Strong Converse for Bi-Static ISAC with Two Detection-Error Exponents

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***Abstract*—The paper considers an information-theoretic bi-static integrated sensing and communication (ISAC) model and provides new results on the fundamental limits of this model. Specifically, we show a strong converse result for memoryless ISAC where the channel transition law depends on an underlying binary hypothesis, which the radar receiver wishes to determine with largest exponential decay rates of the probabilities of detection error under the two hypotheses. In this sense, the channel is a compound channel, and we establish a strong converse under maximum probability of error criteria. We further prove that the fundamental limits of our ISAC system remain unchanged under average probability of error criteria as long as the admissible channel decoding errors are bounded by $1/2$.**

***Index Terms*—Integrated sensing and communication, bi-static radar, detection error exponent.**

I. Introduction

Huge technological efforts are being made to integrate radar systems with communication systems, in particular in view of the future 6G mobile communication standard [1], [2] and its deployment for autonomous navigation or smart manufacturing sites. In *integrated sensing and communication systems (ISAC)*, the idea is to use a common waveform for both tasks: the emitted signals are modulated so as to achieve reliable data communication while the backscatters of these signals are used to sense the environment, detect hazardous events, or infer properties of other terminals (e.g., velocities or directions of other cars).

ISAC has already inspired a plethora of works in the signal processing and communications communities, see for example [3]–[10] and references therein, as well as (to a lesser extent) in the information-theoretic community [11]–[21]. The results in [17]–[20] and the present manuscript all focus on the system model in Figure 1 consisting of a transmitter (Tx) sending a message to a receiver (Rx) over a state-dependent discrete memoryless channel (SDMC). A bi-static radar receiver close to the Tx receives the backscattered signal, and due to the proximity to the Tx, this radar receiver also knows the Tx's channel inputs and compares them to the backscatterers.

We follow the model in [17]–[20], where the channel is memoryless and stationary with a transition law that depends on a binary hypothesis. The goal of the radar receiver is to detect the underlying hypothesis. In a real-world application, the hypothesis can correspond to the presence or absence of an obstacle, which the radar receiver wishes to determine. As in [18], we measure sensing performance in terms of the

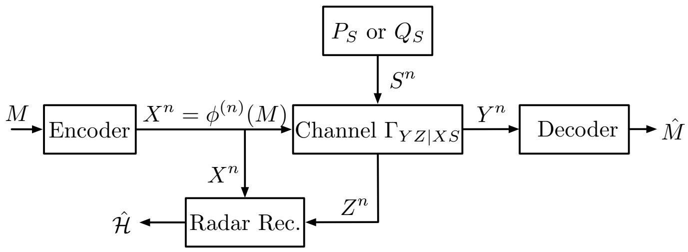

Fig. 1: Bi-static Radar ISAC Model

two exponential decay-rates of the detection error probabilities under the two hypotheses. The difference between [18] and the present work is that we allow for positive probabilities of decoding error in the communication and determine the fundamental limits of ISAC under this assumption. In particular, we prove a strong converse by showing that even when one allows for positive decoding error probabilities neither the achievable rate nor the achievable sensing performance can be improved. Our results are based on maximum error-probability criteria. In fact, since the communication channel is a compound channel (on a set of two channels) it is known that a geneal strong converse fails under average-probability of error criteria even for only the channel coding part [22], [23].

The converse proof in this paper is an extension of the channel coding strong converse proof in [24] to incorporate also the sensing bounds, see also [20] for a strong converse for bi-static ISAC when sensing performance is measured in terms of Stein's exponent or distortion. Strong converse proofs based on change-of-measure arguments go back to Gu and Effros [25], [26] and can be also found in various other works, e.g., [27]. The proof method was formalized and first applied to channel coding by Tyagi and Watanabe [28].

Notation: Upper-case letters are used for random quantities and lower-case letters for deterministic realizations. Calligraphic font is used for sets. All random variables are assumed finite and discrete. We abbreviate the n-tuples $(X_1, \ldots, X_n)$ and $(x_1, \ldots, x_n)$ as X^n and x^n. We further abbreviate *independent and identically distributed* as *i.i.d.* and *probability mass function* as *pmf.* Pmfs of i.i.d. random tuples are denoted by $P^{\otimes}$.

Entropy, conditional entropy, and mutual information functionals are written as $H(\cdot)$, $H(\cdot|\cdot)$, and $I(\cdot;\cdot)$, potentially with pmfs in the subscripts. The Kullback-Leibler divergence is denoted by $D(\cdot\|\cdot)$. We denote by $\boldsymbol{\pi}_{x^n}$ the type of a sequence x^n. We also use Landau notation, where $o(1)$ denotes a function that tends to 0 as $n \to \infty$.

II. Setup and Main Results

Consider the bi-static radar receiver model over a memoryless channel in Fig. 1. A transmitter (Tx) that wishes to communicate a random message M to a receiver (Rx) over a state-dependent channel. The message M is uniformly distributed over the set $\mathcal{M} = \{1, \ldots, 2^{nR}\}$ with $R > 0$ and $n > 0$ denoting the rate and blocklength of communication, respectively. The channel from the Tx to the Rx depends on a state-sequence $S^n = (S_1, \ldots, S_n)$ which depends on a binary hypothesis $\mathcal{H} \in \{0, 1\}$. Under the null hypothesis $\mathcal{H} = 0$ it is i.i.d. according to the pmf P_S and under the alternative hypothesis $\mathcal{H} = 1$ it is i.i.d. according to the pmf Q_S. For a given blocklength n, the Tx thus produces the n-length sequence of channel inputs

$$X^n = \phi^{(n)}(M) \tag{1}$$

for some choice of the encoding function $\phi^{(n)} \colon \{1, \ldots, 2^{nR}\} \to \mathcal{X}^n$.

Based on X^n and S^n the channel produces the sequences Y^n observed at the Rx and the backscattered signal Z^n. The channel is assumed memoryless and described by the stationary transition law $\Gamma_{YZ|SX}$ implying that the pair (Y_t, Z_t) is produced according to the channel law $\Gamma_{YZ|SX}$ based on the time-t symbols (X_t, S_t).

The Rx attempts to guess message M based on the sequence of channel outputs Y^n:

$$\hat{M} = g^{(n)}(Y^n) \tag{2}$$

using a decoding function of the form $g^{(n)} \colon \mathcal{Y}^n \to \{1, \ldots, 2^{nR}\}$.

Performance of communication is measured in terms of maximum error probability

$$p^{(n)}(\text{error}) := \max_{\mathsf{H} \in \{0,1\}} \max_{m \in \{1,\ldots,2^{nR}\}} \Pr\left[\hat{M} \neq M | \mathcal{H} = \mathsf{H}, M = m\right]. \tag{3}$$

We assume a radar receiver close to the Tx, that wishes to guess the underlying hypothesis based on the inputs and backscattered signals. I.e., it produces a guess of the form

$$\hat{\mathcal{H}} = h^{(n)}(X^n, Z^n) \in \{0, 1\}. \tag{4}$$

Radar sensing performance is measured in terms of error-exponent pairs. That means, it is required that the type-I and type-II error probabilities

$$\alpha_n := \max_{m \in \mathcal{M}} \Pr\left[\hat{\mathcal{H}} = 1 \middle| \mathcal{H} = 0, M = m\right] \tag{5}$$

and

$$\beta_n := \max_{m \in \mathcal{M}} \Pr\left[\hat{\mathcal{H}} = 0 \middle| \mathcal{H} = 1, M = m\right] \tag{6}$$

decay exponentially fast to 0 with largest possible exponents.

Definition 1: A rate-exponent pair (R, θ) is (ϵ, r)-achievable over the state-dependent DMC $(\mathcal{X}, \mathcal{Y}, \Gamma_{YZ|XS})$ with state-distributions P_S and Q_S, if there exists a sequence of encoding, decoding, and estimation functions $\{(\phi^{(n)}, g^{(n)}, h^{(n)})\}$ such that for each blocklength n the maximum probability of error (over the two hypotheses) satisfies

$$\overline{\lim_{n\to\infty}}\, p^{(n)}(\text{error}) \leq \epsilon, \tag{7}$$

while the detection error probabilities satisfy:

$$-\varliminf_{n\to\infty} \frac{1}{n} \log \alpha_n \geq r, \tag{8}$$

$$-\varliminf_{n\to\infty} \frac{1}{n} \log \beta_n \geq \theta. \tag{9}$$

We use the abbreviations:

$$P_{YZ|X}(y, z|x) := \sum_{s \in \mathcal{S}} P_S(s) \Gamma_{YZ|SX}(y, z|s, x) \tag{10}$$

$$Q_{YZ|X}(y, z|x) := \sum_{s \in \mathcal{S}} Q_S(s) \Gamma_{YZ|SX}(y, z|s, x). \tag{11}$$

Let also $P_{Y|X}, P_{Z|X}$ and $Q_{Z|X}, Q_{Y|X}$ denote the respective conditional marginals.

Theorem 1: For any $\epsilon \in [0, 1)$ and $r > 0$, a rate-exponent triple (R, θ, r) is (ϵ, r)-achievable, if and only if, there exists a pmf P_X satisfying

$$R \leq \min\{I_{P_X P_{Y|X}}(X; Y), I_{P_X Q_{Y|X}}(X; Y)\}, \tag{12}$$

and

$$\theta \leq \min_{\substack{\bar{P}_{Z|X}: \\ \mathbb{E}_{P_X}[D(\bar{P}_{Z|X} \| P_{Z|X})] \leq r}} \mathbb{E}_{P_X}\left[D(\bar{P}_{Z|X} \| Q_{Z|X})\right]. \tag{13}$$

Proof: Achievability follows by standard random coding arguments for a compound channel, where the transmitter uniformly picks the codewords over the set of n-length sequences of a fixed type P_X and the decoder uses a universal decoding rule such as a maximum mutual information (MMI) decoder. The radar receiver checks the conditional type of the received sequence z^n given the transmitted codeword x^n and decides on $\hat{\mathcal{H}} = 0$ if the conditional type $\boldsymbol{\pi}_{z^n|x^n}$ satisfies

$$\mathbb{E}_{P_X}[D(\boldsymbol{\pi}_{z^n|x^n} \| P_{Z|X})] \leq r. \tag{14}$$

The converse, which is the main contribution of this paper, is proved in Section III. ∎

Remark 1: Above Theorem 1 applies to a setup with *maximum* probabilities of error, see (3), (5), and (6). The theorem however applies unchanged also when the definitions (3), (5), and (6) are replaced by the following *average* probabilities of error:

$$p^{(n)}(\text{error}) := \max_{\mathsf{H} \in \{0,1\}} \Pr\left[\hat{M} \neq M | \mathcal{H} = \mathsf{H}\right]. \tag{15a}$$

$$\alpha_n := \Pr\left[\hat{\mathcal{H}} = 1 \middle| \mathcal{H} = 0\right] \tag{15b}$$

$$\beta_n := \Pr\left[\hat{\mathcal{H}} = 0 \middle| \mathcal{H} = 1\right] \tag{15c}$$

under the condition that $\epsilon \in [0, 1/2)$. The converse proof under these average probability of error criteria is obtained by first applying expurgation arguments and then following similar steps as in Section III.

III. Strong Converse Proof

Fix a sequence of encoding, decoding, and estimation functions $\{(\phi^{(n)}, g^{(n)}, h^{(n)})\}_{n=1}^{\infty}$. Assume that (7) and (8) are satisfied. For readability, we will also write $x^n(\cdot)$ for the function $\phi^{(n)}(\cdot)$. Choose a sequence of small positive numbers $\{\mu_n\}_{n=1}^{\infty}$ satisfying

$$\lim_{n\to\infty} \mu_n = 0 \tag{16}$$

$$\lim_{n\to\infty} n \cdot \mu_n^2 = \infty. \tag{17}$$

Let T be uniform over $\{1, \ldots, n\}$ independent of all other quantities and consider an increasing subsequence of blocklengths $\{n_i\}$ so that the expected type $\mathbb{E}_M[\boldsymbol{\pi}_{x^n(M)}(x)]$ converges and denote the convergence point by $P_X(x)$:

$$P_X(x) := \lim_{i\to\infty} \mathbb{E}_M[\boldsymbol{\pi}_{x^{n_i}(M)}(x)], \qquad x \in \mathcal{X}. \tag{18}$$

In the remainder of this proof, we restrict attention to this subsequence of blocklengths $\{n_i\}_{i=1}^{\infty}$.

Proof of Channel Coding Bound: We first prove the converse bound for channel coding. We start by considering the case $\mathcal{H} = 0$ and channel transition law $P_{Y|X}$.

Fix a blocklength $n \in \{n_i\}$. Based on the two conditions

$$g^{(n)}(y^n) = m \tag{19a}$$

$$\left|\pi_{x^n(m)y^n}(a, b) - \pi_{x^n(m)}(a) P_{Y|X}(b|a)\right| \le \mu_n, \tag{19b}$$

define for each message $m \in \mathcal{M}$ the set

$$\mathcal{D}_{\mathcal{C},m} := \{y^n : \quad \text{(19a) and (19b)}\}. \tag{20}$$

Introduce the new random tuple $Y_{\mathcal{C}}^n$ with the following conditional pmf given the message M:

$$P_{Y_{\mathcal{C}}^n|M}(y^n|m) = \frac{P_{Y|X}^{\otimes n}(y^n|x^n(m))}{\Delta_{\mathcal{C},m}} \cdot \mathbb{1}\{y^n \in \mathcal{D}_{\mathcal{C},m}\}, \tag{21}$$

for

$$\Delta_{\mathcal{C},m} := \sum_{y^n} P_{Y|X}^{\otimes n}(y^n|x^n(m)) \cdot \mathbb{1}\{y^n \in \mathcal{D}_{\mathcal{C},m}\}. \tag{22}$$

By the union bound, by (7), and by [29, Remark to Lemma 2.12], we have:

$$\Delta_{\mathcal{C},m} \ge 1 - \epsilon - \frac{|\mathcal{X}||\mathcal{Y}|}{4\mu_n^2 n}, \quad \forall m \in \mathcal{M}. \tag{23}$$

Continue to notice that:

$$R \overset{(a)}{=} \frac{1}{n} I(M; Y_{\mathcal{C}}^n) \tag{24}$$

$$\le \frac{1}{n}\sum_{i=1}^{n} H(Y_{\mathcal{C},i}) - \frac{1}{n} H\left(Y_{\mathcal{C}}^n \middle| M\right) \tag{25}$$

$$= H(Y_{\mathcal{C},T}|T) - \frac{1}{n} H\left(Y_{\mathcal{C}}^n \middle| M\right) \tag{26}$$

$$\le H(Y_{\mathcal{C},T}) - \frac{1}{n} H\left(Y_{\mathcal{C}}^n \middle| M\right), \tag{27}$$

where we defined the random variable T to be uniform over $\{1, \ldots, n\}$ independent of the other random variables. Here, (a) holds because $M = g(Y_{\mathcal{C}}^n)$ by Condition (19a).

Define next $X_T = x_T(M)$ (the T-th symbol of codeword $x^n(M)$), and notice that

$$P_{X_T Y_{\mathcal{C},T}}(x, y) = \frac{1}{n}\sum_{t=1}^{n} P_{X_t Y_{\mathcal{C},t}}(x, y) \tag{28}$$

$$= \frac{1}{n}\sum_{t=1}^{n} \mathbb{E}\left[\mathbb{1}\{(X_t, Y_{\mathcal{C},t}) = (x, y)\}\right] \tag{29}$$

$$= \mathbb{E}\left[\pi_{x^n(M)Y_{\mathcal{C}}^n}(x, y)\right] \tag{30}$$

$$= \mathbb{E}_M\left[\pi_{x^n(M)}(x)\right] \cdot P_{Y|X}(y|x) + o(1), \tag{31}$$

where the last equality holds by Condition (19b). By continuity of the entropy functional and the definition in (18):

$$\lim_{n_i\to\infty} H(Y_{\mathcal{C},T}) = H_{P_X P_{Y|X}}(Y). \tag{32}$$

Next, by definition and by (21):

$$\frac{1}{n} H(Y_{\mathcal{C}}^n | M = m)$$

$$= -\frac{1}{n}\sum_{y^n \in \mathcal{D}_{\mathcal{C},m}} P_{Y_{\mathcal{C}}^n|M=m}(y^n) \log P_{Y_{\mathcal{C}}^n|M=m}(y^n) \tag{33}$$

$$\ge -\frac{1}{n}\sum_{y^n \in \mathcal{D}_{\mathcal{C},m}} P_{Y_{\mathcal{C}}^n|M=m}(y^n) \log \frac{P_{Y|X}^{\otimes n}(y^n|x^n(m))}{\Delta_{\mathcal{C},m}} \tag{34}$$

$$= -\frac{1}{n}\sum_{t=1}^{n}\sum_{y_t\in\mathcal{Y}} P_{Y_{\mathcal{C},t}|M=m}(y_t) \log P_{Y|X}(y_t|x_t(m)) + \frac{1}{n}\log \Delta_{\mathcal{C},m}, \tag{35}$$

$$= -\frac{1}{n}\sum_{t=1}^{n}\sum_{y\in\mathcal{Y}} \mathbb{E}\left[\mathbb{1}\{Y_{\mathcal{C},t} = y\} \middle| M = m\right] \log P_{Y|X}(y|x_t(m)) + \frac{1}{n}\log \Delta_{\mathcal{C},m}, \tag{36}$$

$$= -\frac{1}{n}\sum_{t=1}^{n}\sum_{y\in\mathcal{Y}} \mathbb{E}\left[\sum_{x\in\mathcal{X}} \mathbb{1}\{x_t(m) = x, Y_{\mathcal{C},t} = y\} \middle| M = m\right] \cdot \log P_{Y|X}(y|x) + \frac{1}{n}\log \Delta_{\mathcal{C},m}, \tag{37}$$

$$= -\sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} \mathbb{E}\left[\pi_{x^n(m)Y_{\mathcal{C}}^n}(x, y) \middle| M = m\right] \cdot \log P_{Y|X}(y|x) + \frac{1}{n}\log \Delta_{\mathcal{C},m} \tag{38}$$

$$= -\sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} \pi_{x^n(m)}(x) P_{Y|X}(y|x) \cdot \log P_{Y|X}(y|x) + o(1), \tag{39}$$

where the last equality holds by Condition (19b) and because the two Inequalities (23) and $\epsilon \in [0, 1)$ imply that $\frac{1}{n}\log \Delta_{\mathcal{C},m}$

vanishes as $n \to \infty$. Taking the average over all messages yields:

$$\frac{1}{n} H(Y_{\mathcal{C}}^n|M) \geq -\sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} \mathbb{E}\left[\pi_{x^n(M)}(x)\right] \log P_{Y|X}(y|x) + o(1). \tag{40}$$

Using the definition of P_X, we obtain:

$$\begin{aligned}\lim_{i\to\infty} \frac{1}{n_i} H(Y_{\mathcal{C}}^n|M) &= -\sum_{x\in\mathcal{X}} P_X(x) \sum_{y\in\mathcal{Y}} P_{Y|X}(y|x)\log P_{Y|X}(y|x)\\ &= H_{P_X P_{Y|X}}(Y|X). \end{aligned}\tag{41}$$

Combining (27) with (32) and (41), we can conclude that

$$R \leq H_{P_X P_{Y|X}}(Y) - H_{P_X P_{Y|X}}(Y|X) \tag{42}$$
$$= I_{P_X P_{Y|X}}(X;Y), \tag{43}$$

for P_X as defined in (18).

Following the same steps (19)–(43) but with the channel law $Q_{Y|X}$ instead of $P_{Y|X}$, one can show that

$$R \leq \min\left\{I_{P_X P_{Y|X}}(X;Y),\ I_{P_X Q_{Y|X}}(X;Y)\right\}. \tag{44}$$

Proof of the Error Exponents: Fix a small value of $\delta > 0$ and consider any conditional type $\bar{P}_{Z|X}$ so that

$$\mathbb{E}_{P_X}\left[D\left(\bar{P}_{Z|X} \,\|\, P_{Z|X}\right)\right] < r - \delta. \tag{45}$$

Fix a sufficiently large blocklength $n \in \{n_i\}$ and a message m, for which the following two inequalities hold:

$$\begin{aligned}&\mathbb{E}_{\boldsymbol{\pi}_{x^n(m)}}\left[D\left(\bar{P}_{Z|X} \,\|\, P_{Z|X}\right)\right]\\ &\quad\leq \mathbb{E}_M\left[\mathbb{E}_{\boldsymbol{\pi}_{x^n(M)}}\left[D\left(\bar{P}_{Z|X} \,\|\, P_{Z|X}\right)\right]\right] &(46)\\ &\quad< r - \delta/2. &(47)\end{aligned}$$

Then, based on the conditions

$$h^{(n)}\left(x^n(m), z^n\right) = 0 \tag{48a}$$
$$\left|\pi_{x^n(m)z^n}(a,b) - \pi_{x^n(m)}(a)\bar{P}_{Z|X}(b|a)\right| \leq \mu_n, \tag{48b}$$

define the set

$$\mathcal{D}_{\mathcal{S},m} \triangleq \{z^n\colon \quad \text{(48a) and (48b)}\}. \tag{49}$$

Define also the new random variable $Z_{\mathcal{S}}^n$ of conditional law

$$P_{Z_{\mathcal{S}}^n}(z^n) = \frac{P_{Z|X}^{\otimes n}(z^n|x^n(m))}{\Delta_{\mathcal{S},m}} \cdot \mathbb{1}\left\{z^n \in \mathcal{D}_{\mathcal{S},m}\right\}, \tag{50}$$

for

$$\Delta_{\mathcal{S},m} := \sum_{z^n} P_{Z|X}^{\otimes n}(z^n|x^n(m))\cdot \mathbb{1}\left\{z^n \in \mathcal{D}_{\mathcal{S},m}\right\}. \tag{51}$$

Defining $D_b(a\|b) := a\log_2\frac{a}{b} + (1-a)\log_2\frac{1-a}{1-b}$, we notice the sequence of (in)equalities:

$$\begin{aligned}&-\frac{1}{n}\log \Pr\left[\hat{\mathcal{H}} = 0\middle|\mathcal{H} = 1, M = m\right]\\ &\leq -\frac{1}{n}\log \sum_{z^n\in\mathcal{D}_{\mathcal{S},m}} Q_{Z|X}^{\otimes n}(z^n|x^n(m)) &(52)\\ &= \frac{1}{n} D_b\left(\sum_{z^n\in\mathcal{D}_{\mathcal{S},m}} P_{Z_{\mathcal{S}}^n}(z^n) \;\Big\|\; \sum_{z^n\in\mathcal{D}_{\mathcal{S},m}} Q_{Z|X}^{\otimes n}(z^n|x^n(m))\right) &(53)\\ &\leq \frac{1}{n} D\left(P_{Z_{\mathcal{S}}^n} \Big\| \, Q_{Z|X}^{\otimes n}(\cdot|x^n(m))\right) &(54)\\ &\leq \frac{1}{n}\sum_{z^n\in\mathcal{Z}^n} P_{Z_{\mathcal{S}}^n}(z^n)\log\frac{P_{Z|X}^{\otimes n}(z^n|x^n(m))}{Q_{Z|X}^{\otimes n}(z^n|x^n)}\\ &\quad -\frac{1}{n}\log\Delta_{\mathcal{S},m} &(55)\\ &= \frac{1}{n}\sum_{x\in\mathcal{X}}\sum_{\substack{t\in\{1,\ldots,n\}:\\ x_t(m)=x}}\sum_{z} P_{\tilde{Z}_{\mathcal{S},t}}(z)\log\frac{P_{Z|X}(z|x)}{Q_{Z|X}(z|x)}\\ &\quad -\frac{1}{n}\log\Delta_{\mathcal{S},m}, &(56)\end{aligned}$$

where Inequality (53) holds because $\sum_{z^n\in\mathcal{D}_{\mathcal{S},m}} P_{Z_{\mathcal{S}}^n}(z^n) = 1$.

To bound the term $\Delta_{Z,m}$, let $\mathcal{T}_{x^n(m)}(\bar{P}_{Z|X})$ be the set of z^n sequences satisfying (48b) (and thus $\mathcal{D}_{Z,m} \subseteq \mathcal{T}_{x^n(m)}(\bar{P}_{Z|X})$) and notice that

$$\begin{aligned}\alpha_n &\geq \Pr\left[\hat{\mathcal{H}} = 1\middle|\mathcal{H} = 0, M = m\right] &(57)\\ &\geq \sum_{z^n\in\mathcal{T}_{x^n(m)}(\bar{P}_{Z|X})\setminus\mathcal{D}_{\mathcal{S},m}} P_{Z|X}^{\otimes n}(z^n|x^n(m)) &(58)\\ &= \sum_{z^n\in\mathcal{T}_{x^n(m)}(\bar{P}_{Z|X})} P_{Z|X}^{\otimes n}(z^n|x^n(m)) - \Delta_{\mathcal{S},m} &(59)\\ &= 2^{-n\left(\mathbb{E}_{\boldsymbol{\pi}_{x^n(m)}(x)}\left[D(\bar{P}_{Z|X}\|P_{Z|X})\right]+o(1)\right)} - \Delta_{\mathcal{S},m}, &(60)\end{aligned}$$

where the last equality holds by a conditional version of Sanov's theorem. Therefore,

$$\Delta_{\mathcal{S},m} \geq 2^{-n\left(\mathbb{E}_{\boldsymbol{\pi}_{x^n(m)}}\left[D(\bar{P}_{Z|X}\|P_{Z|X})\right]+o(1)\right)} - \alpha_n, \tag{61}$$

and by the condition on the type-I error probability (8) and our assumption (46)–(47):

$$-\frac{1}{n}\log\Delta_{\mathcal{S},m} \leq \mathbb{E}_{\boldsymbol{\pi}_{x^n(m)}}\left[D(\bar{P}_{Z|X}\|P_{Z|X})\right] + o(1). \tag{62}$$

We next observe that

$$\begin{aligned}\frac{1}{n}\sum_{\substack{t\in\{1,\ldots,n\}:\\ x_t(\tilde{M})=x}} P_{\tilde{Z}_t}(z) &= \frac{1}{n}\sum_{\substack{t\in\{1,\ldots,n\}:\\ x_t(\tilde{M})=x}} \mathbb{1}\{\tilde{Z}_t = z\} &(63)\\ &= \frac{1}{n}\sum_{t\in\{1,\ldots,n\}} \mathbb{1}\{x_t(\tilde{M}) = x, \tilde{Z}_t = z\} &(64)\\ &= \boldsymbol{\pi}_{x^n(m)\tilde{Z}^n}(x,z) &(65)\\ &= \boldsymbol{\pi}_{x^n(m)}(x)\bar{P}_{Z|X}(z|x) + o(1), &(66)\end{aligned}$$

where the last equation holds by the type-condition (48b).

From the definition of β_n and by combining (56), (62), and (66), we obtain:

$$-\frac{1}{n}\log\beta_n$$

$$\leq -\frac{1}{n}\log \Pr\left[\hat{\mathcal{H}}=0 \middle| \mathcal{H}=1, M=m\right] \tag{67}$$

$$\leq \sum_x \boldsymbol{\pi}_{x^n(m)}(x) \sum_z \bar{P}_{Z|X}(z|x) \frac{P_{Z|X}(z|x)}{Q_{Z|X}(z|x)}$$

$$+\mathbb{E}_{\boldsymbol{\pi}_{x^n(m)}}\left[D(\bar{P}_{Z|X} \| P_{Z|X})\right] + o(1) \tag{68}$$

$$\leq \sum_x \boldsymbol{\pi}_{x^n(m)}(x) D(\bar{P}_{Z|X}(\cdot|x) \| Q_{Z|X}(\cdot|x)) + o(1) \tag{69}$$

$$\leq \sum_x \mathbb{E}_M[\boldsymbol{\pi}_{x^n(M)}(x)] D(\bar{P}_{Z|X}(\cdot|x) \| Q_{Z|X}(\cdot|x)) + o(1), \tag{70}$$

where the last step holds by (47).

The desired converse is then immediately established by considering the accumulation point of the increasing blocklengths $\{n_i\}$, and by using definition (18).

IV. Conclusion and Future Directions

We established the strong converse for a memoryless ISAC problem with bi-static radar when the sensing performance is measured in terms of the largest exponential decay rates of the detection error probabilities under the two hypotheses. Notice that our model also includes as special case the setups where the receiver has perfect or imperfect channel-state information by including this state-information as part of the output.

Interesting future research directions include extensions to mono-static radar systems where the transmitter can apply closed-loop encodings depending also on past generalized feedback systems or systems with memory. Analyzing other sensing criteria is also of interest, such as the estimation error when the distribution of the state-sequence depends on a single continuous-valued parameter as for example the Doppler shift in a radar application.

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References

[1] O. Li, J. He, K. Zeng, Z. Yu, X. Du, Z. Zhou, Y. Liang, G. Wang, Y. Chen, P. Zhu, W. Tong, D. Lister, and L. Ibbetson, "Integrated sensing and communication in 6G: a prototype of high resolution multichannel THz sensing on portable device," *EURASIP Journal on Wireless Communications and Networking*, vol. 2022, no. 1, p. 106, 2022.

[2] M. Chafii, L. Bariah, S. Muhaidat, and M. Debbah, "Twelve scientific challenges for 6G: Rethinking the foundations of communications theory," 2023. [Online]. Available: https://arxiv.org/abs/2207.01843

[3] S. H. Dokhanchi, M. B. Shankar, M. Alaee-Kerahroodi, T. Stifter, and B. Ottersten, "Adaptive waveform design for automotive joint radar-communications system," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*. IEEE, 2019, pp. 4280–4284.

[4] F. Liu, Y. Cui, C. Masouros, J. Xu, T. X. Han, Y. C. Eldar, and S. Buzzi, "Integrated sensing and communications: Toward dual-functional wireless networks for 6G and beyond," *IEEE Journal on Selected Areas in Communications*, vol. 40, no. 6, pp. 1728–1767, 2022.

[5] C. Sturm and W. Wiesbeck, "Waveform design and signal processing aspects for fusion of wireless communications and radar sensing," *Proceedings of the IEEE*, vol. 99, no. 7, pp. 1236–1259, 2011.

[6] M. P. Roberton and E. Brown, "Integrated radar and communications based on chirped spread-spectrum techniques," *IEEE MTT-S International Microwave Symposium Digest, 2003*, vol. 1, pp. 611–614, 2003.

[7] G. N. Saddik, R. S. Singh, and E. R. Brown, "Ultra-wideband multifunctional communications/radar system," *IEEE Transactions on Microwave Theory and Techniques*, vol. 55, no. 7, pp. 1431–1437, 2007.

[8] A. Liu, Z. Huang, M. Li, Y. Wan, W. Li, T. X. Han, C. Liu, R. Du, D. K. P. Tan, J. Lu, Y. Shen, F. Colone, and K. Chetty, "A survey on fundamental limits of integrated sensing and communication," *IEEE Communications Surveys & Tutorials*, vol. 24, no. 2, pp. 994–1034, 2022.

[9] F. Liu, L. Zheng, Y. Cui, C. Masouros, A. P. Petropulu, H. Griffiths, and Y. C. Eldar, "Seventy years of radar and communications: The road from separation to integration," *IEEE Signal Processing Magazine*, vol. 40, no. 5, pp. 106–121, 2023.

[10] C. Ouyang, Y. Liu, H. Yang, and N. Al-Dhahir, "Integrated sensing and communications: A mutual information-based framework," *IEEE Communications Magazine*, vol. 61, no. 5, pp. 26–32, 2023.

[11] A. Liu, M. Li, M. Kobayashi, and G. Caire, *Fundamental Limits for ISAC: Information and Communication Theoretic Perspective*. Singapore: Springer Nature Singapore, 2023, pp. 23–52.

[12] M. Kobayashi, G. Caire, and G. Kramer, "Joint state sensing and communication: Optimal tradeoff for a memoryless case," in *2018 IEEE International Symposium on Information Theory (ISIT)*, 2018, pp. 111–115.

[13] M. Kobayashi, H. Hamad, G. Kramer, and G. Caire, "Joint state sensing and communication over memoryless multiple access channels," in *2019 IEEE International Symposium on Information Theory (ISIT)*, 2019, pp. 270–274.

[14] M. Ahmadipour, M. Kobayashi, M. Wigger, and G. Caire, "An information-theoretic approach to joint sensing and communication," *IEEE Transactions on Information Theory*, pp. 1–1, 2022.

[15] M. Ahmadipour, M. Wigger, and M. Kobayashi, "Coding for sensing: An improved scheme for integrated sensing and communication over MACs," in *2022 IEEE International Symposium on Information Theory (ISIT)*, 2022, pp. 3025–3030.

[16] O. Günlü, M. R. Bloch, R. F. Schaefer, and A. Yener, "Secure integrated sensing and communication," *IEEE Journal on Selected Areas in Information Theory*, vol. 4, pp. 40–53, 2023.

[17] H. Joudeh and F. M. J. Willems, "Joint communication and binary state detection," *IEEE Journal on Selected Areas in Information Theory*, vol. 3, no. 1, pp. 113–124, 2022.

[18] H. Wu and H. Joudeh, "On joint communication and channel discrimination," in *2022 IEEE International Symposium on Information Theory (ISIT)*, 2022, pp. 3321–3326.

[19] M.-C. Chang, S.-Y. Wang, T. Erdoğan, and M. R. Bloch, "Rate and detection-error exponent tradeoff for joint communication and sensing of fixed channel states," *IEEE Journal on Selected Areas in Information Theory*, vol. 4, pp. 245–259, 2023.

[20] M. Ahmadipour, M. Wigger, and S. Shamai, "Strong converses for memoryless bi-static ISAC," in *2023 IEEE International Symposium on Information Theory (ISIT)*, 2023, pp. 1818–1823.

[21] G. Choi and N. Lee, "Information-theoretical approach to integrated pulse-doppler radar and communication systems," 2023. [Online]. Available: https://arxiv.org/abs/2302.05046

[22] R. Ahlswede, "Certain results in coding theory for compound channels," in *Proc. Coll. Information Theory*, vol. 1, 1967, pp. 35–60.

[23] A. Lapidoth and P. Narayan, "Reliable communication under channel uncertainty," *IEEE Transactions on Information Theory*, vol. 44, no. 6, pp. 2148–2177, 1998.

[24] M. Hamad, M. Wigger, and M. Sarkiss, "Strong converses using typical changes of measures and asymptotic markov chains," *IEEE Transactions on Information Theory*, pp. 1–1, 2023.

[25] W. Gu and M. Effros, "A strong converse for a collection of network source coding problems," in *2009 IEEE International Symposium on Information Theory*. IEEE, 2009, pp. 2316–2320.

[26] ——, "A strong converse in source coding for super-source networks," in *2011 IEEE International Symposium on Information Theory*. IEEE, 2011, pp. 395–399.

[27] T. Han and K. Kobayashi, "Exponential-type error probabilities for multiterminal hypothesis testing," *IEEE Transactions on Information Theory*, vol. 35, no. 1, pp. 2–14, 1989.

[28] H. Tyagi and S. Watanabe, "Strong converse using change of measure arguments," *IEEE Transactions on Information Theory*, vol. 66, no. 2, pp. 689–703, 2019.

[29] I. Csiszár and J. Körner, *Information theory: coding theorems for discrete memoryless systems*. Cambridge University Press, 2011.