n=1}^{\infty}$ so that $\frac{\epsilon_n}{n} \to 0$ as $n \to \infty$ and

$$\begin{aligned} &\sum_{k=1}^{K}\left(R_k^{(F)} + R_k^{(S)}\right) \\ &= \frac{1}{n}\left[H(\mathbf{M}_{\text{odd}}) + H(\mathbf{M}_{\text{even}})\right] \\ &\leq \frac{1}{n}\left[I(\mathbf{M}_{\text{odd}}; \mathbf{Y}_{\text{odd}}, \mathbf{Q}_{\text{odd}}) + I(\mathbf{M}_{\text{even}}; \mathbf{Y}_{\text{even}}, \mathbf{Q}_{\text{even}} | \mathbf{M}_{\text{odd}})\right] \\ &\quad + \frac{\epsilon_n}{n} \\ &= \frac{1}{n}\left[I(\mathbf{M}_{\text{odd}}; \mathbf{Y}_{\text{odd}}) + I(\mathbf{M}_{\text{even}}; \mathbf{Y}_{\text{even}} | \mathbf{M}_{\text{odd}})\right. \\ &\qquad \left. + I(\mathbf{M}_{\text{odd}}; \mathbf{Q}_{\text{odd}} | \mathbf{Y}_{\text{odd}}) + I(\mathbf{M}_{\text{even}}; \mathbf{Q}_{\text{even}} | \mathbf{M}_{\text{odd}}, \mathbf{Y}_{\text{even}})\right] \\ &\quad + \frac{\epsilon_n}{n} \\ &\leq \frac{1}{n}\left[h(\mathbf{Y}_{\text{odd}}) - h(\mathbf{Y}_{\text{odd}} | \mathbf{M}_{\text{odd}}) + h(\mathbf{Y}_{\text{even}} | \mathbf{M}_{\text{odd}}) - h(\mathbf{Z}_{\text{even}})\right] \\ &\qquad \left. + H(\mathbf{Q}_{\text{odd}}) + I(\mathbf{M}_{\text{even}}; \mathbf{Q}_{\text{even}} | \mathbf{M}_{\text{odd}}, \mathbf{Y}_{\text{even}})\right] + \frac{\epsilon_n}{n} \\ &\leq \frac{1}{n}\left[h(\mathbf{Y}_{\text{odd}}) - h(\mathbf{Y}_{\text{odd}} | \mathbf{M}_{\text{odd}})\right. \\ &\qquad + h(\mathbf{Y}_{\text{even}} | \mathbf{M}_{\text{odd}}) - h(\mathbf{Z}_{\text{even}}) + H(\mathbf{Q}_{\text{odd}}) \\ &\qquad + I(\mathbf{M}_{\text{even}}; \mathbf{Q}_{\text{even}} | \mathbf{M}_{\text{odd}}, \mathbf{Y}_{\text{even}}, \mathbf{Z}_{\text{even}} - \alpha^{-1}\mathbf{Z}_{\text{odd}}) \\ &\qquad \left. + I(\mathbf{M}_{\text{even}}; \mathbf{Z}_{\text{even}} - \alpha^{-1}\mathbf{Z}_{\text{odd}} | \mathbf{M}_{\text{odd}}, \mathbf{Y}_{\text{even}})\right] \\ &+ \frac{\epsilon_n}{n} \\ &\stackrel{(a)}{\leq} \left(\frac{K}{2} + 1\right) \cdot \frac{1}{2}\log(1 + (1+\alpha^2)P) \\ &\quad + \frac{K-2}{2} \cdot \max\{-\log|\alpha|, 0\} \\ &\quad + \pi K + \frac{K}{2}\frac{1}{2}\log(1+\alpha^2) + \frac{\epsilon_n}{n}, \end{aligned} \tag{45}$$

where we where (a) holds because:

- By the entropy maximizing property of the Gaussian distribution:
$$h(\mathbf{Y}_{\text{odd}}) - h(\mathbf{Z}_{\text{even}}) \leq n\frac{K}{2} \cdot \frac{1}{2}\log(1 + (1+\alpha^2)P); \tag{46}$$
- By (41) and (42):
$$\begin{aligned} &h(\mathbf{Y}_{\text{odd}} | \mathbf{M}_{\text{odd}}) - h(\mathbf{Y}_{\text{even}} | \mathbf{M}_{\text{odd}}) \\ &\quad = h(Y_K^n | M_{k-1}) - h(Y_1^n | M_1) \\ &\qquad + \sum_{i=1}^{K/2-1}\left[h(X_{2i}^n + Z_{2i}^n) - h(\alpha X_{2i}^n + Z_{2i+1}^n)\right] \\ &\quad \leq n\frac{1}{2}\log(1 + (1+\alpha^2)P) \\ &\qquad + n\left(\frac{K}{2} - 1\right)\max\{-\log|\alpha|, 0\}; \end{aligned} \tag{47}$$

- By the rate-limitation of the conferencing links:

$$H(\mathbf{Q}_{\text{odd}}) \leq n\pi K; \tag{48}$$

- From the tuple $(\mathbf{M}_{\text{odd}}, \mathbf{Y}_{\text{even}}, \mathbf{Z}_{\text{even}} - \alpha^{-1}\mathbf{Z}_{\text{odd}})$ it is possible to compute also $\mathbf{Y}_{\text{odd}}$ and thus $\mathbf{Q}_{\text{even}}$:

$$I(\mathbf{M}_{\text{even}}; \mathbf{Q}_{\text{even}}|\mathbf{M}_{\text{odd}}, \mathbf{Y}_{\text{even}}, \mathbf{Z}_{\text{even}} - \alpha^{-1}\mathbf{Z}_{\text{odd}}) = 0; \tag{49}$$

- By the fact that conditioning reduces entropy:

$$\begin{aligned} &I(\mathbf{M}_{\text{even}}; \mathbf{Z}_{\text{even}} - \alpha^{-1}\mathbf{Z}_{\text{odd}}|\mathbf{M}_{\text{odd}}, \mathbf{Y}_{\text{even}}) \\ &\leq h(\mathbf{Z}_{\text{even}} - \alpha^{-1}\mathbf{Z}_{\text{odd}}) - h(\mathbf{Z}_{\text{even}} - \alpha^{-1}\mathbf{Z}_{\text{odd}}|\mathbf{Z}_{\text{even}}) \\ &= n\frac{K}{2} \cdot \frac{1}{2}\log(1+\alpha^2) \end{aligned} \tag{50}$$

Taking $n \to \infty$ establishes the proof.

VII. Proof of Theorem 3

The converse follows directly from Theorem 2. Achievability is proved in the following.

When transmitting only "fast" messages or only "slow" messages, the setup in this paper coincides with the setup in [7] with 0 tranmitter conferencing rounds and either 0 or $\mathsf{D}_{\max}$ receiver-conferencing rounds. Thus, by [7], the multiplexing gain pairs

$$\left(\mathsf{S}^{(F)} = \frac{1}{2},\ \mathsf{S}^{(S)} = 0\right) \tag{51}$$

and

$$\left(\mathsf{S}^{(F)} = 0,\ \mathsf{S}^{(S)} = \min\left\{\frac{1}{2} + \mu,\ \ \frac{2\mathsf{D}_{\max}+1}{2\mathsf{D}_{\max}+2}\right\}\right) \tag{52}$$

are achievable.

Recall that in the case of only receiver-conferencing, the coding scheme in [7] periodically silences every $2\mathsf{D}_{\max}+2$nd transmitter. This splits the network into smaller subnets of $2\mathsf{D}_{\max}+1$ active transmitters and $2\mathsf{D}_{\max}+2$ receivers, where each active transmitter can send a message at prelog 1. A close inspection of the coding scheme in [7] reveals that the decoding of the source messages sent at the left-most transmitter of each subnetwork does not rely on the conferencing messages. We can thus easily adopt the coding scheme in [7] to our setup with "fast" and "slow" messages by letting the left-most transmitter of any subnetwork only send a "fast" message, and letting all other active transmitters only send "slow" messages. With conferencing prelog

$$\mu_{\max} = \frac{\mathsf{D}_{\max}}{2\mathsf{D}_{\max}+2} \tag{53}$$

this scheme achieves the pair

$$\left(\mathsf{S}^{(F)} = \frac{1}{2\mathsf{D}_{\max}+2},\ \mathsf{S}^{(S)} = \frac{2\mathsf{D}_{\max}}{2\mathsf{D}_{\max}+2}\right). \tag{54}$$

For a given conferencing prelog $\mu \leq \mu_{\max}$, we timeshare this scheme with the scheme achieving (51). Choosing the timesharing parameter

$$\beta := \frac{\mu}{\mu_{\max}} = \mu\frac{2\mathsf{D}_{\max}+2}{\mathsf{D}_{\max}} \tag{55}$$

ensures that the conferencing prelog constraint (21) is satisfied. We obtain that for all $\mu \leq \mu_{\max}$, the following pair of multiplexing gains is achievable:

$$\begin{aligned} \mathsf{S}^{(F)} &:= \beta \cdot \frac{1}{2\mathsf{D}_{\max}+2} + (1-\beta)\cdot\frac{1}{2} \\ &= \frac{1}{2} - \beta\cdot\frac{\mathsf{D}_{\max}}{2\mathsf{D}_{\max}+2} = \frac{1}{2} - \mu \end{aligned} \tag{56a}$$

$$\mathsf{S}^{(S)} := \beta\cdot\frac{2\mathsf{D}_{\max}}{2\mathsf{D}_{\max}+2} + (1-\beta)\cdot 0 = 2\mu. \tag{56b}$$

Timesharing finally the schemes achieving the pairs in (51), (52), (54) establishes the direct part of the theorem.

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