Constrained intra-cell and inter-cell cooperation in cellular networks

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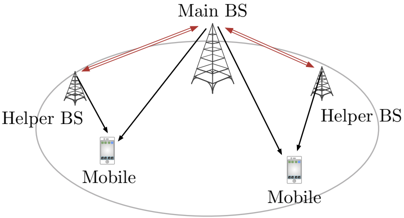
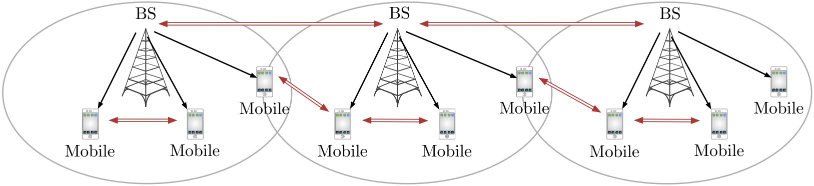
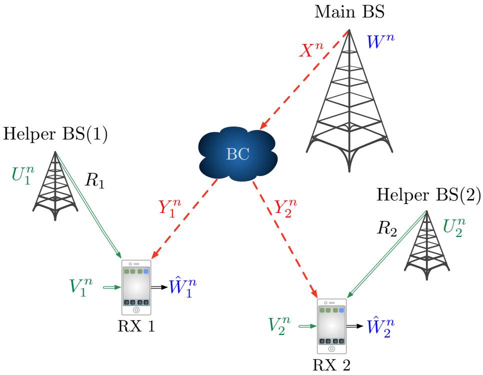


BS-BS cooperation inside a cell



BS-BS or mobile-mobile cooperation across cells

BS-to-BS cooperation inside a cell: downlink



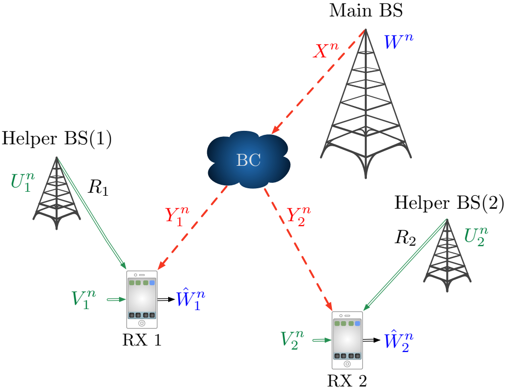
(Almost) lossless joint source-channel coding

Reliable communications for rates (R_1, R_2) possible, if $\exists$ encodings and decodings s.t.

$$\mathbb{P}\Big[\{\hat{W}_1^n \neq W^n\} \cup \{\hat{W}_2^n \neq W^n\}\Big] \longrightarrow 0$$

[1] R. Timo and M. Wigger, "Slepian-Wolf coding for broadcasting with cooperative basestations," *IEEE Trans. Communications*, 2015.

Two scenarios for helper side-informations U_1^n and U_2^n



Scenario 1 : Scalar quantisation

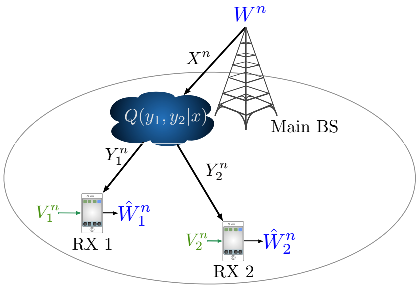
$$U_{k,t} = \phi_k(X_t)$$

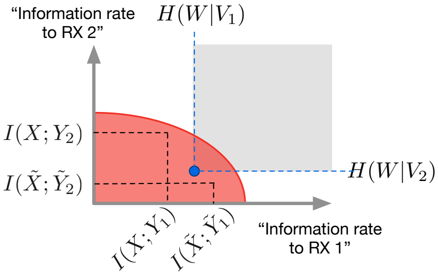
Scenario 2 : Correlated sources

$$W^n, V_1^n, V_2^n, U_1^n, U_2^n \text{ i.i.d.} \sim P_{WV_1V_2U_1U_2}$$

[2] D. Gunduz, E. Erkip, A. Goldsmith, and H. V. Poor, "Reliable joint source-channel cooperative transmission over relay networks," *IEEE Trans. Inform. Theory*, 2013.

Background: No helper basestations, $R_1 = R_2 = 0$





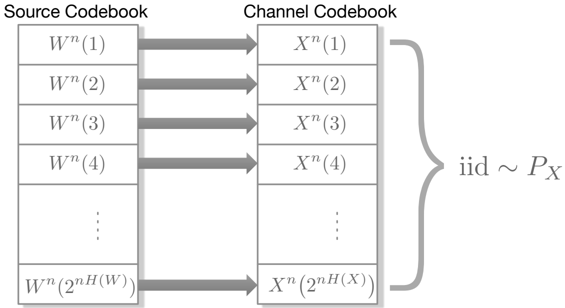
Theorem [3]:

Reliable communications possible iff $\exists X \sim P_X$ s.t.

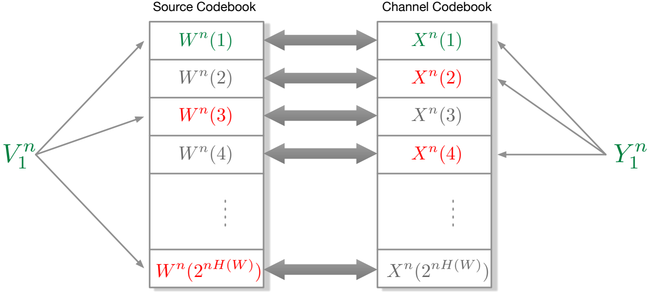
$$H(W|V_1) \le I(X;Y_1) \qquad \text{and} \qquad H(W|V_2) \le I(X;Y_2)$$

[3] E. Tuncel, "Slepian-Wolf coding over broadcast channels," *IEEE Trans. Inform. Theory*, 2006.

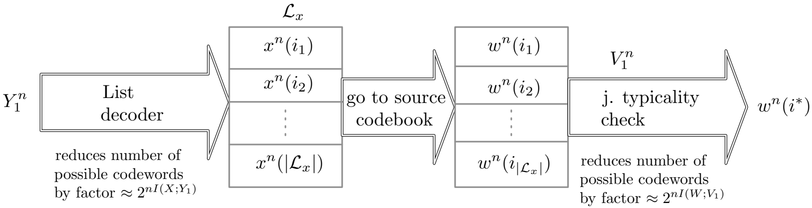
Source codebook, channel codebook & encoding



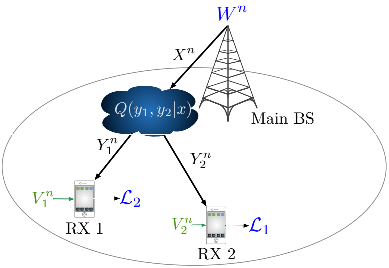
Decoding at Receiver 1



- Choose unique $w^n(i)$ common to both lists



Extension: Slepian-Wolf coding over BC with list decoding

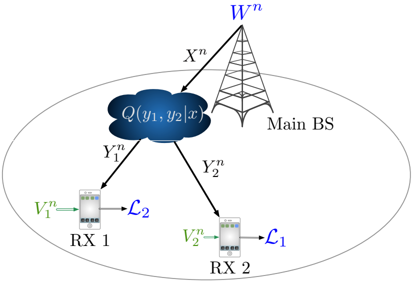


- **Receiver 1 outputs $\mathcal{L}_1 \subseteq \mathcal{W}^n$**
- **Receiver 2 outputs $\mathcal{L}_2 \subseteq \mathcal{W}^n$**

(D_1, D_2)-*achievable* if for $|\mathcal{L}_1| \leq 2^{nD_1}$ and $|\mathcal{L}_2| \leq 2^{nD_2}$:

$$\mathbb{P}\Big[\{W^n \notin \mathcal{L}_1\} \bigcup \{W^n \notin \mathcal{L}_2\}\Big] \to 0 \quad \text{as} \quad n \to \infty$$

Slepian-Wolf coding over BC with list decoding

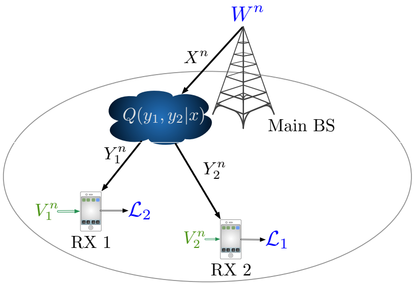


Lemma:

$(D_1 \geq 0, D_2 \geq 0)$-achievable iff $\exists X \sim P_X$ s.t.

$$D_1 \geq H(W|V_1) - I(X; Y_1)$$
$$D_2 \geq H(W|V_2) - I(X; Y_2)$$

Slepian-Wolf coding over BC with list decoding



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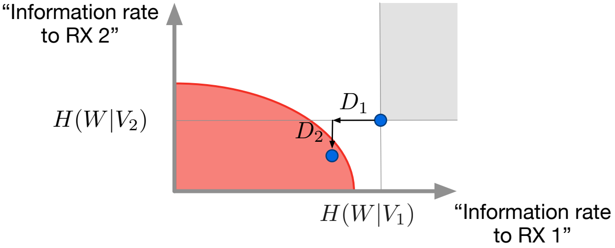
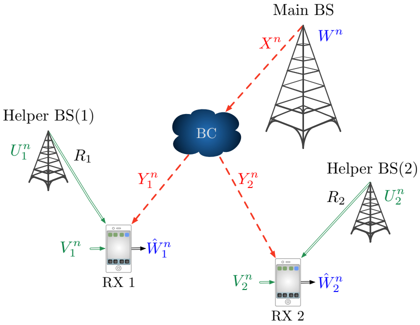


Base-station cooperation Model 2: $(W^n, V_1^n, V_2^n, U_1^n, U_2^n)$ i.i.d.



Theorem:

Reliable communication possible with helper rates (R_1, R_2) iff $\exists X, A_1, A_2$ s.t.

$$H(W|V_1, A_1) \leq I(X; Y_1) \qquad R_1 \geq I(U_1; A_1|V_1)$$
$$H(W|V_2, A_2) \leq I(X; Y_2) \qquad R_2 \geq I(U_2; A_2|V_2)$$

and $(W, V_1) - U_1 - A_1$ and $(W, V_2) - U_2 - A_2$

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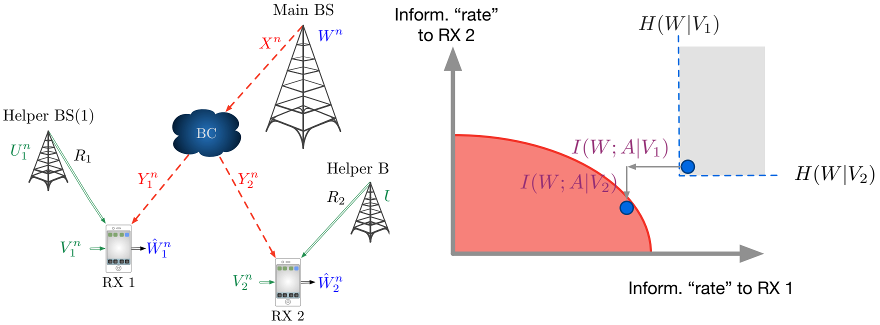

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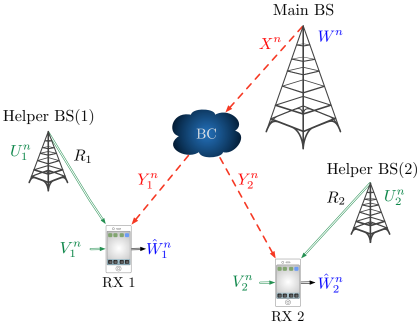
$$H(W|V_2, A_2) \leq I(X; Y_2) \qquad R_2 \geq I(U_2; A_2|V_2) \geq I(W; A_2|V_2)$$

and $(W, V_1) - U_1 - A_1$ and $(W, V_2) - U_2 - A_2$

Base-station cooperation Model 1: $U_{1,k} = \phi_1(X_k)$



Theorem:

Reliable communications possible with helper rates (R_1, R_2) iff $\exists X \sim P_X$ s.t.

$$H(W|V_1) < I(X;Y_1) + \min\left\{R_1,\ I(X;U_1|Y_1)\right\}$$

$$H(W|V_2) < I(X;Y_2) + \min\left\{R_2,\ I(X;U_2|Y_2)\right\}$$

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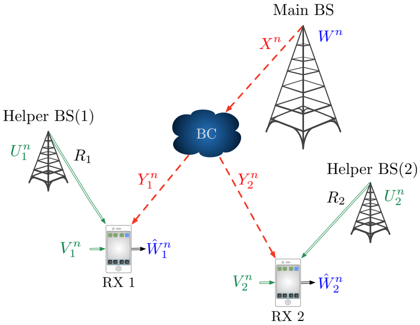

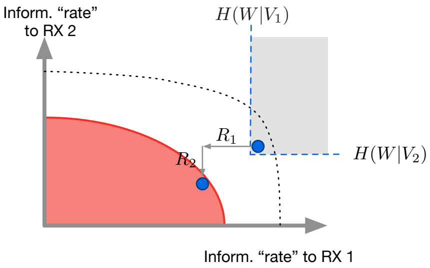

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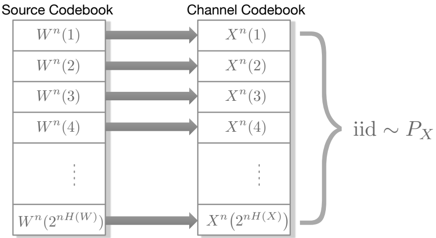
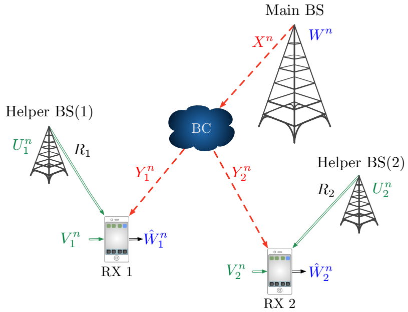
Encoding at Main BS



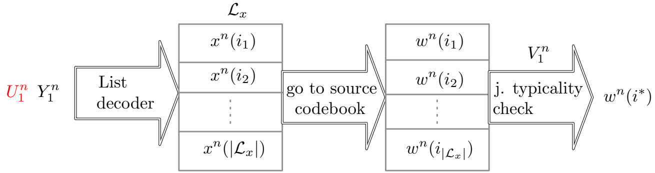
- Choose i s.t. $W^n = w^n(i)$
- Send $x^n(i)$

Receiver 1 and Helper BS in Model 1 ($U_{k,t} = \phi_k(X_t)$)

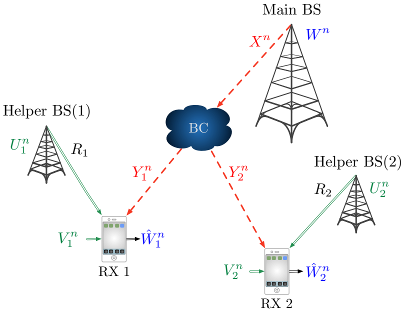


- Helper BS 1:
  randomly hash U_1^n
  (see deterministic relay channel)

- Mobile 1: recover U_1^n from helper bits, side-info V_1^n, and outputs Y_1^n, and do:

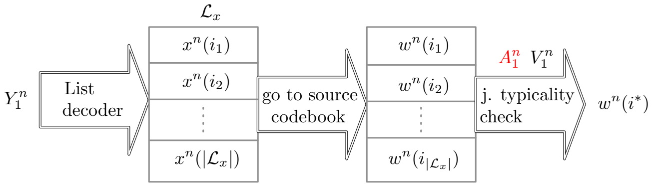


Decoding at Receiver 1 in Model 2: $(W^n, V_1^n, V_2^n, U_1^n, U_2^n)$ i.i.d.



- Helper BS 1:
  use Wyner's helper source code to compress V_1^n into A_1^n

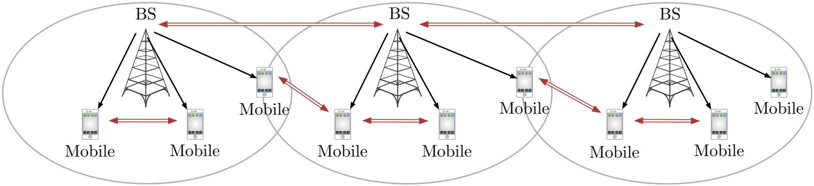
- Mobile 1: recover A_1^n from helper bits and side-info Y_1^n, and do:



Summary on intra-cell cooperation

- Slepian-Wolf coding over BCs revisited with list-decoding
- BS cooperation Model 1: $U_1^n = \phi_1(X^n)$
  $\rightarrow$ *List decoding* and *hash & forward* optimal
- BS cooperation Model 2: U_1^n directly correlated with W^n
  $\rightarrow$ *List decoding* and *Wyner-Ziv coding* optimal
- Results extend to bandwidth mismatched case and many receivers

Intercell cooperation: BS-to-BS or mobile-to-mobile cooperation



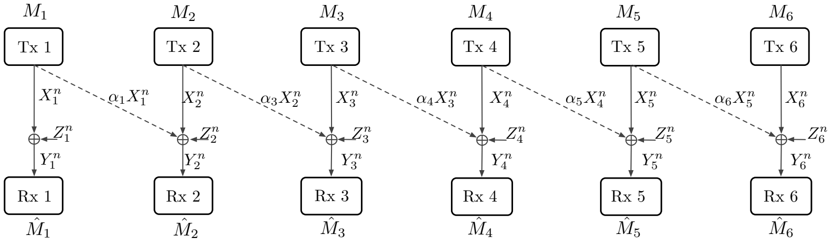
- Cooperation over digital links of given capacities

\# of conferencing rounds limited due to latency or complexity constraints

- For small networks: 1 or 2 rounds sufficient

[4] R. Timo, S. Shamai, M. Wigger, "Conferencing in Wyner's asymmetric interference network: effect of number of rounds," in *Proc. of ITW*, 2015.

Wyner's Asymmetric Soft-Handoff Model

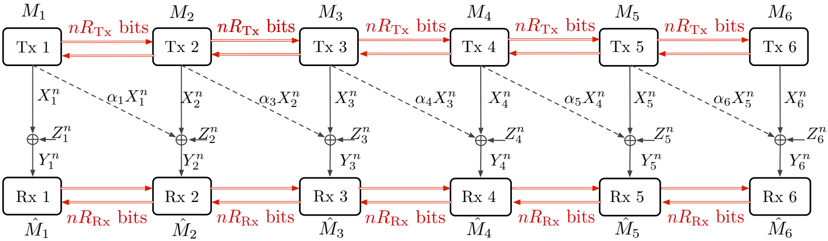


- K transmitter/receiver pairs ($K = 6$ above)
- Channel gains $\{\alpha_k\}$ fixed, constant, non-zero
- Memoryless Gaussian noises of variance σ^2 and equal power constraints P

Goal

Determine message rates $R_1, \ldots, R_K$ s.t. $\forall k\colon \Pr(\hat{M}_k \neq M_k) \to 0$ as $n \to \infty$

Wyner's Asymmetric Soft-Handoff Model



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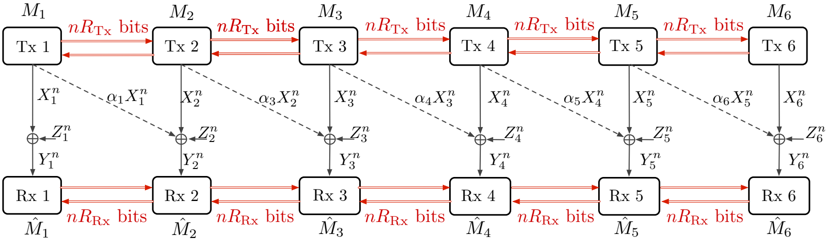
Goal

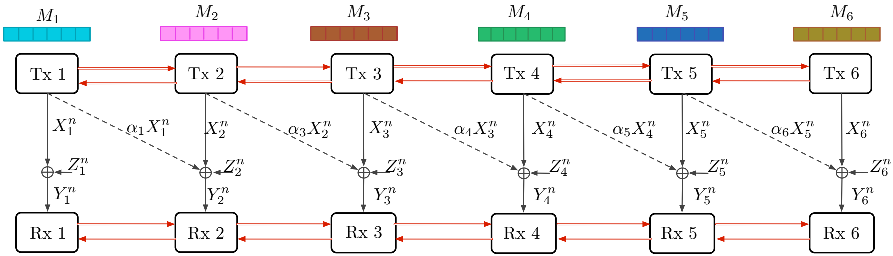
Determine message rates $R_1, \ldots, R_K$ s.t. $\forall k$: $\Pr(\hat{M}_k \neq M_k) \to 0$ as $n \to \infty$

Communication takes place in 4 phases



- Transmitter-conferencing in $\kappa_{\mathsf{Tx}} \geq 1$ rounds
- Cooperative communication over network
- Receiver-conferencing over $\kappa_{\mathsf{Rx}} \geq 1$ rounds
- Cooperative decoding

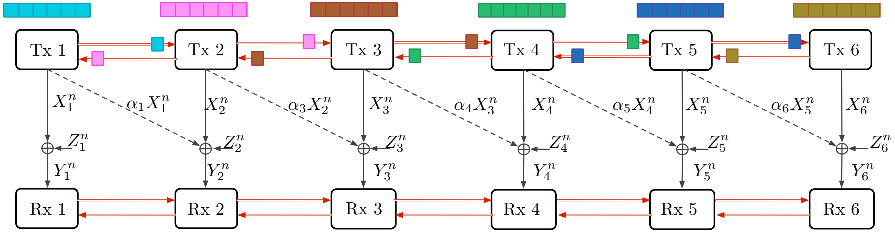
Example for 4 phases: $\kappa_{\mathsf{Tx}} = \kappa_{\mathsf{Rx}} = 2$



- Tx-conferencing: Messages can spread over κ_{Tx} transmitters to left & right
- Rx-conferencing: Output signals can spread over κ_{Rx} receivers to left & right

Example for 4 phases: $\kappa_{\text{Tx}} = \kappa_{\text{Rx}} = 2$

Phase 1: Transmitter-conferencing



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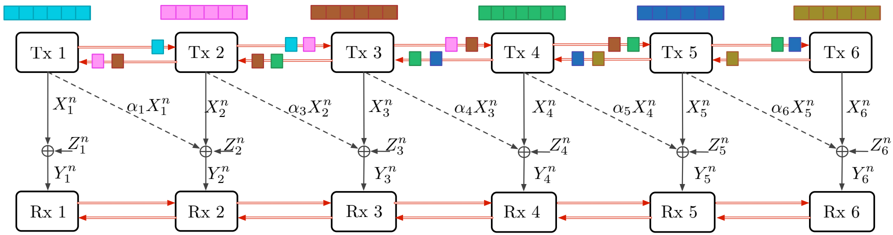

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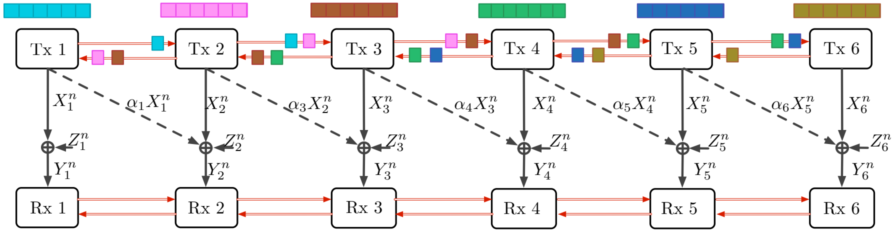
Phase 2: Cooperative communication over network



- Tx-conferencing: Messages can spread over κ_{Tx} transmitters to left & right
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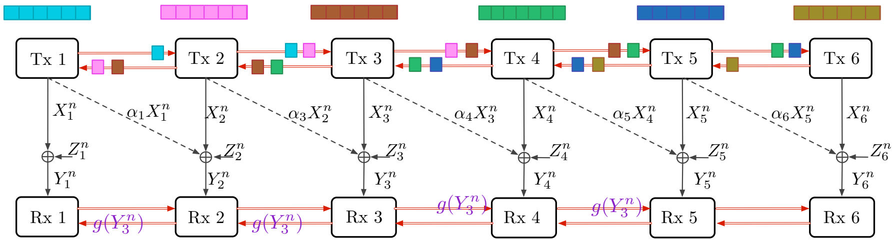
Example for 4 phases: $\kappa_{\mathsf{Tx}} = \kappa_{\mathsf{Rx}} = 2$

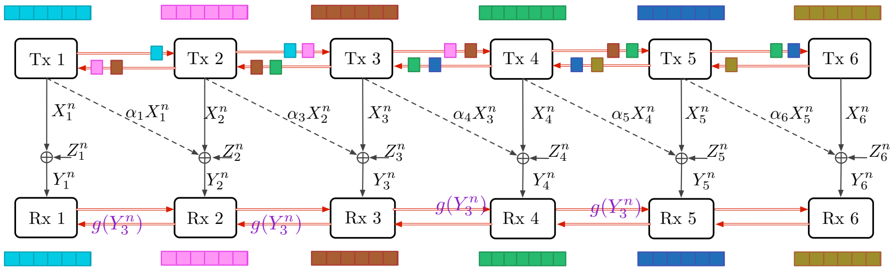
Phase 3: Receiver-conferencing



- Tx-conferencing: Messages can spread over κ_{Tx} transmitters to left & right
- Rx-conferencing: Output signals can spread over κ_{Rx} receivers to left & right

Example for 4 phases: $\kappa_{\text{Tx}} = \kappa_{\text{Rx}} = 2$

Phase 4: Clustered decoding



- Tx-conferencing: Messages can spread over κ_{Tx} transmitters to left & right
- Rx-conferencing: Output signals can spread over κ_{Rx} receivers to left & right

High-SNR Performance: Multiplexing-Gain Per User

- Sum-capacity: C_Σ maximum sum of rates $R_1 + R_2 + \cdots + R_K$ s.t. $p(error) \to 0$

- Asymptotic multiplexing gain per user $\mathcal{S}$:

$$\text{Sum-capacity: } C_\Sigma \approx \mathcal{S} \cdot \frac{K}{2} \log(1 + P/\sigma^2), \qquad P/\sigma^2 \gg 1$$

- Conferencing prelogs μ_{Tx} and μ_{Rx}:

$$R_{\mathrm{Tx}} = \mu_{\text{Tx}} \cdot \frac{1}{2} \log(1 + P/\sigma^2) \qquad \text{and} \qquad R_{\mathrm{Rx}} = \mu_{\text{Rx}} \cdot \frac{1}{2} \log(1 + P/\sigma^2)$$

Result I : Converse

Theorem

$$\mathcal{S} \leq \min\left\{1\,,\ \frac{2\mu_{\mathsf{Tx}} + 2\mu_{\mathsf{Rx}} + 1}{2}\,,\ \frac{2\kappa_{\mathsf{Tx}} + 2\kappa_{\mathsf{Rx}} + 1}{2\kappa_{\mathsf{Tx}} + 2\kappa_{\mathsf{Rx}} + 2}\right\}$$

$$\mathcal{S} \leq \min\left\{1\,,\ \frac{2\mu_{\mathsf{Rx}} + 1}{2}\,,\ \frac{2\kappa_{\mathsf{Rx}} + 1}{2\kappa_{\mathsf{Rx}} + 2}\right\}, \qquad \text{if } \mu_{\mathsf{Tx}} = 0$$

$$\mathcal{S} \leq \min\left\{1\,,\ \frac{2\mu_{\mathsf{Tx}} + 1}{2}\,,\ \frac{2\kappa_{\mathsf{Tx}} + 1}{2\kappa_{\mathsf{Tx}} + 2}\right\}, \qquad \text{if } \mu_{\mathsf{Rx}} = 0$$

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[5] A. Lapidoth, N. Levy, S. Shamai (Shitz), and M. Wigger, "Cognitive Wyner networks with clustered decoding," IEEE Trans. on Information Theory, Oct. 2014.

Result II : Achievability

Theorem

Let

$$\frac{\mu_{\mathrm{Tx}}}{\kappa_{\mathrm{Tx}}} \geq \frac{\mu_{\mathrm{Rx}}}{\kappa_{\mathrm{Rx}}}$$

Then,

$$\mathcal{S} \geq \mathcal{S}^{\mathrm{Ach}} = \begin{cases} \dfrac{2\mu_{\mathrm{Tx}} + 2\mu_{\mathrm{Rx}} + 1}{2} & \text{if} \quad 2\frac{\mu_{\mathrm{Tx}}}{\kappa_{\mathrm{Tx}}}(1+\kappa_{\mathrm{Tx}}) + 2\mu_{\mathrm{Rx}} \leq 1 \\ \dfrac{2\kappa_{\mathrm{Tx}} + 2\mu_{\mathrm{Rx}} + 1}{2\kappa_{\mathrm{Tx}} + 2} & \text{otherwise} \\ \dfrac{2\kappa_{\mathrm{Tx}} + 2\kappa_{\mathrm{Rx}} + 1}{2\kappa_{\mathrm{Tx}} + 2\kappa_{\mathrm{Rx}} + 2} & \text{if} \quad 2\frac{\mu_{\mathrm{Rx}}}{\kappa_{\mathrm{Rx}}}(1+\kappa_{\mathrm{Tx}}) + 2\mu_{\mathrm{Rx}} > 1. \end{cases}$$

Tight when μ_{Tx} *and* μ_{Rx} *small or large and when* $\mu_{\mathrm{Tx}} = 0$ *or* $\mu_{\mathrm{Rx}} = 0$

Result III : Without limitation on number of conferencing rounds

Theorem

$$\mathcal{S}_\infty = \min\left\{1, \frac{2\mu_{\mathsf{Tx}} + 2\mu_{\mathsf{Rx}} + 1}{2}\right\}$$

Comparison for $\kappa_{\mathrm{Tx}} = \kappa_{\mathrm{Rx}}$ and $\mu_{\mathrm{Tx}} = 2\mu_{\mathrm{Rx}} = \mu$

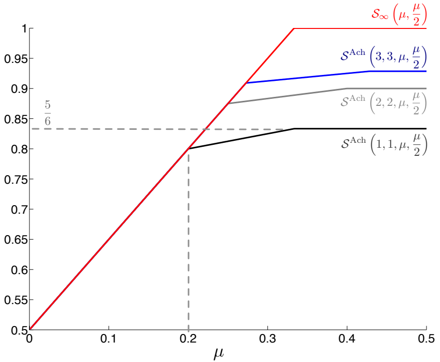
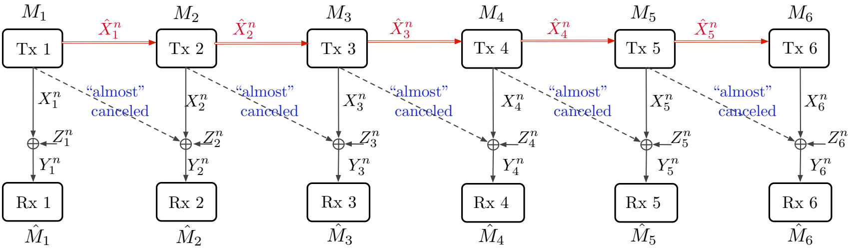


Figure 1: $\mathcal{S}^{\mathrm{Ach}}(\kappa_{\mathrm{Tx}}, \kappa_{\mathrm{Rx}}, \mu_{\mathrm{Tx}}, \mu_{\mathrm{Rx}})$

- For small conferencing prelogs $\kappa = 1$ suffices!
- For finite κ, multiplexing gain per user saturates below 1

Coding scheme for unlimited conferencing rounds

- $\mu_{\mathsf{Tx}} = 1/2$ or $\mu_{\mathsf{Rx}} = 1/2 \implies \mathcal{S}_{\kappa=\infty} = 1$



- With Tx-conferencing: $\hat{X}_k = f_k(M_1, \ldots, M_k)$ or $\hat{X}_k = f_k(M_{k+1}, \ldots, M_K)$

[5] V. Ntranos, M.A. Maddah-Ali and G. Caire, "Cellular interference alignment," *IEEE Trans. Inform. Theory*, Mar. 2015

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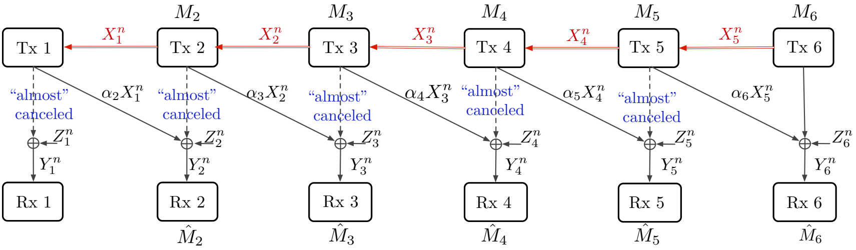

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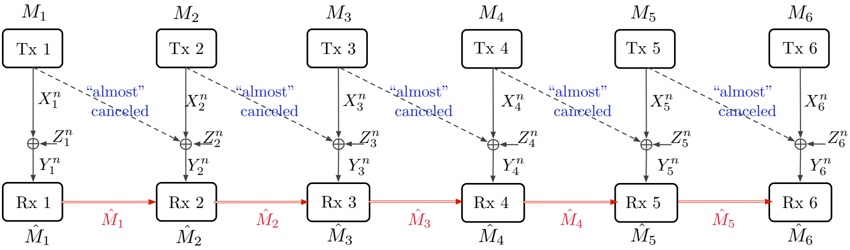
- $\mu_{\text{Tx}} = 1/2$ or $\mu_{\text{Rx}} = 1/2 \implies \mathcal{S}_{\kappa=\infty} = 1$



- With Tx-conferencing: $\hat{X}_k = f_k(M_1, \ldots, M_k)$ or $\hat{X}_k = f_k(M_{k+1}, \ldots, M_K)$

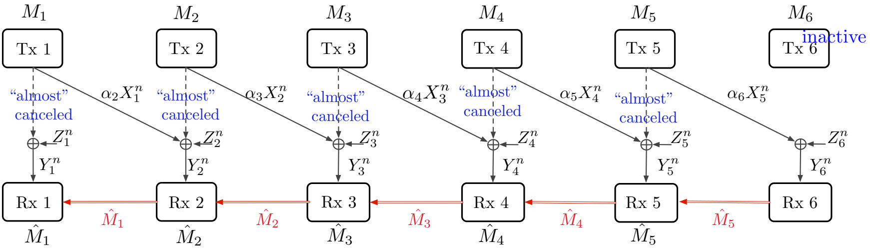
- With Rx-conferencing: $\hat{M}_k = g_k(Y_1^n, \ldots, Y_k^n)$ or $\hat{M}_k = g_k(Y_{k+1}^n, \ldots, Y_K^n)$

$\rightarrow$ Interference cancelation technique causes interference to propagate!

[5] V. Ntranos, M.A. Maddah-Ali and G. Caire, "Cellular interference alignment," *IEEE Trans. Inform. Theory*, Mar. 2015

Coding scheme for unlimited conferencing rounds

- $\mu_{\mathsf{Tx}} = 1/2$ or $\mu_{\mathsf{Rx}} = 1/2 \implies \mathcal{S}_{\kappa=\infty} = 1$



- With Tx-conferencing: $\hat{X}_k = f_k(M_1, \ldots, M_k)$ or $\hat{X}_k = f_k(M_{k+1}, \ldots, M_K)$

- With Rx-conferencing: $\hat{M}_k = g_k(Y_1^n, \ldots, Y_k^n)$ or $\hat{M}_k = g_k(Y_{k+1}^n, \ldots, Y_K^n)$

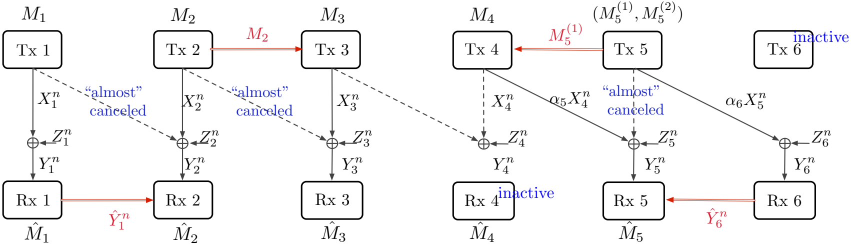
[5] V. Ntranos, M.A. Maddah-Ali and G. Caire, "Cellular interference alignment," *IEEE Trans. Inform. Theory*, Mar. 2015

Coding Scheme for $\kappa = 1$

Interference mitigation causes interference to propagate

For finite κ_{Tx} and κ_{Rx}: need to switch off transmitters
$\Rightarrow$ saturation of $\mathcal{S}$ even when $\mu_{\mathsf{Tx}}, \mu_{\mathsf{Rx}} \to \infty$

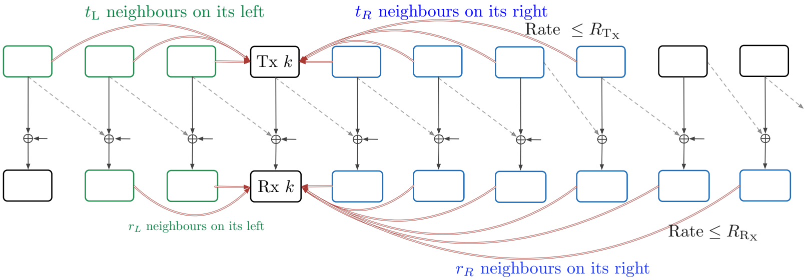
- $\mu_{\mathsf{Tx}} = 1/2$ or $\mu_{\mathsf{Rx}} = 1/2 \implies \mathcal{S} = 5/6$



- Tx-conferencing: M_k and Rx-conferencing: $\hat{Y}_k^n = g_k(Y_k^n)$

$\to$ achievable also with oblivious-codebooks

Extension: Conferencing Links to Multiple Left-& Right-Neighbours



- Combination of presented schemes
- $\mathcal{S} = \frac{\kappa_{\mathsf{Tx}}(t_{\mathsf{L}}+t_{\mathsf{R}})+\kappa_{\mathsf{Rx}}(r_{\mathsf{L}}+r_{\mathsf{R}})+1}{\kappa_{\mathsf{Tx}}(t_{\mathsf{L}}+t_{\mathsf{R}})+\kappa_{\mathsf{Rx}}(r_{\mathsf{L}}+r_{\mathsf{R}})+2}$ even when $\mu_{\mathsf{Tx}}, \mu_{\mathsf{Rx}} \to \infty$

Summary and Outlook on: Cooperation across cells

- For small conferencing prelogs $\kappa_{\mathsf{Tx}} = \kappa_{\mathsf{Rx}} = 1$ suffice!
- For finite $\kappa_{\mathsf{Tx}}, \kappa_{\mathsf{Rx}}$, multiplexing gain per user saturates below 1
- Duality between transmitter-cooperation and receiver-cooperation
- In future: analyze different cooperation constraints
  - oblivious codebooks
  - more accurate latency and complexity constraints? (mobiles!)