Joint Coding of eMBB and URLLC in Vehicle-to-Everything (V2X) Communications

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***Abstract*—A point-to-point communication is considered where a roadside unite (RSU) wishes to simultaneously send messages of enhanced mobile broadband (eMBB) and ultra-reliable low-latency communication (URLLC) services to a vehicle. The eMBB message arrives at the beginning of a block and its transmission lasts over the entire block. During each eMBB transmission block, random arrivals of URLLC messages are assumed. To improve the reliability of the URLLC transmissions, the RSU reinforces their transmissions by mitigating the interference of eMBB transmission by means of dirty paper coding (DPC). In the proposed coding scheme, the eMBB messages are decoded based on two approaches: treating interference as noise, and successive interference cancellation. Rigorous bounds are derived for the error probabilities of eMBB and URLLC transmissions achieved by our scheme. Numerical results illustrate that they are lower than bounds for standard time-sharing.**

I. Introduction

Enhanced mobile broadband (eMBB) and ultra-reliable low-latency communication (URLLC) services enabled by 5G new radio (NR) are considered as key enablers of the vehicle-to-everything (V2X) technology [1]–[6]. Particularly, eMBB services aim to provide high data rate for content delivery and therefore improve the quality of experience (QoE) of in-vehicle entertainment applications. URLLC services, however, are key to guarantee the delivery of critical road safety information and thus enable fully autonomous driving of connected vehicles [7], [8].

Coexistence of eMBB and URLLC services in V2X communications has been studied in the literature [9]–[11]. In [9], a novel URLLC and eMBB coexistence mechanism for the cellular V2X framework is proposed where at the begining of the transmission interval eMBB users are associated with a V2X base station, whereas, URLLC users are allowed to puncture the eMBB transmissions upon arrival. The work in [10] formulates an optimization problem for joint scheduling of punctured eMBB and URLLC traffic to maximize the aggregate utility of the eMBB users subject to latency constraints for the URLLC users. Related to this work is [11], where resources are allocated jointly between eMBB and URLLC messages for a one-way highway vehicular network in which a vehicle receives an eMBB message from the nearest roadside unit (RSU) and URLLC messages from the nearest vehicle. During each eMBB transmission interval, random arrivals of URLLC messages are assumed. The eMBB time slot is thus divided into mini-slots and the newly arrived URLLC messages are immediately scheduled in the next mini-slot by puncturing the on-going eMBB transmissions. To guarantee the reliability of the URLLC transmission, guard zones are deployed around the vehicle and the eMBB transmissions are not allowed inside such zones.

In this work, the RSU wishes to transmit both eMBB and URLLC messages to a vehicle. The eMBB message arrives at the beginning of a block and its transmission lasts over the entire block. The eMBB blocklength is again divided into mini-slots and URLLC messages arrive randomly at the beginning of these mini-slots. Specifically, at the beginning of each of these mini-slots a URLLC message arrives with probability $\rho \in [0, 1]$ and the RSU simultaneously sends the eMBB message as well as the newly arrived URLLC message over this mini-slot. With probability $1-\rho$ no URLLC message arrives at the beginning of the mini-slot and the RSU only sends the eMBB message. In our work, we do not use guard zones, but instead the RSU reinforces transmission of URLLC messages by mitigating the interference of eMBB transmission by means of dirty paper coding [12]–[14]. After each mini-slot, the receiving vehicle attempts to decode a URLLC message, and after the entire transmission interval it decodes the eMBB message. Given that the URLLC transmissions interfere with the transmission of eMBB, we employ two different eMBB decoding approaches. The first approach, known as *treating interference as noise (TIN)*, is to treat the URLLC interference as noise. The second approach, known as *successive interference cancellation (SIC)*, is to first subtract the decoded URLLC message and then decode the eMBB message based on the received signal. Rigorous bounds are derived for achievable error probabilities of eMBB (in both approaches) and URLLC transmissions. Numerical results illustrate that our proposed scheme significantly outperforms the standard time-sharing scheme.

II. Problem Setup

Consider a point-to-point setup with one RSU (transmitter) and one vehicle (receiver) communicating over a n_{e} uses of an AWGN channel. The transmitter (Tx) sends a single, so called *eMBB*-type message $M^{(\mathsf{e})}$, over the entire blocklength n_{e}, where $M^{(\mathsf{e})}$ is uniformly distributed over a given set $\mathcal{M}^{(\mathsf{e})} := \{1, \ldots, L_{\mathsf{e}}\}$. Message $M^{(\mathsf{e})}$ is thus available at the Tx at time $t = 1$ (and remains until time n_{e}). Additionally, prior to each channel use in

$$\mathcal{T}^{(\mathsf{U})} := \{1, 1 + n_{\mathsf{U}}, 1 + 2n_{\mathsf{U}}, \ldots, 1 + (\eta - 1)\, n_{\mathsf{U}}\}, \quad (1)$$

where

$$\eta := \left\lfloor \frac{n_{\mathsf{e}}}{n_{\mathsf{U}}} \right\rfloor, \tag{2}$$

the Tx generates with probability ρ an additional, so called, *URLLC*-type message that it wishes to convey to the Rx. With probability $1-\rho$ no URLLC-type message is generated. For each $b \in [\eta]$, if a URLLC message is generated at time $t = (b-1)n_{\mathsf{U}}+1$, then we set $A_b = 1$, and otherwise we set $A_b = 0$. Denote the time-instances from $(b-1)\cdot n_{\mathsf{U}}+1$ to $b \cdot n_{\mathsf{U}}$ by block b. If in block b a message is generated we denote it by $M_b^{(\mathsf{U})}$ and assume that it is uniformly distributed over the set $\mathcal{M}^{(\mathsf{U})} := \{1,\ldots,L_{\mathsf{U}}\}$.

During block b, the Tx computes its inputs as:

$$X_t = \begin{cases} f_t^{(\mathsf{U})}\left(M_b^{(\mathsf{U})}, M^{(\mathsf{e})}\right), & \text{if } A_b = 1, \\ f_t^{(\mathsf{e})}\left(M^{(\mathsf{e})}\right), & \text{if } A_b = 0, \end{cases} \tag{3}$$

for $t = (b-1)\cdot n_{\mathsf{U}}+1,\ldots,b\cdot n_{\mathsf{U}}$ and some encoding functions $f_t^{(\mathsf{U})}$ and $f_t^{(\mathsf{e})}$ on appropriate domains. After the last URLLC block, i.e. at times $t = \eta n_{\mathsf{U}}+1,\ldots,n_{\mathsf{e}}$, the Tx produces the inputs

$$X_t = f_t^{(\mathsf{e})}\left(M^{(\mathsf{e})}\right), \quad t = \eta n_{\mathsf{U}}+1,\ldots,n_{\mathsf{e}}. \tag{4}$$

The sequence of channel inputs $X_1,\ldots,X_{n_{\mathsf{e}}}$ has to satisfy the average block-power constraint

$$\frac{1}{n_{\mathsf{e}}}\sum_{t=1}^{n_{\mathsf{e}}} X_t^2 \leq \mathsf{P}, \qquad \text{almost surely.} \tag{5}$$

The input-output relation of the network is described as

$$Y_t = hX_t + Z_t, \tag{6}$$

where $\{Z_t\}$ are independent and identically distributed (i.i.d.) standard Gaussian for all t and independent of all messages; $h > 0$ is the fixed channel coefficient between the Tx and Rx.

After each URLLC block b the receiver (Rx) decodes the transmitted URLLC message $M_b^{(\mathsf{U})}$ if $A_b = 1$. Moreover, at the end of the entire n_{e} channel uses it decodes the eMBB message $M^{(\mathsf{e})}$. Thus, if $A_b = 1$ it produces

$$\hat{M}_b^{(\mathsf{U})} = g^{(n_{\mathsf{U}})}\left(Y_{(b-1)n_{\mathsf{U}}+1},\ldots,Y_{bn_{\mathsf{U}}}\right), \tag{7}$$

for some decoding function $g^{(n_{\mathsf{U}})}$ on appropriate domains. Otherwise, it sets $\hat{M}_b^{(\mathsf{U})} = 0$. We define the average error probability for each message $M_b^{(\mathsf{U})}$ as:

$$\begin{aligned}\epsilon_b^{(\mathsf{U})} := {} & \rho\mathbb{P}\left[\hat{M}_b^{(\mathsf{U})} \neq M_b^{(\mathsf{U})} \middle| A_b = 1\right] \\ & + (1-\rho)\mathbb{P}\left[\hat{M}_b^{(\mathsf{U})} \neq 0 \middle| A_b = 0\right]. \end{aligned} \tag{8}$$

At the end of the n_{e} channel uses, the Rx decodes its desired eMBB message as:

$$\hat{M}^{(\mathsf{e})} = \psi^{(n_{\mathsf{e}})}\left(\boldsymbol{Y}^{n_{\mathsf{e}}}\right), \tag{9}$$

where $\boldsymbol{Y}^{n_{\mathsf{e}}} := (Y_1,\ldots,Y_{n_{\mathsf{e}}})$ and $\psi^{(n_{\mathsf{e}})}$ is a decoding function on appropriate domains. We define the average error probability for message $M^{(\mathsf{e})}$ as

$$\epsilon^{(\mathsf{e})} := \mathbb{P}\left[\hat{M}^{(\mathsf{e})} \neq M^{(\mathsf{e})}\right]. \tag{10}$$

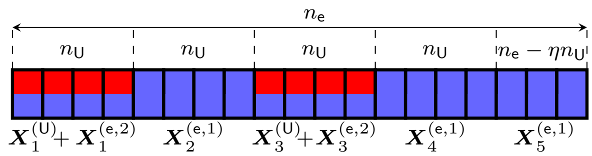

Fig. 1: Example of the coding scheme with $\eta = 4$ and $\mathcal{B}_{\text{sent}} = \{1,3\}$.

The goal is to propose a coding scheme that simultaneously has small error probabilities $\epsilon_b^{(\mathsf{U})}$ and $\epsilon^{(\mathsf{e})}$.

III. Joint Transmission of URLLC and eMBB Messages

A. Construction of Codebooks

Define

$$\mathcal{B}_{\text{arrival}} := \{b \in [\eta] : A_b = 1\}. \tag{11}$$

Choose β_{U} and $\beta_{\mathsf{e}} \in [0,1]$ such that:

$$\beta_{\mathsf{U}} + \beta_{\mathsf{e}} = 1. \tag{12}$$

Fix a value of $\alpha \in [0,1]$. For each block $b \in [\eta]$, for each $j \in [L_v]$ and each realization $m \in [L_{\mathsf{U}}]$, generate codewords $\boldsymbol{V}_b(m,j)$ by picking them uniformly over a centered n_{U}-dimensional sphere of radius $\sqrt{n_{\mathsf{U}}\beta_{\mathsf{v}}\mathsf{P}}$ independently of each other and of all other codewords, for

$$\beta_{\mathsf{v}} := \beta_{\mathsf{U}} + \alpha^2\beta_{\mathsf{e}}. \tag{13}$$

For each $\ell \in [L_{\mathsf{e}}]$ randomly draw a codeword $\boldsymbol{X}_b^{(\mathsf{e},2)}(\ell)$ uniformly distributed on the centered n_{U}-dimensional sphere of radius $\sqrt{n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}}$ and a codeword $\boldsymbol{X}_b^{(\mathsf{e},1)}(\ell)$ uniformly distributed on the centered n_{U}-dimensional sphere of radius $\sqrt{n_{\mathsf{U}}\mathsf{P}}$. All codewords are chosen independently of each other.

B. Encoding

1) Encoding at Blocks $b \in \mathcal{B}_{arrival}$: In each block $b \in \mathcal{B}_{\text{arrival}}$, the Tx has both an eMBB and an URLLC message to send. It first picks the codeword $\boldsymbol{X}_b^{(\mathsf{e},2)}(M^{(\mathsf{e})})$ and then employs DPC to encode $M_b^{(\mathsf{U})}$ while precanceling the interference of its own eMBB codeword $\boldsymbol{X}_b^{(\mathsf{e},2)}(M^{(\mathsf{e})})$. Specifically, it chooses an index j such that the sequence

$$\boldsymbol{X}_b^{(\mathsf{U})} := \boldsymbol{V}_b(M_b^{(\mathsf{U})}, j) - \alpha\boldsymbol{X}_b^{(\mathsf{e},2)} \tag{14}$$

lies in the set

$$\mathcal{D}_b := \left\{\boldsymbol{x}_b^{(\mathsf{U})} : n_{\mathsf{U}}\beta_{\mathsf{U}}\mathsf{P} - \delta_b \leq \left\|\boldsymbol{x}_b^{(\mathsf{U})}\right\|^2 \leq n_{\mathsf{U}}\beta_{\mathsf{U}}\mathsf{P}\right\} \tag{15}$$

for a given $\delta_b > 0$. If multiple such codewords exist, the index $j^\star$ is chosen at random from this set, and the Tx sends:

$$\boldsymbol{X}_b = \boldsymbol{X}_b^{(\mathsf{U})} + \boldsymbol{X}_b^{(\mathsf{e},2)}. \tag{16}$$

We also set $A_{b,\text{sent}} = 1$.

If no appropriate codeword exists, the Tx discards the arrived URLLC message by setting $A_{b,\text{sent}} = 0$ and sends only the eMBB message

$$\boldsymbol{X}_b = \boldsymbol{X}_b^{(\mathsf{e},1)}(M^{(\mathsf{e})}) \tag{17}$$

over this block.

Define

$$\mathcal{B}_{\text{sent}} := \{b \in \mathcal{B}_{\text{arrival}} : A_{b,\text{sent}} = 1\}, \tag{18}$$

where $\mathcal{B}_{\text{sent}} \subseteq \mathcal{B}_{\text{arrival}}$ and represents the set of blocks in which an URLLC message is sent. See Figure 1.

2) Encoding at Blocks $b \in [\eta]\backslash\mathcal{B}_{arrival}$ and in Block $\eta+1$ when $n_{\mathsf{e}} > \eta n_{\mathsf{U}}$: In each Block $b \in [\eta]\backslash\mathcal{B}_{\text{arrival}}$, the Tx sends only eMBB message $M^{(\mathsf{e})}$:

$$\boldsymbol{X}_b = \boldsymbol{X}_{b,1}^{(\mathsf{e})}(M^{(\mathsf{e})}). \tag{19}$$

Over Block b, the Tx thus transmits

$$\boldsymbol{X}_b = \begin{cases} \boldsymbol{X}_b^{(\mathsf{U})} + \boldsymbol{X}_b^{(\mathsf{e},2)} & \text{if } b \in \mathcal{B}_{\text{sent}}, \\ \boldsymbol{X}_b^{(\mathsf{e},1)} & \text{o.w.} \end{cases} \tag{20}$$

C. Decoding

After each block $b \in [\eta]$, the Rx attempts to decode a URLLC message, and after the entire block of n_{e} channel uses it decodes the transmitted eMBB message. Given that the URLLC transmissions interfere with the transmission of eMBB, the Rx envisions two different approaches to decode the eMBB message. The first approach, termed *TIN approach*, is to treat the URLLC interference as noise. The second approach, termed *SIC approach*, is to first subtract the decoded URLLC message and then decode the eMBB message based on the received signal.

1) Decoding of URLLC Messages: At the end of each block $b \in [\eta]$, the Rx observes the following channel outputs $\boldsymbol{Y}_b := \{Y_{(b-1)n_{\mathsf{U}}+1}, \ldots, Y_{bn_{\mathsf{U}}}\}$:

$$\boldsymbol{Y}_b = \begin{cases} h\boldsymbol{X}_b^{(\mathsf{U})} + h\boldsymbol{X}_b^{(\mathsf{e},2)} + \boldsymbol{Z}_b & \text{if } b \in \mathcal{B}_{\text{sent}} \\ h\boldsymbol{X}_b^{(\mathsf{e},1)} + \boldsymbol{Z}_b & \text{o.w.} \end{cases} \tag{21}$$

with $\boldsymbol{Z}_b \sim \mathcal{N}(0, I_{n_{\mathsf{U}}})$. Define the information density metric between $\boldsymbol{y}_b$ and $\boldsymbol{v}_b$ by:

$$i_b^{(\mathsf{U})}(\boldsymbol{v}_b; \boldsymbol{y}_b) := \ln \frac{f_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{v}_b)}{f_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)}. \tag{22}$$

After observing $\boldsymbol{Y}_b$, the Rx chooses the pair

$$(m', j') = \arg\max_{m,j} i_b^{(\mathsf{U})}(\boldsymbol{v}_b(m,j); \boldsymbol{Y}_b). \tag{23}$$

If for this pair

$$i_b^{(\mathsf{U})}(\boldsymbol{v}_b(m',j'); \boldsymbol{Y}_b) > \gamma^{(\mathsf{U})} \tag{24}$$

where $\gamma^{(\mathsf{U})}$ is a threshold over which we optimize, the Rx chooses $(\hat{M}_b^{(\mathsf{U})}, \hat{j}) = (m', j')$ and sets $A_{b,\text{detection}} = 1$. Otherwise the receiver declares that no URLLC message has been sent and indicates it by setting $\hat{M}_b^{(\mathsf{U})} = 0$ and $A_{b,\text{detection}} = 0$.

Define

$$\mathcal{B}_{\text{detect}} := \{b \in [\eta] : A_{b,\text{detection}} = 1\} \tag{25}$$

that is the set of blocks in which an URLLC message is detected. A detection error happens if $\mathcal{B}_{\text{detect}} \neq \mathcal{B}_{\text{sent}}$.

In each block $b \in \mathcal{B}_{\text{detect}}$, set $A_{b,\text{decode}} = 1$ if $(\hat{M}_b^{(\mathsf{U})}, \hat{j}) = (M_b^{(\mathsf{U})}, j)$, otherwise set $A_{b,\text{decode}} = 0$. Define

$$\mathcal{B}_{\text{decode}} := \{b \in \mathcal{B}_{\text{detect}} : A_{b,\text{decode}} = 1\} \tag{26}$$

that is the set of blocks in which an URLLC message is decoded correctly.

2) Decoding the eMBB Message under the TIN approach: To decode its desired eMBB message under this approach, the Rx treats URLLC transmissions as noise. Therefore, the decoding of the eMBB message depends on the detection of URLLC messages sent over the η blocks.

Let B_{dt} be the realization of the set $\mathcal{B}_{\text{detect}}$ defined in (25). Given B_{dt}, the Rx decodes its desired eMBB message based on the outputs of the entire n_{e} channel uses by looking for an index m such that its corresponding codewords $\left\{\{\boldsymbol{x}_b^{(\mathsf{e},1)}(m)\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}(m)\}_{b\in B_{\text{dt}}}\right\}$ maximize

$$\begin{aligned} & i_{\text{TIN}}^{(\mathsf{e})}\left(\{\boldsymbol{x}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \boldsymbol{y}^{n_{\mathsf{e}}}|\mathcal{B}_{\text{detect}} = B_{\text{dt}}\right) \\ & := \ln\prod_{b\notin B_{\text{dt}}} \frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},1)}}(\boldsymbol{y}_b|\boldsymbol{x}_{b,1}^{(\mathsf{e})})}{f_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} + \ln\prod_{b\in B_{\text{dt}}} \frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)}}(\boldsymbol{y}_b|\boldsymbol{x}_{b,2}^{(\mathsf{e})})}{f_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} \end{aligned} \tag{27}$$

among all codewords $\{\{\boldsymbol{x}_b^{(\mathsf{e},1)}(m')\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}(m')\}_{b\in B_{\text{dt}}}\}$.

3) Decoding the eMBB Message under the SIC approach: Under this approach, before decoding the desired eMBB message, the Rx mitigates the interference of the correctly decoded URLLC messages from its observed output signal. Therefore, the decoding of the eMBB message depends not only on the detection of the sent URLLC messages but also on the decoding of such messages.

For each Block $b \in \mathcal{B}_{\text{detect}}$, we define $A_{b,\text{decode}} = 1$ if $(\hat{M}_b^{(\mathsf{U})}, \hat{j}) = (M_b^{(\mathsf{U})}, j)$, otherwise set $A_{b,\text{decode}} = 0$. Define the set of blocks in which an URLLC message is decoded correctly:

$$\mathcal{B}_{\text{decode}} := \{b \in \mathcal{B}_{\text{detect}} : A_{b,\text{decode}} = 1\}. \tag{28}$$

Let B_{dt} be a realization of the set $\mathcal{B}_{\text{detect}}$ and B_{dc} be a realization of the set $\mathcal{B}_{\text{decode}}$. After observing the channel outputs of the entire n_{e} channel uses, the Rx decodes its desired eMBB message by looking for an index m such that its corresponding codewords $\left\{\{\boldsymbol{x}_b^{(\mathsf{e},1)}(m)\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}(m)\}_{b\in B_{\text{dt}}}\right\}$ maximize

$$\begin{aligned} & i_{\text{SIC}}^{(\mathsf{e})}\left(\{\boldsymbol{x}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \boldsymbol{y}^{n_{\mathsf{e}}}|B_{\text{dt}}, B_{\text{dc}}, \{\boldsymbol{V}_b\}_{b\in B_{\text{dc}}}\right) \\ & := \ln\prod_{b\notin B_{\text{dt}}} \frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},1)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},1)})}{f_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} + \ln\prod_{b\in B_{\text{dt}}\backslash B_{\text{dc}}} \frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},2)})}{f_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} \\ & + \ln\prod_{b\in B_{\text{dc}}} \frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)},\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},2)}, \boldsymbol{v}_b)}{f_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{v}_b)} \end{aligned} \tag{29}$$

among all codewords $\{\{\boldsymbol{x}_b^{(\mathsf{e},1)}(m')\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}(m')\}_{b\in B_{\text{dt}}}\}$.

IV. Main Results

Define $\sigma^2 := h^2\mathsf{P}+1$, $\sigma_2^2 := h^2\beta_{\mathsf{v}}\mathsf{P}+1$, $\sigma_3^2 := h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P}+1$ and

$$\lambda(x) := \frac{x}{2}+\frac{u^2}{4}-\frac{u}{2}\sqrt{x+\frac{u^2}{4}}, \tag{31a}$$

$$\tilde{\lambda}(x) := \frac{x}{2}+\frac{u^2}{4}+\frac{u}{2}\sqrt{x+\frac{u^2}{4}}, \tag{31b}$$

$$u := \frac{2\sqrt{n_{\mathsf{U}}\mathsf{P}}\left(\sigma_3^2(\sqrt{\beta_{\mathsf{U}}}+\sqrt{\beta_{\mathsf{e}}})+\sigma^2\sqrt{\beta_{\mathsf{e}}}(1-\alpha)\right)}{h(\sigma^2-\sigma_3^2)}, \tag{31c}$$

$$\tau := \frac{\sqrt{n_{\mathsf{U}}\mathsf{P}}\left(\sqrt{\beta_{\mathsf{v}}}(\sigma^2+\sigma_2^2)+(1-\alpha)\sqrt{\beta_{\mathsf{e}}}\sigma_2^2\right)}{\sigma^2\sigma_2^2}, \tag{31d}$$

and for all integer values $n = 1, 2, \ldots$:

$$\kappa_n(x) := \frac{x(1-x^2)^n}{2n+1}+\frac{2n}{2n+1}\kappa_{n-1}(x) \tag{31e}$$

where $\kappa_0(x) := x$. By employing the scheme proposed in Section III, we have the following theorem on the upper bounds on the URLLC and eMBB error probabilities $\epsilon_b^{(\mathsf{U})}$, $\epsilon_{\text{TIN}}^{(\mathsf{e})}$, and $\epsilon_{\text{SIC}}^{(\mathsf{e})}$.

Theorem 1: For fixed β_{e} , $\beta_{\mathsf{U}} \in [0,1]$ and message set sizes L_{U} and L_{e}, the average error probabilities $\epsilon_b^{(\mathsf{U})}$, $\epsilon_{\text{TIN}}^{(\mathsf{e})}$, and $\epsilon_{\text{SIC}}^{(\mathsf{e})}$ are bounded by

$$\epsilon_b^{(\mathsf{U})} \le \rho\left((1-\zeta)^{L_v}+q+1-q_2\right)+(1-\rho)q_1 \tag{32}$$

$$\epsilon_{\text{TIN}}^{(\mathsf{e})} \le \sum_{k=0}^{\eta}\binom{\eta}{k}q_3^k(1-\rho_{\mathsf{U}}q_2)^{\eta-k}\left(1-\Delta+T\right) \tag{33}$$

$$\epsilon_{\text{SIC}}^{(\mathsf{e})} \le \sum_{k=0}^{\eta}\binom{\eta}{k}q_4^k(1-\rho_{\mathsf{U}}q_2)^{\eta-k} \cdot\left(1-\Delta+\sum_{\tilde{k}=0}^{k}\binom{k}{\tilde{k}}q^{\tilde{k}}(1-q)^{k-\tilde{k}}\left(\frac{\mu T}{\tilde{\mu}}-\nu\right)\right), \tag{34}$$

where $\gamma^{(\mathsf{U})}, \gamma^{(\mathsf{e})}, \tilde{\gamma}^{(\mathsf{e})}$ are arbitrary positive parameters, $G(\cdot,\cdot)$ denotes the regularized gamma function, $k := |B_{\text{dt}}|$, $\tilde{k} = |B_{\text{dc}}|$, $\rho_{\mathsf{U}} := \rho\left(1-(1-\zeta)^{L_v}\right)$, $q_3 := \rho_{\mathsf{U}}q_4+(1-\rho_{\mathsf{U}})q_1$, and

$$q := \sqrt[L_vL_{\mathsf{U}}]{1-q_2}+(L_vL_{\mathsf{U}}-1)e^{-\gamma^{(\mathsf{U})}}, \tag{35a}$$

$$q_1 := 1-\left(1-e^{-\gamma^{(\mathsf{U})}}\right)^{L_vL_{\mathsf{U}}}, \tag{35b}$$

$$q_2 := 1-\left(1-G\left(\frac{n_{\mathsf{U}}}{2},\lambda(\mu_{\mathsf{U}})\right)+G\left(\frac{n_{\mathsf{U}}}{2},\tilde{\lambda}(\mu_{\mathsf{U}})\right)\right)^{L_vL_{\mathsf{U}}} \tag{35c}$$

$$q_4 := 1-\left(1-G\left(\frac{n_{\mathsf{U}}}{2},\tilde{\lambda}(\tilde{\mu}_{\mathsf{U}})\right)+G\left(\frac{n_{\mathsf{U}}}{2},\lambda(\tilde{\mu}_{\mathsf{U}})\right)\right)^{L_vL_{\mathsf{U}}} \tag{35d}$$

$$\Delta := \frac{\rho_{\mathsf{U}}^k(1-\rho_{\mathsf{U}})^{\eta-k}q_2^k(1-q_1)^{\eta-k}}{(\rho_{\mathsf{U}}\cdot q_3+(1-\rho_{\mathsf{U}})\cdot q_1)^k(1-\rho_{\mathsf{U}}\cdot q_2)^{\eta-k}} \tag{35e}$$

$$J_{\mathsf{U}} := \frac{\pi\sqrt{\beta_{\mathsf{v}}\beta_{\mathsf{e}}}2^{\frac{n_{\mathsf{U}}+1}{2}}e^{-\frac{h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P}n_{\mathsf{U}}}{2}}}{9h^2(1-\alpha)(\beta_{\mathsf{v}}+(1-\alpha)^2\beta_{\mathsf{e}})}, \tag{35f}$$

$$\tilde{J}_{\mathsf{U}} := \frac{27\sqrt{\pi}(1+h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P})e^{n_{\mathsf{U}}h^2\mathsf{P}(\beta_{\mathsf{v}}+(1-\alpha)^2\beta_{\mathsf{e}})}}{2(h^2(1-\alpha))^{n_{\mathsf{U}}-2}\sqrt{8(1+2h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P}}}. \tag{35g}$$

and $J_e, \tilde{J}_e, \zeta, \mu_{\mathsf{U}}, \tilde{\mu}_{\mathsf{U}}, \mu, \tilde{\mu}, T$ and ν are defined in (30).

Proof: See Section VI. ■

$$J_{\mathsf{e}} := \left(\frac{\pi 2^{\frac{n_{\mathsf{U}}+1}{2}}e^{\frac{-h^2\beta_{\mathsf{v}}\mathsf{P}n_{\mathsf{U}}}{2}}\sqrt{\beta_{\mathsf{v}}\beta_{\mathsf{e}}}}{9h^2(1-\alpha)^{n_{\mathsf{U}}-1}(\beta_{\mathsf{v}}+(1-\alpha)^2\beta_{\mathsf{e}})}\right)^k\cdot\left(\frac{\sqrt{8(1+2h^2\mathsf{P})}}{27\sqrt{\pi}(1+h^2\mathsf{P})}\right)^{\eta-k} \tag{30a}$$

$$\tilde{J}_{\mathsf{e}} := \left(\frac{\pi 2^{\frac{n_{\mathsf{U}}+1}{2}}e^{\frac{-h^2\beta_{\mathsf{v}}\mathsf{P}n_{\mathsf{U}}}{2}}\sqrt{\beta_{\mathsf{v}}\beta_{\mathsf{e}}}}{9h^2(1-\alpha)^{n_{\mathsf{U}}-1}(\beta_{\mathsf{v}}+(1-\alpha)^2\beta_{\mathsf{e}})}\right)^{k-\tilde{k}}\cdot\left(\frac{\sqrt{8(1+2h^2\mathsf{P})}}{27\sqrt{\pi}(1+h^2\mathsf{P})}\right)^{\eta-k}\cdot\left(\frac{\sqrt{8(1+2h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P})}}{27\sqrt{\pi}(1+h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P})}\right)^{\tilde{k}} \tag{30b}$$

$$\zeta := \frac{1}{\sqrt{\pi}}\frac{\Gamma(\frac{n_{\mathsf{U}}}{2})}{\Gamma(\frac{n_{\mathsf{U}}-1}{2})}\left(\kappa_{\frac{n_{\mathsf{U}}-3}{2}}\left(\alpha\sqrt{\beta_{\mathsf{e}}/\beta_{\mathsf{v}}}+\delta_b/(2\alpha n_{\mathsf{U}}\mathsf{P}\sqrt{\beta_{\mathsf{v}}\beta_{\mathsf{e}}})\right)-\kappa_{\frac{n_{\mathsf{U}}-3}{2}}\left(\alpha\sqrt{\beta_{\mathsf{e}}/\beta_{\mathsf{v}}}\right)\right) \tag{30c}$$

$$\mu_{\mathsf{U}} := \frac{2\sigma^2\sigma_3^2}{h^2(\sigma^2-\sigma_3^2)}\left(\frac{n_{\mathsf{U}}}{2}\ln\frac{\sigma^2}{\sigma_3^2}-\gamma^{(\mathsf{U})}+\ln J_{\mathsf{U}}\right)+\frac{\sigma_3^2}{\sigma^2-\sigma_3^2}\left(n_{\mathsf{U}}\mathsf{P}(\sqrt{\beta_{\mathsf{U}}}-\sqrt{\beta_{\mathsf{e}}})^2-\delta_b\right)-\frac{\sigma^2n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}(1-\alpha)^2}{\sigma^2-\sigma_3^2} \tag{30d}$$

$$\tilde{\mu}_{\mathsf{U}} := \frac{2\sigma^2\sigma_3^2}{h^2(\sigma^2-\sigma_3^2)}\left(\frac{n_{\mathsf{U}}}{2}\ln\frac{\sigma^2}{\sigma_3^2}-\gamma^{(\mathsf{U})}+\ln\tilde{J}_{\mathsf{U}}\right)+\frac{\sigma_3^2}{\sigma^2-\sigma_3^2}\left(n_{\mathsf{U}}\mathsf{P}(\sqrt{\beta_{\mathsf{U}}}+\sqrt{\beta_{\mathsf{e}}})^2\right)-\frac{\sigma^2n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}(1-\alpha)^2}{\sigma^2-\sigma_3^2} \tag{30e}$$

$$\mu := \frac{n_{\mathsf{e}}}{2}\ln\sigma^2-\frac{kn_{\mathsf{U}}}{2}\ln\sigma_2^2-\frac{\eta-k}{2\sigma^2}n_{\mathsf{U}}\mathsf{P}+\frac{k}{2\sigma_2^2}\beta_{\mathsf{v}}n_{\mathsf{U}}\mathsf{P}-\frac{k}{2\sigma^2}\left(\sqrt{\beta_{\mathsf{v}}}+(1-\alpha)\sqrt{\beta_{\mathsf{e}}}\right)^2n_{\mathsf{U}}\mathsf{P}-\gamma^{(\mathsf{e})}+\ln J_{\mathsf{e}} \tag{30f}$$

$$\tilde{\mu} := \frac{n_{\mathsf{e}}}{2}\ln\sigma^2+n_{\mathsf{U}}\mathsf{P}\left(\frac{k-\tilde{k}}{2}\left(\frac{\beta_{\mathsf{v}}}{\sigma_2^2}-\frac{(\sqrt{\beta_{\mathsf{v}}}+(1-\alpha)\sqrt{\beta_{\mathsf{e}}})^2}{\sigma^2}-\frac{\ln\sigma_2^2}{\mathsf{P}}\right)+\frac{\tilde{k}}{2\mathsf{P}}\ln\frac{\sigma_3^2}{\sigma^2}-\frac{\eta-k}{2\sigma^2}-\frac{\tilde{k}(1-\alpha)^2\beta_{\mathsf{e}}}{2\sigma_3^2}\right)+\ln e^{-\tilde{\gamma}^{(\mathsf{e})}}\tilde{J}_{\mathsf{e}} \tag{30g}$$

$$T := \frac{(n_{\mathsf{e}}-kn_{\mathsf{U}})(\sigma^2-1)}{2\sigma^2\mu}+\frac{(\eta+1-k)\sqrt{n_{\mathsf{U}}\mathsf{P}}}{\sigma^2\mu}\frac{\sqrt{2}\Gamma\left(\frac{n_{\mathsf{U}}+1}{2}\right)}{\Gamma\left(\frac{n_{\mathsf{U}}}{2}\right)}+\frac{k\tau}{\mu}\frac{\sqrt{2}\Gamma\left(\frac{n_{\mathsf{U}}+1}{2}\right)}{\Gamma\left(\frac{n_{\mathsf{U}}}{2}\right)}+\frac{kn_{\mathsf{U}}(\sigma^2-\sigma_2^2)}{2\sigma^2\sigma_2^2\mu}+(L_e-1)e^{-\gamma^{(\mathsf{e})}} \tag{30h}$$

$$\nu := \frac{\tilde{k}}{\tilde{\mu}}\left(\frac{\sqrt{2}\Gamma\left(\frac{n_{\mathsf{U}}+1}{2}\right)}{\Gamma\left(\frac{n_{\mathsf{U}}}{2}\right)}\left(\tau-\frac{(1-\alpha)\sqrt{n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}}}{\sigma_3^2}\right)+n_{\mathsf{U}}\left(\frac{\sigma^2-\sigma_2^2}{2\sigma^2\sigma_2^2}-\frac{\sigma_3^2-1}{2\sigma_3^2}\right)\right)+(L_e-1)\left(\frac{\mu}{\tilde{\mu}}e^{-\tilde{\gamma}^{(\mathsf{e})}}+e^{-\tilde{\gamma}^{(\mathsf{e})}}\right) \tag{30i}$$

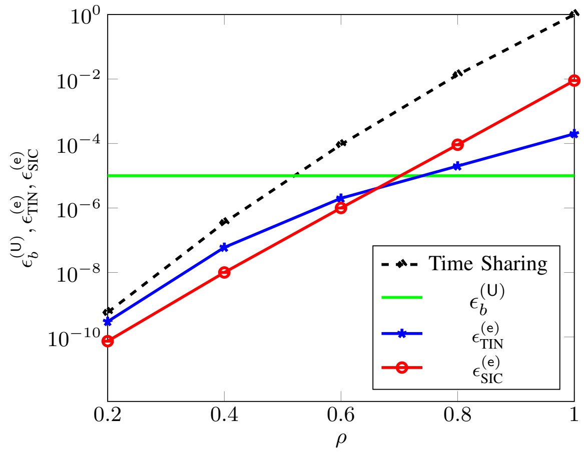

Fig. 2: Upper bounds on $\epsilon_{\text{TIN}}^{(\text{e})}, \epsilon_{\text{SIC}}^{(\text{e})}$ for $\mathsf{P} = 5, n_{\text{e}} = 600$ and $n_{\text{U}} = 200$ and for maximum value of $\epsilon_b^{(\text{U})}$ fixed at 10^{-5}.

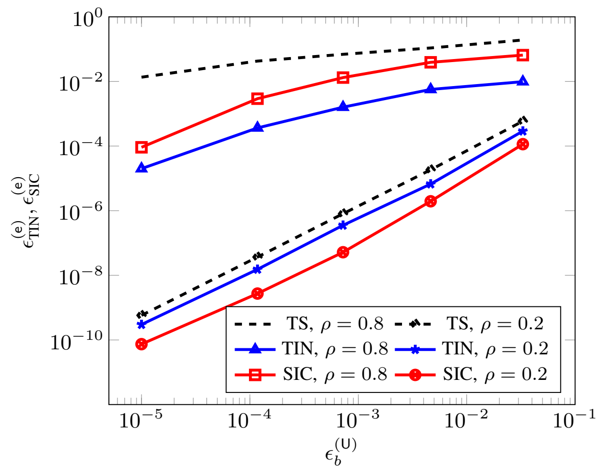

Fig. 3: Upper bounds on $\epsilon_{\text{TIN}}^{(\text{e})}, \epsilon_{\text{SIC}}^{(\text{e})}, \epsilon_b^{(\text{U})}$ for $\mathsf{P} = 5$ and $n_{\text{U}} = 20 \cdot b$ and $n_{\text{e}} = 3n_{\text{U}}$ for values of b in $\{10, 8, 6, 4, 2\}$.

V. Numerical Analysis

In Figure 2, we numerically compare the bounds in Theorem 1 with the time-sharing scheme where URLLC transmissions puncture the eMBB transmission upon arrival. In this figure, we set the maximum error probability of URLLC transmission to be equal to 10^{-5}. For each value of $\rho \in \{0.2, 0.4, 0.6, 0.8, 1\}$, we then optimize the parameters α, β_{e} and β_{U} to minimize the eMBB error probability under both TIN and SIC approaches. As can be seem from this figure, our schemes outperform the time-sharing scheme specifically for large values of ρ, i.e., in regions with dense URLLC arrivals.

In Figure 3, we numerically compare the bounds in Theorem 1 for $\rho = 0.2$ and $\rho = 0.8$. In this plot, $n_{\text{U}} = 20 \cdot b$ and $n_{\text{e}} = 3n_{\text{U}}$ and the value of b varies from 10 to 2 with step size 2. The values of α, β_{e} and β_{U} are optimized to minimize $\epsilon_{\text{TIN}}^{(\text{e})}$ and $\epsilon_{\text{SIC}}^{(\text{e})}$ for a given maximum $\epsilon_b^{(\text{U})}$. As can be seen from this figure, when ρ is high, the TIN scheme outperforms the SIC and the time-sharing schemes. For low values of ρ, however, the SIC scheme outperforms the other two schemes. The reason is that for high values of ρ, more subtracted URLLC interference will be wrong which introduces error in the eMBB decoding under the SIC scheme.

VI. Proof of Theorem 1

A. Bounding $\epsilon_b^{(\text{U})}$

Recall the definition of the sets $\mathcal{B}_{\text{arrival}}$, $\mathcal{B}_{\text{sent}}$ and $\mathcal{B}_{\text{detect}}$ from (11), (18) and (25), respectively. Given that URLLC message $M_b^{(\text{U})}$ arrives at the beginning of Block b, i.e., $b \in \mathcal{B}_{\text{arrival}}$, we have the following error events:

$$\mathcal{E}_{\text{U},1} := \{b \notin \mathcal{B}_{\text{sent}}\} \tag{36}$$
$$\mathcal{E}_{\text{U},2} := \{b \notin \mathcal{B}_{\text{detect}}\} \tag{37}$$
$$\mathcal{E}_{\text{U},3} := \left\{\left(\hat{M}_b^{(\text{U})}, \hat{j}\right) \neq \left(M_b^{(\text{U})}, j\right)\right\}. \tag{38}$$

Given that no URLLC message is sent over Block b, i.e., $b \notin \mathcal{B}_{\text{sent}}$, we have the following error event:

$$\mathcal{E}_{\text{U},4} := \{b \in \mathcal{B}_{\text{detect}}\}. \tag{39}$$

The error probability of decoding URLLC message $M_b^{(\text{U})}$ of Block b thus is bounded by

$$\begin{aligned}\epsilon_b^{(\text{U})} \leq\ & \mathbb{P}[b \in \mathcal{B}_{\text{arrival}}]\mathbb{P}[\mathcal{E}_{\text{U},1}|b \in \mathcal{B}_{\text{arrival}}] \\ & + \mathbb{P}[b \in \mathcal{B}_{\text{arrival}}]\mathbb{P}[\mathcal{E}_{\text{U},2}|\mathcal{E}_{\text{U},1}^c, b \in \mathcal{B}_{\text{arrival}}] \\ & + \mathbb{P}[b \in \mathcal{B}_{\text{arrival}}]\mathbb{P}[\mathcal{E}_{\text{U},3}|\mathcal{E}_{\text{U},2}^c, \mathcal{E}_{\text{U},1}^c, b \in \mathcal{B}_{\text{arrival}}] \\ & + \mathbb{P}[b \notin \mathcal{B}_{\text{arrival}}]\mathbb{P}[\mathcal{E}_{\text{U},4}|b \notin \mathcal{B}_{\text{arrival}}].\end{aligned} \tag{40}$$

1) Analyzing $\mathbb{P}[\mathcal{E}_{\text{U},1}|b \in \mathcal{B}_{arrival}]$: From (15) we notice that $\left(\boldsymbol{V}_b - \alpha \boldsymbol{X}_b^{(\text{e},2)}\right) \in \mathcal{D}_b$ if and only if

$$n_{\text{U}}\beta_{\text{U}}\mathsf{P} - \delta_b \leq ||\boldsymbol{V}_b - \alpha \boldsymbol{X}_b^{(\text{e},2)}||^2 \leq n_{\text{U}}\beta_{\text{U}}\mathsf{P}. \tag{41}$$

Recall that $||\boldsymbol{V}_k||^2 = n_{\text{U}}\beta_{\text{v}}\mathsf{P}$ almost surely.

Lemma 1: We can prove that

$$\mathbb{P}[(\boldsymbol{V}_b - \alpha \boldsymbol{X}_b^{(\text{e},2)}) \in \mathcal{D}_b] = \zeta \tag{42}$$

where ζ is defined in (30c).

Proof: see Appendix A. ∎

Since the L_v codewords are generated independently:

$$\mathbb{P}[\mathcal{E}_{\text{U},1}|b \in \mathcal{B}_{\text{arrival}}] = (1 - \zeta)^{L_v}. \tag{43}$$

To analyze the remaining error events, we employ the following lemma.

Lemma 2: For any $\gamma^{(\text{U})} > 0$:

$$\begin{aligned}&\mathbb{P}[i_b^{(\text{U})}(\boldsymbol{V}_b(m,j); \boldsymbol{Y}_b) \leq \gamma^{(\text{U})}] \\ &\leq 1 - G\left(\frac{n_{\text{U}}}{2}, \lambda(\mu_{\text{U}})\right) + G\left(\frac{n_{\text{U}}}{2}, \tilde{\lambda}(\mu_{\text{U}})\right),\end{aligned} \tag{44}$$

where $G(\cdot,\cdot)$ is the regularized gamma function and $\lambda(\cdot)$ and $\tilde{\lambda}(\cdot)$ are defined in (31) and μ_{U} is defined in (30).

Proof: See Appendix B. ∎

2) Analyzing $\mathbb{P}[\mathcal{E}_{\mathsf{U},2}|\mathcal{E}^c_{\mathsf{U},1}, b \in \mathcal{B}_{arrival}]$*:* This error event is equivalent to the probability that for all $j \in [L_v]$ and for all $m \in [L_{\mathsf{U}}]$ there is no codeword $V_b(m,i)$ such that $i(\boldsymbol{V}_b(m,i);\boldsymbol{Y}_b) > \gamma^{(\mathsf{U})}$. Therefore,

$$\mathbb{P}[\mathcal{E}_{\mathsf{U},2}|\mathcal{E}^c_{\mathsf{U},1}, b \in \mathcal{B}_{\text{arrival}}]$$

$$= \left(\mathbb{P}\left[i(\boldsymbol{V}_b(m,j);\boldsymbol{Y}_b) \leq \gamma^{(\mathsf{U})}\right]\right)^{L_v L_{\mathsf{U}}} \tag{45}$$

$$\leq \left(1 - G\left(\frac{n_{\mathsf{U}}}{2}, \lambda(\mu_{\mathsf{U}})\right) + G\left(\frac{n_{\mathsf{U}}}{2}, \tilde{\lambda}(\mu_{\mathsf{U}})\right)\right)^{L_v L_{\mathsf{U}}} \tag{46}$$

where the last inequality holds by Lemma 2.

3) Analyzing $\mathbb{P}[\mathcal{E}_{\mathsf{U},3}|\mathcal{E}^c_{\mathsf{U},2}, \mathcal{E}^c_{\mathsf{U},1}, b \in \mathcal{B}_{arrival}]$*:* To evaluate this probability, we use the threshold bound for maximum-metric decoding. For any given threshold $\gamma^{(\mathsf{U})}$:

$$\begin{aligned}&\mathbb{P}[\mathcal{E}_{\mathsf{U},3}|\mathcal{E}^c_{\mathsf{U},2}, \mathcal{E}^c_{\mathsf{U},1}, b \in \mathcal{B}_{\text{arrival}}] \\ &\leq \mathbb{P}[i(\boldsymbol{V}_b(M_b^{(\mathsf{U})}, j);\boldsymbol{Y}_b) \leq \gamma^{(\mathsf{U})}] \\ &\quad + (L_v L_{\mathsf{U}} - 1)\mathbb{P}[i(\bar{\boldsymbol{V}}_b(m',j');\boldsymbol{Y}_b) > \gamma^{\mathsf{U}}]\end{aligned} \tag{47}$$

where $m' \in \{1,\ldots,L_{\mathsf{U}}\}$, $j' \in \{1,\ldots,L_v\}$, $(M_b^{(\mathsf{U})}, j) \neq (m',j')$, $\bar{\boldsymbol{V}}_b \sim f_{\boldsymbol{V}_b}$ and is independent of $(\boldsymbol{V}_b, \boldsymbol{Y}_b)$.

Lemma 3: For any $\gamma^{(\mathsf{U})} > 0$:

$$\mathbb{P}[i(\bar{\boldsymbol{V}}_b;\boldsymbol{Y}_b) > \gamma^{(\mathsf{U})}] \leq e^{-\gamma^{(\mathsf{U})}}. \tag{48}$$

Proof: See Appendix C. ■

By Lemmas 2 and 3, we have

$$\begin{aligned}&\mathbb{P}[\mathcal{E}_{\mathsf{U},3}|\mathcal{E}^c_{\mathsf{U},2}, \mathcal{E}^c_{\mathsf{U},1}, b \in \mathcal{B}_{\text{arrival}}] \\ &\leq 1 - G\left(\frac{n_{\mathsf{U}}}{2}, \lambda(\mu_{\mathsf{U}})\right) + G\left(\frac{n_{\mathsf{U}}}{2}, \tilde{\lambda}(\mu_{\mathsf{U}})\right) + (L_v L_{\mathsf{U}} - 1)e^{-\gamma^{(\mathsf{U})}}.\end{aligned} \tag{49}$$

4) Analyzing $\mathbb{P}[\mathcal{E}_{\mathsf{U},4}|b \notin \mathcal{B}_{arrival}]$*:* This error event is equivalent to the probability that given no URLLC is arrived, there exists at least one codeword $V_b(m,i)$ with $m \in [L_{\mathsf{U}}]$ and $j \in [L_v]$ such that $i(\boldsymbol{V}_b(m,j);\boldsymbol{Y}_b) > \gamma^{(\mathsf{U})}$. Therefore,

$$\mathbb{P}[\mathcal{E}_{\mathsf{U},4}|b \notin \mathcal{B}_{\text{arrival}}]$$

$$= 1 - \left(\mathbb{P}\left[i(\boldsymbol{V}_b(m,j);\boldsymbol{Y}_b) \leq \gamma^{(\mathsf{U})}\right]\right)^{L_v L_{\mathsf{U}}} \tag{50}$$

$$\leq 1 - \left(1 - e^{-\gamma^{(\mathsf{U})}}\right)^{L_v L_{\mathsf{U}}}. \tag{51}$$

where the last inequality follows by Lemma 3.

By combining (43), (49), (46) and (51) we prove bound (32).

B. Bounding $\epsilon^{(\mathsf{e})}_{TIN}$

Define

$$\rho_{\mathsf{U}} := \mathbb{P}[b \in \mathcal{B}_{\text{sent}}], \tag{52a}$$

$$\rho_{\text{det},0} := \mathbb{P}[b \in \mathcal{B}_{\text{detect}}|b \in \mathcal{B}_{\text{sent}}], \tag{52b}$$

$$\rho_{\text{det},1} := \mathbb{P}[b \in \mathcal{B}_{\text{detect}}|b \notin \mathcal{B}_{\text{sent}}]. \tag{52c}$$

Lemma 4: We prove that

$$\rho_{\mathsf{U}} = \rho\left(1 - (1-\zeta)^{L_v}\right), \quad \rho_{\text{det},1} \leq q_1, \quad q_2 \leq \rho_{\text{det},0} \leq q_3, \tag{53}$$

where q_1, q_2 and q_3 are defined in (35) and ζ in (30c).

Proof: See Appendix D. ■

Given $\mathcal{B}_{\text{detect}} = B_{\text{dt}}$, we have the following two error events:

$$\mathcal{E}_{\text{TIN},1} = \{\mathcal{B}_{\text{detect}} \neq \mathcal{B}_{\text{sent}}\} \tag{54}$$

$$\mathcal{E}_{\text{TIN},2} = \{\hat{M}^{(\mathsf{e})} \neq M^{(\mathsf{e})}\}. \tag{55}$$

The eMBB decoding error probability under the TIN approach thus is bounded by

$$\begin{aligned}\epsilon^{\text{TIN}}_{\mathsf{e}} \leq \sum_{B_{\text{dt}}} &\mathbb{P}[\mathcal{B}_{\text{detect}} = B_{\text{dt}}] \\ &\cdot \left(\mathbb{P}[\mathcal{E}_{\text{TIN},1}|\mathcal{B}_{\text{detect}} = B_{\text{dt}}] + \mathbb{P}[\mathcal{E}_{\text{TIN},2}|\mathcal{B}_{\text{detect}} = B_{\text{dt}}, \mathcal{E}^c_{\text{TIN},1}]\right).\end{aligned} \tag{56}$$

1) Analyzing $\mathbb{P}[\mathcal{B}_{detect} = B_{dt}]$*:* Define

$$\rho_{\text{det}} := \mathbb{P}[b \in \mathcal{B}_{\text{detect}}, b \in \mathcal{B}_{\text{sent}}] + \mathbb{P}[b \in \mathcal{B}_{\text{detect}}, b \notin \mathcal{B}_{\text{sent}}] \tag{57}$$

$$= \rho_{\mathsf{U}}\rho_{\text{det},0} + (1-\rho_{\mathsf{U}})\rho_{\text{det},1}, \tag{58}$$

where ρ_{U}, $\rho_{\text{det},0}$ and $\rho_{\text{det},1}$ are defined in (52). By Lemma 4:

$$\rho_{\mathsf{U}} \cdot q_2 \leq \rho_{\text{det}} \leq \rho_{\mathsf{U}} \cdot q_3 + (1-\rho_{\mathsf{U}}) \cdot q_1, \tag{59}$$

and thus by the independence of the blocks:

$$\mathbb{P}[\mathcal{B}_{\text{detect}} = B_{\text{dt}}] \tag{60}$$

$$= \rho_{\text{det}}^{|B_{\text{dt}}|}(1-\rho_{\text{det}})^{\eta - |B_{\text{dt}}|} \tag{61}$$

$$\leq (\rho_{\mathsf{U}} \cdot q_3 + (1-\rho_{\mathsf{U}}) \cdot q_1)^{|B_{\text{dt}}|}(1-\rho_{\mathsf{U}} \cdot q_2)^{\eta - |B_{\text{dt}}|} \tag{62}$$

2) Analyzing $\mathbb{P}[\mathcal{E}_{TIN,1}|\mathcal{B}_{detect} = B_{dt}]$*:* Notice that the values of $\rho_{\mathsf{U}}, \rho_{\text{det},0}$ and $\rho_{\text{det},1}$ stay the same for all blocks in $[\eta]$. Thus

$$\mathbb{P}[\mathcal{B}_{\text{detect}} \neq \mathcal{B}_{\text{sent}}|\mathcal{B}_{\text{detect}} = B_{\text{dt}}] \tag{63}$$

$$= 1 - \mathbb{P}[\mathcal{B}_{\text{sent}} = B_{\text{dt}}|\mathcal{B}_{\text{detect}} = B_{\text{dt}}] \tag{64}$$

$$= 1 - \frac{\mathbb{P}[\mathcal{B}_{\text{sent}} = B_{\text{dt}}, \mathcal{B}_{\text{detect}} = B_{\text{dt}}]}{\mathbb{P}[\mathcal{B}_{\text{detect}} = B_{\text{dt}}]} \tag{65}$$

$$= 1 - \frac{\mathbb{P}[\mathcal{B}_{\text{sent}} = B_{\text{dt}}]\mathbb{P}[\mathcal{B}_{\text{detect}} = B_{\text{dt}}|\mathcal{B}_{\text{sent}} = B_{\text{dt}}]}{\rho_{\text{det}}^{|B_{\text{dt}}|}(1-\rho_{\text{det}})^{\eta - |B_{\text{dt}}|}} \tag{66}$$

$$= 1 - \frac{\rho_{\mathsf{U}}^{|B_{\text{dt}}|}(1-\rho_{\mathsf{U}})^{\eta - |B_{\text{dt}}|}\rho_{\text{det},0}^{|B_{\text{dt}}|}(1-\rho_{\text{det},1})^{\eta - |B_{\text{dt}}|}}{\rho_{\text{det}}^{|B_{\text{dt}}|}(1-\rho_{\text{det}})^{\eta - |B_{\text{dt}}|}} \tag{67}$$

$$\leq 1 - \frac{\rho_{\mathsf{U}}^{|B_{\text{dt}}|}(1-\rho_{\mathsf{U}})^{\eta - |B_{\text{dt}}|}q_2^{|B_{\text{dt}}|}(1-q_1)^{\eta - |B_{\text{dt}}|}}{(\rho_{\mathsf{U}} \cdot q_3 + (1-\rho_{\mathsf{U}}) \cdot q_1)^{|B_{\text{dt}}|}(1-\rho_{\mathsf{U}} \cdot q_2)^{\eta - |B_{\text{dt}}|}} \tag{68}$$

where $\rho_{\mathsf{U}}, q_1, q_2$ and q_3 are defined in (35). The inequality in (68) follows by Lemma 4.

3) Analyzing $\mathbb{P}[\mathcal{E}_{TIN,2}|\mathcal{B}_{detect} = B_{dt}, \mathcal{E}^c_{TIN,1}]$*:* To bound $\mathbb{P}[\hat{M}^{(\mathsf{e})} \neq M^{(\mathsf{e})}|\mathcal{B}_{\text{detect}} = B_{\text{dt}}, \mathcal{E}^c_{\text{TIN},1}]$, we use the threshold bound for maximum-metric decoding. For any given threshold $\gamma^{(\mathsf{e})}$:

$$\begin{aligned}&\mathbb{P}[\hat{M}^{(\mathsf{e})} \neq M^{(\mathsf{e})}|\mathcal{B}_{\text{detect}} = B_{\text{dt}}, \mathcal{E}^c_{\text{TIN},1}] \\ &\leq \mathbb{P}\left[i^{(\mathsf{e})}_{\text{TIN}}\left(\{\boldsymbol{X}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{X}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \boldsymbol{Y}^{n_{\mathsf{e}}}|B_{\text{dt}}\right) < \gamma^{(\mathsf{e})}\right] \\ &\quad + \mathbb{P}\left[i^{(\mathsf{e})}_{\text{TIN}}\left(\{\bar{\boldsymbol{X}}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\bar{\boldsymbol{X}}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \boldsymbol{Y}^{n_{\mathsf{e}}}|B_{\text{dt}}\right) \geq \gamma^{(\mathsf{e})}\right] \\ &\quad \cdot (L_{\mathsf{e}} - 1)\end{aligned} \tag{69}$$

where for each b, $\bar{\boldsymbol{X}}_b^{(\mathsf{e},1)} \sim f_{\boldsymbol{X}_b^{(\mathsf{e},1)}}$ and $\bar{\boldsymbol{X}}_b^{(\mathsf{e},2)} \sim f_{\boldsymbol{X}_b^{(\mathsf{e},2)}}$ and are independent of $(\boldsymbol{X}_b^{(\mathsf{e},1)}, \boldsymbol{X}_b^{(\mathsf{e},2)}, \boldsymbol{Y}_b)$. We use the following two lemmas to bound the above two probability terms.

Lemma 5: For any $\gamma^{(\mathsf{e})} > 0$:

$$\mathbb{P}\left[i_{\text{TIN}}^{(\mathsf{e})}\left(\{\boldsymbol{X}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{X}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; Y^{n_\mathsf{e}}|B_{\text{dt}}\right) < \gamma^{(\mathsf{e})}\right] \leq T - (L_v - 1)e^{-\gamma^{(\mathsf{e})}} \tag{70}$$

where T is defined in (30h).

Proof: See Appendix E. ■

Lemma 6: For any $\gamma^{(\mathsf{e})} > 0$:

$$\mathbb{P}\left[i_{\text{TIN}}^{(\mathsf{e})}\left(\{\bar{\boldsymbol{X}}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\bar{\boldsymbol{X}}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \{\boldsymbol{Y}_b\}_{b=1}^{\eta+1}|B_{\text{dt}}\right) \geq \gamma^{(\mathsf{e})}\right] \leq e^{-\gamma^{(\mathsf{e})}}. \tag{71}$$

Proof: The proof is similar to the proof of Lemma 3 and omitted. ■

Combining Lemmas 5 and 6 with (69) and defining $k := |B_{\text{dt}}|$ proves the bound in (33).

C. Bounding $\epsilon_{SIC}^{(\mathsf{e})}$

Recall the definition of the sets $\mathcal{B}_{\text{arrival}}$, $\mathcal{B}_{\text{sent}}$, $\mathcal{B}_{\text{detect}}$ and $\mathcal{B}_{\text{decode}}$ from (11), (18), (25), and (28), respectively. Let B_{dt} be a realization of the set $\mathcal{B}_{\text{detect}}$, and B_{dc} be a realization of the set $\mathcal{B}_{\text{decode}}$. We have the following two error events:

$$\mathcal{E}_{\text{SIC},1} = \{\mathcal{B}_{\text{detect}} \neq \mathcal{B}_{\text{sent}}\} \tag{72}$$
$$\mathcal{E}_{\text{SIC},2} = \{\hat{M}^{(\mathsf{e})} \neq M^{(\mathsf{e})}\} \tag{73}$$

The eMBB decoding error probability under the SIC approach thus is given by

$$\epsilon_{\mathsf{e}}^{\text{SIC}} \leq \sum_{B_{\text{dt}}} \mathbb{P}[\mathcal{B}_{\text{detect}} = B_{\text{dt}}] \Big(\mathbb{P}[\mathcal{E}_{\text{SIC},1}|\mathcal{B}_{\text{detect}} = B_{\text{dt}}] + \sum_{B_{\text{dc}}} \mathbb{P}[\mathcal{B}_{\text{decode}} = B_{\text{dc}}|\mathcal{E}_{\text{SIC},1}^c, \mathcal{B}_{\text{detect}} = B_{\text{dt}}] \cdot \mathbb{P}[\mathcal{E}_{\text{SIC},2}|\mathcal{B}_{\text{detect}} = B_{\text{dt}}, \mathcal{B}_{\text{decode}} = B_{\text{dc}}, \mathcal{E}_{\text{SIC},1}^c]\Big). \tag{74}$$

1) Analyzing $\mathbb{P}[\mathcal{B}_{decode} = B_{dc}|\mathcal{E}_{SIC,1}^c, \mathcal{B}_{detect} = B_{dt}]$: For any subset $B_c \subseteq B_d$ we have:

$$\mathbb{P}[\mathcal{B}_{\text{decode}} = B_{\text{dc}}|\mathcal{B}_{\text{detect}} = \mathcal{B}_{\text{sent}} = B_{\text{dt}}] \tag{75}$$
$$= \prod_{b\in B_{\text{dc}}} \mathbb{P}[\hat{M}_b^{(\mathsf{U})} = M_b^{(\mathsf{U})}|\mathcal{B}_{\text{detect}} = \mathcal{B}_{\text{sent}} = B_{\text{dt}}] \cdot \prod_{b\in B_{\text{dt}}\setminus B_{\text{dc}}} \left(1 - \mathbb{P}[\hat{M}_b^{(\mathsf{U})} = M_b^{(\mathsf{U})}|\mathcal{B}_{\text{detect}} = \mathcal{B}_{\text{sent}} = B_{\text{dt}}]\right) \tag{76}$$
$$\leq q^{|B_{\text{dc}}|}(1-q)^{|B_{\text{dt}}|-|B_{\text{dc}}|} \tag{77}$$

where q is defined in (35). Inequality (77) holds by (49) and by the independence of the blocks.

2) Analyzing $\mathbb{P}[\mathcal{E}_{SIC,2}|\mathcal{B}_{detect} = B_{dt}, \mathcal{B}_{decode} = B_{dc}, \mathcal{E}_{SIC,1}^c]$: To bound this probability, we use the threshold bound for maximum-metric decoding. For any given threshold $\tilde{\gamma}^{(\mathsf{e})}$:

$$\mathbb{P}[\hat{M}^{(\mathsf{e})} \neq M^{(\mathsf{e})}|\mathcal{B}_{\text{detect}} = B_{\text{dt}}, \mathcal{B}_{\text{decode}} = B_{\text{dc}}, \mathcal{E}_{\text{SIC},1}^c] \tag{78}$$
$$\leq \mathbb{P}\Big[i_{\text{SIC}}^{(\mathsf{e})}(\{\boldsymbol{X}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{X}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \boldsymbol{Y}^{n_\mathsf{e}}|B_{\text{dt}}, B_{\text{dc}}, \{\boldsymbol{V}_b\}_{b\in B_{\text{dc}}}) < \tilde{\gamma}^{(\mathsf{e})}\Big] + (L_\mathsf{e} - 1)\mathbb{P}\Big[i_{\text{SIC}}^{(\mathsf{e})}(\{\bar{\boldsymbol{X}}_{b,1}^{(\mathsf{e})}\}_{b\notin B_{\text{dt}}}, \{\bar{\boldsymbol{X}}_{b,2}^{(\mathsf{e})}\}_{b\in B_{\text{dt}}}; \boldsymbol{Y}_b^{n_\mathsf{e}}|B_{\text{dt}}, B_{\text{dc}}, \{\boldsymbol{V}_b\}_{b\in B_{\text{dc}}}) \geq \tilde{\gamma}^{(\mathsf{e})}\Big] \tag{79}$$

where for each b, $\bar{\boldsymbol{X}}_b^{(\mathsf{e},1)} \sim f_{\boldsymbol{X}_b^{(\mathsf{e},1)}}$ and $\bar{\boldsymbol{X}}_b^{(\mathsf{e},2)} \sim f_{\boldsymbol{X}_b^{(\mathsf{e},2)}}$ and are independent of $(\boldsymbol{X}_b^{(\mathsf{e},1)}, \boldsymbol{X}_b^{(\mathsf{e},2)}, \boldsymbol{Y}^{n_\mathsf{e}})$. We use the following two lemmas to bound the above two probability terms.

Lemma 7: Given $\tilde{\gamma}^{(\mathsf{e})}$, we prove that

$$\mathbb{P}\Big[i_{\text{SIC}}^{(\mathsf{e})}(\{\boldsymbol{X}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{X}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \{\boldsymbol{Y}_b\}_{b=1}^{\eta+1} |B_{\text{dt}}, \{\boldsymbol{V}_b\}_{b\in B_{\text{dt}}}) < \tilde{\gamma}^{(\mathsf{e})}\Big] \leq \frac{\mu T}{\tilde{\mu}} - \nu \tag{80}$$

where T, ν, μ and $\tilde{\mu}$ are defined in (30).

Proof: See Appendix F. ■

Lemma 8: We can prove that

$$\mathbb{P}\Big[i_{\text{SIC}}^{(\mathsf{e})}(\{\bar{\boldsymbol{X}}_{b,1}^{(\mathsf{e})}\}_{b\notin B_{\text{dt}}}, \{\bar{\boldsymbol{X}}_{b,2}^{(\mathsf{e})}\}_{b\in B_{\text{dt}}}; \{\boldsymbol{Y}_b\}_{b=1}^{\eta+1}|B_{\text{dt}}, \{\boldsymbol{V}_b\}_{b\in B_{\text{dt}}}) \geq \tilde{\gamma}^{(\mathsf{e})}\Big] \leq e^{-\tilde{\gamma}^{(\mathsf{e})}}. \tag{81}$$

Proof: The proof is based on the argument provided in the proof of Lemma 3. ■

Combining Lemmas 7 and 8 with (77) and defining $\tilde{k} = |B_{\text{dc}}|$ proves the bound in (34).

VII. Conclusions

We considered a point-to-point scenario where a roadside unite (RSU) wishes to simultaneously send eMBB and URLLC messages to a vehicle. During each eMBB transmission interval, random arrivals of URLLC messages are assumed. To improve the reliability of the URLLC transmissions, we proposed a coding scheme that mitigates the interference of eMBB transmission by means of dirty paper coding (DPC). We derived rigorous upper bounds on the error probabilities of eMBB and URLLC transmissions achieved by our scheme. Our numerical analysis shows that the proposed scheme significantly improves over the standard time-sharing.

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Appendix A
Proof of Lemma 1

By (41) and since $\boldsymbol{X}_b^{(\mathsf{e},2)}$ and $\boldsymbol{V}_b$ are drawn uniformly on the n_{U}-dimensional spheres of radii $\sqrt{n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}}$ and $\sqrt{n_{\mathsf{U}}(\beta_{\mathsf{U}}+\alpha^2\beta_{\mathsf{e}})\mathsf{P}}$, the error event $\mathcal{E}_{b,v}$ holds whenever the following condition is violated:

$$\alpha\beta_{\mathsf{e}}n_{\mathsf{U}}\mathsf{P} \le \langle \boldsymbol{V}_b, \boldsymbol{X}_b^{(\mathsf{e},2)}\rangle \le \alpha\beta_{\mathsf{e}}n_{\mathsf{U}}\mathsf{P} + \frac{\delta_b}{2\alpha}. \tag{82}$$

The distribution of $\langle \boldsymbol{V}_b, \boldsymbol{X}_b^{(\mathsf{e},2)}\rangle$ depends on $\boldsymbol{V}_b$ only through its magnitude, because $\boldsymbol{X}_b^{(\mathsf{e},2)}$ is uniform over a sphere and applying an orthogonal transformation to $\boldsymbol{V}_b$ and $\boldsymbol{X}_b^{(\mathsf{e},2)}$ does neither change the inner product of the two vectors nor the distribution of $\boldsymbol{X}_b^{(\mathsf{e},2)}$. In the following we therefore assume that $\boldsymbol{V}_b = (||\boldsymbol{V}_b||, 0, \ldots, 0)$, in which case (82) is equivalent to:

$$\frac{\alpha\beta_{\mathsf{e}}n_{\mathsf{U}}\mathsf{P}}{\sqrt{\beta_{\mathsf{v}}n_{\mathsf{U}}\mathsf{P}}} \le X_{b,2,1}^{(\mathsf{e})} \le \frac{\alpha\beta_{\mathsf{e}}n_{\mathsf{U}}\mathsf{P}}{\sqrt{\beta_{\mathsf{v}}n_{\mathsf{U}}\mathsf{P}}} + \frac{\delta_b}{2\alpha\sqrt{\beta_{\mathsf{v}}n_{\mathsf{U}}\mathsf{P}}} \tag{83}$$

where $X_{b,2,1}^{(\mathsf{e})}$ is the first entry of the vector $\boldsymbol{X}_b^{(\mathsf{e},2)}$.

The distribution of a given symbol in a length-n_{U} random sequence distributed uniformly on the sphere is [15]

$$f_{X_{b,2,1}^{(\mathsf{e})}}\left(x_{b,2,1}^{(\mathsf{e})}\right) = \frac{1}{\sqrt{\pi n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}}}\frac{\Gamma(\frac{n_{\mathsf{U}}}{2})}{\Gamma(\frac{n_{\mathsf{U}}-1}{2})}\left(1-\frac{(x_{b,2,1}^{(\mathsf{e})})^2}{n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}}\right)^{\frac{n_{\mathsf{U}}-3}{2}} \times \mathbb{1}\{(x_{b,2,1}^{(\mathsf{e})})^2 \le n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}\}. \tag{84}$$

Thus,

$$\begin{aligned}
&\mathbb{P}\left[\boldsymbol{V}_b - \alpha\boldsymbol{X}_b^{(\mathsf{e},2)} \in \mathcal{D}_k\right] \\
&= \int_{\frac{\alpha\beta_{\mathsf{e}}n_{\mathsf{U}}\mathsf{P}}{\sqrt{\beta_{\mathsf{v}}n_{\mathsf{U}}\mathsf{P}}}}^{\frac{\alpha\beta_{\mathsf{e}}n_{\mathsf{U}}\mathsf{P}}{\sqrt{\beta_{\mathsf{v}}n_{\mathsf{U}}\mathsf{P}}}+\frac{\delta_b}{2\alpha\sqrt{\beta_{\mathsf{v}}n_{\mathsf{U}}\mathsf{P}}}} f_{X_{b,2,1}^{(\mathsf{e})}}\left(x_{b,2,1}^{(\mathsf{e})}\right) dx_{b,2,1}^{(\mathsf{e})} &(85)\\
&= \frac{1}{\sqrt{\pi}}\frac{\Gamma(\frac{n_{\mathsf{U}}}{2})}{\Gamma(\frac{n_{\mathsf{U}}-1}{2})}\kappa_{\frac{n_{\mathsf{U}}-3}{2}}\left(\frac{2\alpha^2 n_{\mathsf{U}}\mathsf{P}\beta_{\mathsf{e}}+\delta_b}{2\alpha n_{\mathsf{U}}\mathsf{P}\sqrt{\beta_{\mathsf{v}}\beta_{\mathsf{e}}}}\right) \\
&\quad -\frac{1}{\sqrt{\pi}}\frac{\Gamma(\frac{n_{\mathsf{U}}}{2})}{\Gamma(\frac{n_{\mathsf{U}}-1}{2})}\kappa_{\frac{n_{\mathsf{U}}-3}{2}}\left(\alpha\sqrt{\frac{\beta_{\mathsf{e}}}{\beta_{\mathsf{v}}}}\right), &(86)
\end{aligned}$$

where

$$\kappa_n(x) = \frac{x(1-x^2)^n}{2n+1} + \frac{2n}{2n+1}\kappa_{n-1}(x) \tag{87}$$

with $\kappa_0(x) = x$. This concludes the proof.

Appendix B
Proof of Lemma 2

Note that $\boldsymbol{Y}_b$ and $\boldsymbol{Y}_b|\boldsymbol{V}_b$ do not follow a Gaussian distribution. Define

$$Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b) = \mathcal{N}(\boldsymbol{y}_{k,1}; \boldsymbol{0}, I_{n_{\mathsf{U}}}\sigma^2) \tag{88}$$

$$Q_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{v}_b) = \mathcal{N}(\boldsymbol{y}_h; h\boldsymbol{V}_b, I_{n_{\mathsf{U}}}\sigma_3^2) \tag{89}$$

with $\sigma^2 = h^2\mathsf{P}+1$ and $\sigma_3^2 = h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P}+1$.

Introduce

$$\tilde{i}_b^{(\mathsf{U})}(\boldsymbol{v}_b;\boldsymbol{y}_b) := \ln\frac{Q_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{v}_b)}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)}. \tag{90}$$

Lemma 9: We can prove that

$$i_b^{(\mathsf{U})}(\boldsymbol{v}_b;\boldsymbol{y}_b) \ge \tilde{i}_b^{(\mathsf{U})}(\boldsymbol{v}_b;\boldsymbol{y}_b) + \ln J_{\mathsf{U}}, \tag{91}$$

where

$$J_{\mathsf{U}} := \frac{\pi\sqrt{\beta_{\mathsf{v}}\beta_{\mathsf{e}}}\,2^{\frac{n_{\mathsf{U}}+1}{2}}e^{-\frac{h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P}n_{\mathsf{U}}}{2}}}{9h^2(1-\alpha)(\beta_{\mathsf{v}}+(1-\alpha)^2\beta_{\mathsf{e}})} \tag{92}$$

Proof: By [16, Propsition 2]:

$$\frac{f_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} \le \frac{9((1-\alpha)h)^{n_{\mathsf{U}}}}{2\pi\sqrt{2}}\frac{\beta_{\mathsf{v}}\mathsf{P}+(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P}}{(1-\alpha)\mathsf{P}\sqrt{\beta_{\mathsf{v}}\beta_{\mathsf{e}}}}. \tag{93}$$

By [17, Lemma 5]:

$$\frac{f_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{v}_b)}{Q_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{v}_b)} \ge 2^{\frac{n_{\mathsf{U}}-2}{2}}\left(h(1-\alpha)\right)^{n_{\mathsf{U}}-2}e^{-\frac{h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P}n_{\mathsf{U}}}{2}} \tag{94}$$

Combining the two bounds concludes the proof. ■

As a result, we have

$$\begin{aligned}
&\mathbb{P}[i_b^{(\mathsf{U})}(\boldsymbol{V}_b;\boldsymbol{Y}_b) \le \gamma^{(\mathsf{U})}] &(95)\\
&\le \mathbb{P}[\tilde{i}(\boldsymbol{V}_b;\boldsymbol{Y}_b) \le \gamma^{(\mathsf{U})} - \ln J_{\mathsf{U}}] &(96)\\
&= \mathbb{P}\left[\ln\frac{Q_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{Y}_b|\boldsymbol{V}_b)}{Q_{\boldsymbol{Y}_b}(\boldsymbol{Y}_b)} \le \gamma^{(\mathsf{U})} - \ln J_{\mathsf{U}}\right] &(97)\\
&= \mathbb{P}\left[\ln\frac{\frac{1}{(\sqrt{2\sigma_3^2\pi})^{n_{\mathsf{U}}}}\exp\left(-\frac{||\boldsymbol{Y}_b-h\boldsymbol{V}_b||^2}{2\sigma_3^2}\right)}{\frac{1}{(\sqrt{2\pi\sigma^2})^{n_{\mathsf{U}}}}\exp\left(-\frac{||\boldsymbol{Y}_b||^2}{2\sigma^2}\right)} \le \gamma^{(\mathsf{U})} - \ln J_{\mathsf{U}}\right] &(98)\\
&= \mathbb{P}\left[\frac{n_{\mathsf{U}}}{2}\ln\frac{\sigma^2}{\sigma_3^2} + \frac{||\boldsymbol{Y}_b||^2}{2\sigma^2} - \frac{||\boldsymbol{Y}_b-h\boldsymbol{V}_b||^2}{2\sigma_3^2} \le \gamma^{(\mathsf{U})} - \ln J_{\mathsf{U}}\right] &(99)\\
&= \mathbb{P}\Bigg[\frac{h^2}{2\sigma^2}||\boldsymbol{X}_b^{(\mathsf{U})}||^2 + \frac{h^2}{2}\left(\frac{1}{\sigma^2}-\frac{(1-\alpha)^2}{\sigma_3^2}\right)||\boldsymbol{X}_b^{(\mathsf{e},2)}||^2 \\
&\quad + \frac{h^2}{2}\left(\frac{1}{\sigma^2}-\frac{1}{\sigma_3^2}\right)||\boldsymbol{Z}_b||^2 + \frac{h}{\sigma^2}\langle\boldsymbol{X}_b^{(\mathsf{U})},\boldsymbol{X}_b^{(\mathsf{e},2)}\rangle \\
&\quad + \frac{h}{\sigma^2}\langle\boldsymbol{X}_b^{(\mathsf{U})},\boldsymbol{Z}_b\rangle + \left(\frac{h}{\sigma^2}+\frac{h(1-\alpha)}{\sigma_3^2}\right)\langle\boldsymbol{X}_b^{(\mathsf{e},2)},\boldsymbol{Z}_b\rangle \\
&\quad \le \gamma^{(\mathsf{U})} - \ln J_{\mathsf{U}} - \frac{n_{\mathsf{U}}}{2}\ln\frac{\sigma^2}{\sigma_3^2}\Bigg] &(100)\\
&\le \mathbb{P}\Bigg[\frac{h^2(n_{\mathsf{U}}\beta_{\mathsf{U}}\mathsf{P}-\delta_b)}{2\sigma^2} + \frac{h^2 n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}}{2}\left(\frac{1}{\sigma^2}-\frac{(1-\alpha)^2}{\sigma_3^2}\right) \\
&\quad + \frac{h^2}{2}\left(\frac{1}{\sigma^2}-\frac{1}{\sigma_3^2}\right)||\boldsymbol{Z}_b||^2 - \frac{hn_{\mathsf{U}}\mathsf{P}\sqrt{\beta_{\mathsf{U}}\beta_{\mathsf{e}}}}{\sigma^2} \\
&\quad - h\sqrt{n_{\mathsf{U}}\mathsf{P}}\left(\frac{\sqrt{\beta_{\mathsf{U}}}}{\sigma^2}+\frac{\sqrt{\beta_{\mathsf{e}}}}{\sigma^2}+\frac{\sqrt{\beta_{\mathsf{e}}}(1-\alpha)}{\sigma_3^2}\right)||\boldsymbol{Z}_b|| \\
&\quad \le \gamma^{(\mathsf{U})} - \ln J_{\mathsf{U}} - \frac{n_{\mathsf{U}}}{2}\ln\frac{\sigma^2}{\sigma_3^2}\Bigg] &(101)\\
&= \mathbb{P}\left[||\boldsymbol{Z}_b||^2 + u||\boldsymbol{Z}_b|| \ge \mu_{\mathsf{U}}\right] &(102)\\
&= \mathbb{P}\left[\left(||\boldsymbol{Z}_b||+\frac{u}{2}\right)^2 \ge \mu_{\mathsf{U}} + \frac{u^2}{4}\right] &(103)\\
&= 1 - F\left(\sqrt{\mu_{\mathsf{U}}+\frac{u^2}{4}}-\frac{u}{2}\right) + F\left(-\sqrt{\mu_{\mathsf{U}}+\frac{u^2}{4}}-\frac{u}{2}\right) &(104)
\end{aligned}$$

where

$$\mu_{\mathsf{U}} := \frac{2\sigma^2\sigma_3^2}{h^2(\sigma^2-\sigma_3^2)}\left(\frac{n_{\mathsf{U}}}{2}\ln\frac{\sigma^2}{\sigma_3^2}-\gamma^{(\mathsf{U})}+\ln J_{\mathsf{U}}\right)$$
$$+\frac{\sigma_3^2}{\sigma^2-\sigma_3^2}\left(n_{\mathsf{U}}\mathsf{P}(\sqrt{\beta_{\mathsf{U}}}-\sqrt{\beta_{\mathsf{e}}})^2-\delta_b\right)$$
$$-\frac{\sigma^2 n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}(1-\alpha)^2}{\sigma^2-\sigma_3^2}$$
$$u := \frac{2\sqrt{n_{\mathsf{U}}\mathsf{P}}\left(\sigma_3^2(\sqrt{\beta_{\mathsf{U}}}+\sqrt{\beta_{\mathsf{e}}})+\sigma^2\sqrt{\beta_{\mathsf{e}}}(1-\alpha)\right)}{h(\sigma^2-\sigma_3^2)}$$

Notice that in (104) we use the fact that $||\boldsymbol{Z}_b||$ follows a chi-distribution with degree n_{U} and $F(\cdot)$ represents its CDF.

APPENDIX C
PROOF OF LEMMA 3

By Bayes' rule we have

$$f_{\boldsymbol{V}_b}(\bar{\boldsymbol{v}}_b) = \frac{f_{\boldsymbol{Y}_b}(\boldsymbol{y}_b) f_{\boldsymbol{V}_b|\boldsymbol{Y}_b}(\bar{\boldsymbol{v}}_b|\boldsymbol{y}_b)}{f_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\bar{\boldsymbol{v}}_b)} \tag{106}$$
$$= f_{\boldsymbol{V}_b|\boldsymbol{Y}_b}(\bar{\boldsymbol{v}}_b|\boldsymbol{y}_b)\exp\left(-i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)\right). \tag{107}$$

By multiplying both sides of the above equation by $\mathbb{1}\{i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)>\gamma\}$ and integrating over all $\bar{\boldsymbol{v}}_b$, we have

$$\int_{\bar{\boldsymbol{v}}_b}\mathbb{1}\{i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)>\gamma\} f_{\boldsymbol{V}_b}(\bar{\boldsymbol{v}}_b)d\bar{\boldsymbol{v}}_b =$$
$$\int_{\bar{\boldsymbol{v}}_b}\mathbb{1}\{i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)>\gamma\} e^{-i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)} f_{\boldsymbol{V}_b|\boldsymbol{Y}_b}(\bar{\boldsymbol{v}}_b|\boldsymbol{y}_b)d\bar{\boldsymbol{v}}_b. \tag{108}$$

Note that the left-hand side of (108) is equivalent to $\mathbb{P}[i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)>\gamma|\boldsymbol{Y}_b=\boldsymbol{y}_b]$. Thus

$$\mathbb{P}[i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)>\gamma|\boldsymbol{Y}_b=\boldsymbol{y}_b] \tag{109}$$
$$= \int_{\bar{\boldsymbol{v}}_b}\mathbb{1}\{i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)>\gamma\} \times \exp\left(-i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)\right) f_{\boldsymbol{V}_b|\boldsymbol{Y}_b}(\bar{\boldsymbol{v}}_b|\boldsymbol{y}_b)d\bar{\boldsymbol{v}}_b \tag{110}$$
$$= \int_{\bar{\boldsymbol{v}}_b}\mathbb{1}\left\{\frac{f_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\bar{\boldsymbol{v}}_b)}{f_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)}e^{-\gamma}>1\right\} \times \exp\left(-i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)\right) f_{\boldsymbol{V}_b|\boldsymbol{Y}_b}(\bar{\boldsymbol{v}}_b|\boldsymbol{y}_b)d\bar{\boldsymbol{v}}_b \tag{111}$$
$$\le \int_{\bar{\boldsymbol{v}}_b}\frac{f_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\bar{\boldsymbol{v}}_b)}{f_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)}e^{-\gamma} \times \exp\left(-i(\bar{\boldsymbol{v}}_b,\boldsymbol{y}_b)\right) f_{\boldsymbol{V}_b|\boldsymbol{Y}_b}(\bar{\boldsymbol{v}}_b|\boldsymbol{y}_b)d\bar{\boldsymbol{v}}_b \tag{112}$$
$$= \int_{\bar{\boldsymbol{v}}_b} e^{-\gamma} f_{\boldsymbol{V}_b|\boldsymbol{Y}_b}(\bar{\boldsymbol{v}}_b|\boldsymbol{y}_b)d\bar{\boldsymbol{v}}_b \tag{113}$$
$$= e^{-\gamma}. \tag{114}$$

APPENDIX D
PROOF OF LEMMA 4

We start by analyzing the quantities in ρ_{U}, $\rho_{\text{det},0}$ and $\rho_{\text{det},1}$ defined in (52a), (52b) and (52c).

1) Analyzing ρ_{U}:

$$\rho_{\mathsf{U}} = \rho\cdot\mathbb{P}[\exists\, j\in[L_v] \text{ s.t. } \boldsymbol{X}_b^{(\mathsf{U})}(\boldsymbol{V}_b(M_b^{(\mathsf{U})},j))\in\mathcal{D}_b] \tag{115}$$
$$= \rho(1-(1-\zeta)^{L_v}) \tag{116}$$

where the last equality is by (43).

2) Bounding $\rho_{det,0}$:

$$\rho_{\text{det},0}$$
$$= \mathbb{P}[b\in\mathcal{B}_{\text{detect}}|b\in\mathcal{B}_{\text{sent}}] \tag{117}$$
$$= 1-\mathbb{P}[\forall m,\forall j: i_b^{(\mathsf{U})}(\boldsymbol{V}_b(m,j);\boldsymbol{Y}_b)\le\gamma^{(\mathsf{U})}|b\in\mathcal{B}_{\text{sent}}] \tag{118}$$
$$\ge 1-\left(1-G\left(\frac{n_{\mathsf{U}}}{2},\lambda(\mu_{\mathsf{U}})\right)+G\left(\frac{n_{\mathsf{U}}}{2},\tilde{\lambda}(\mu_{\mathsf{U}})\right)\right)^{L_vL_{\mathsf{U}}} \tag{119}$$

where (119) is by (46).

Lemma 10: For any $\gamma^{(\mathsf{U})}>0$:

$$\mathbb{P}[i_b^{(\mathsf{U})}(\boldsymbol{V}_b(m,j);\boldsymbol{Y}_b)\le\gamma^{(\mathsf{U})}]$$
$$\ge 1-G\left(\frac{n_{\mathsf{U}}}{2},\tilde{\lambda}(\tilde{\mu}_{\mathsf{U}})\right)+G\left(\frac{n_{\mathsf{U}}}{2},\lambda(\tilde{\mu}_{\mathsf{U}})\right) \tag{120}$$

where $G(\cdot,\cdot)$ is the regularized gamma function, $\lambda(\cdot)$ and $\tilde{\lambda}(\cdot)$ are defined in (31) and $\tilde{\mu}_{\mathsf{U}}$ is defined in (30).

Proof: The proof is similar to the proof of Lemma 2. We present a sketch of the proof.

We start by upper bounding

$$i_b^{(\mathsf{U})}(\boldsymbol{v}_b;\boldsymbol{y}_b)\le\tilde{i}_b^{(\mathsf{U})}(\boldsymbol{v}_b;\boldsymbol{y}_b)+\ln\tilde{J}_{\mathsf{U}}, \tag{121}$$

where by [16, Propsition 2] and [17, Lemma 6] we can prove that

$$\tilde{J}_{\mathsf{U}} := \frac{27\sqrt{\pi}(1+h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P})e^{n_{\mathsf{U}}h^2\mathsf{P}(\beta_{\mathsf{v}}+(1-\alpha)^2\beta_{\mathsf{e}})}}{2(h^2(1-\alpha))^{n_{\mathsf{U}}-2}\sqrt{8(1+2h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P}}}. \tag{122}$$

Thus

$$\mathbb{P}[i_b^{(\mathsf{U})}(\boldsymbol{V}_b;\boldsymbol{Y}_b)\le\gamma^{(\mathsf{U})}] \tag{123}$$
$$\ge\mathbb{P}[\tilde{i}(\boldsymbol{V}_b;\boldsymbol{Y}_b)\le\gamma^{(\mathsf{U})}-\ln\tilde{J}_{\mathsf{U}}] \tag{124}$$
$$=\mathbb{P}\left[||\boldsymbol{Z}_b||^2-u||\boldsymbol{Z}_b||\ge\tilde{\mu}_{\mathsf{U}}\right] \tag{125}$$
$$=\mathbb{P}\left[\left(||\boldsymbol{Z}_b||-\frac{u}{2}\right)^2\ge\tilde{\mu}_{\mathsf{U}}+\frac{u^2}{4}\right] \tag{126}$$
$$=1-F\left(\sqrt{\tilde{\mu}_{\mathsf{U}}+\frac{u^2}{4}}+\frac{u}{2}\right)+F\left(-\sqrt{\mu_{\mathsf{U}}+\frac{u^2}{4}}+\frac{u}{2}\right) \tag{127}$$

■

where

$$\tilde{\mu}_{\mathsf{U}} := \frac{2\sigma^2\sigma_3^2}{h^2(\sigma^2-\sigma_3^2)}\left(\frac{n_{\mathsf{U}}}{2}\ln\frac{\sigma^2}{\sigma_3^2}-\gamma^{(\mathsf{U})}+\ln\tilde{J}_{\mathsf{U}}\right)$$
$$+\frac{\sigma_3^2}{\sigma^2-\sigma_3^2}\left(n_{\mathsf{U}}\mathsf{P}(\sqrt{\beta_{\mathsf{U}}}+\sqrt{\beta_{\mathsf{e}}})^2\right)$$
$$-\frac{\sigma^2 n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}(1-\alpha)^2}{\sigma^2-\sigma_3^2} \tag{128}$$

By Lemma 10:

$$\rho_{\text{det},0}\le 1-\left(1-G\left(\frac{n_{\mathsf{U}}}{2},\tilde{\lambda}_1\right)+G\left(\frac{n_{\mathsf{U}}}{2},\tilde{\lambda}_2\right)\right)^{L_vL_{\mathsf{U}}}. \tag{129}$$

3) Upper Bounding $\rho_{det,1}$:

$$\rho_{\text{det},1}$$
$$=\mathbb{P}[b\in\mathcal{B}_{\text{detect}}|b\notin\mathcal{B}_{\text{sent}}] \tag{130}$$
$$=\mathbb{P}[\exists m\in[L_{\mathsf{U}}],j\in[L_v]: i_b^{(\mathsf{U})}(\boldsymbol{V}_b(m,j);\boldsymbol{Y}_b)\ge\gamma^{(\mathsf{U})}|b\notin\mathcal{B}_{\text{sent}}]$$
$$=1-\mathbb{P}[\forall m,\forall j: i_b^{(\mathsf{U})}(\boldsymbol{V}_b(m,j);\boldsymbol{Y}_b)\le\gamma^{(\mathsf{U})}|b\in\mathcal{B}_{\text{sent}}] \tag{131}$$
$$\le 1-\left(1-e^{-\gamma^{(\mathsf{U})}}\right)^{L_vL_{\mathsf{U}}} \tag{132}$$

where (132) is by (51).

Appendix E
Proof of Lemma 5

Notice that for each $b \in [1:\eta+1]$, $\boldsymbol{Y}_b$ and for $b \in B_{\text{dt}}$, $\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)}$ do not follow a Gaussian distribution. Define $Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)$ as in (88) and

$$Q_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},2)}) = \mathcal{N}(\boldsymbol{y}_b; h(1-\alpha)\boldsymbol{X}_b^{(\mathsf{e},2)}, I_{n_\mathsf{U}}\sigma_2^2) \tag{133}$$

with $\sigma_2^2 = h^2\beta_\mathsf{v}\mathsf{P}+1$.

Introduce

$$\tilde{i}_{\text{TIN}}^{(\mathsf{e})}\left(\{\boldsymbol{x}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \{\boldsymbol{y}_b\}_{b=1}^{\eta+1}|B_{\text{dt}}\right)$$
$$:= \ln\prod_{b\notin B_{\text{dt}}}\frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},1)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},1)})}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} + \ln\prod_{b\in B_{\text{dt}}}\frac{Q_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},2)})}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} \tag{134}$$

Lemma 11: We can prove that

$$i_{\text{TIN}}^{(\mathsf{e})}\left(\{\boldsymbol{x}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \{\boldsymbol{y}_b\}_{b=1}^{\eta+1}|B_{\text{dt}}\right)$$
$$\geq \tilde{i}_{\text{TIN}}^{(\mathsf{e})}\left(\{\boldsymbol{x}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \{\boldsymbol{y}_b\}_{b=1}^{\eta+1}|B_{\text{dt}}\right) + \ln J_\mathsf{e}, \tag{135}$$

where

$$J_\mathsf{e} := \left(\frac{\pi 2^{\frac{n_\mathsf{U}+1}{2}} e^{\frac{-h^2\beta_\mathsf{v}\mathsf{P}n_\mathsf{U}}{2}}\sqrt{\beta_\mathsf{v}\beta_\mathsf{e}}}{9h^2(1-\alpha)^{n_\mathsf{U}-1}(\beta_\mathsf{v}+(1-\alpha)^2\beta_\mathsf{e})}\right)^k \cdot \left(\frac{\sqrt{8(1+2h^2\mathsf{P})}}{27\sqrt{\pi}(1+h^2\mathsf{P})}\right)^{\eta-k} \tag{136}$$

Proof: similar to the proof of Lemma 9 and by [16, Proposition 2], for $b \notin B_{\text{dt}}$:

$$\frac{f_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} \leq \frac{27\sqrt{\pi}(1+h^2\mathsf{P})}{\sqrt{8(1+2h^2\mathsf{P})}}. \tag{137}$$

■

As a result, we have

$$\mathbb{P}\left[i_{\text{TIN}}^{(\mathsf{e})}\left(\{\boldsymbol{X}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{X}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \boldsymbol{Y}^{n_\mathsf{e}}|B_{\text{dt}}\right) < \gamma^{(\mathsf{e})}\right]$$
$$\leq \mathbb{P}\Big[\tilde{i}_{\text{TIN}}^{(\mathsf{e})}\left(\{\boldsymbol{X}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{X}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \boldsymbol{Y}^{n_\mathsf{e}}|B_{\text{dt}}\right) < \gamma^{(\mathsf{e})} - \ln J_\mathsf{e}\Big]$$
$$= \mathbb{P}\Bigg[\ln\prod_{b\notin B_{\text{dt}}}\frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},1)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},1)})}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} + \ln\prod_{b\in B_{\text{dt}}}\frac{Q_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},2)})}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} < \gamma^{(\mathsf{e})} - \ln J_\mathsf{e}\Bigg]$$
$$= \mathbb{P}\Bigg[\ln\prod_{b\notin B_{\text{dt}}\setminus\eta+1}\frac{\frac{1}{(\sqrt{2\pi})^{n_\mathsf{U}}}e^{-\frac{||\boldsymbol{Z}_b||^2}{2}}}{\frac{1}{(\sqrt{2\pi\sigma^2})^{n_\mathsf{U}}}e^{-\frac{||\boldsymbol{X}_b^{(\mathsf{e},1)}+\boldsymbol{Z}_b||^2}{2\sigma^2}}} + \ln\prod_{b\in B_{\text{dt}}}\frac{\frac{1}{(\sqrt{2\pi\sigma_2^2})^{n_\mathsf{U}}}e^{-\frac{||\boldsymbol{V}_b+\boldsymbol{Z}_b||^2}{2\sigma_2^2}}}{\frac{1}{(\sqrt{2\pi\sigma^2})^{n_\mathsf{U}}}e^{-\frac{||\boldsymbol{X}_b^{(\mathsf{U})}+\boldsymbol{X}_b^{(\mathsf{e},2)}+\boldsymbol{Z}_b||^2}{2\sigma^2}}}$$
$$+\ln\frac{\frac{1}{(\sqrt{2\pi})^{n_\mathsf{e}-\eta n_\mathsf{U}}}e^{-\frac{||\boldsymbol{Z}_{\eta+1}||^2}{2}}}{\frac{1}{(\sqrt{2\pi\sigma^2})^{n_\mathsf{e}-\eta n_\mathsf{U}}}e^{-\frac{||\boldsymbol{X}_{\eta+1,1}^{(\mathsf{e})}+\boldsymbol{Z}_{\eta+1}||^2}{2\sigma^2}}} < \gamma^{(\mathsf{e})} - \ln J_\mathsf{e}\Bigg]$$
$$= \mathbb{P}\Bigg[\frac{1}{2}\sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b||^2 - \frac{1}{2\sigma^2}||\boldsymbol{X}_b^{(\mathsf{e},1)}+\boldsymbol{Z}_b||^2 + \sum_{b\in B_{\text{dt}}}\frac{||\boldsymbol{V}_b+\boldsymbol{Z}_b||^2}{2\sigma_2^2} - \frac{||\boldsymbol{V}_b+(1-\alpha)\boldsymbol{X}_b^{(\mathsf{e},2)}+\boldsymbol{Z}_b||^2}{2\sigma^2}$$
$$> -\gamma^{(\mathsf{e})} + \ln J_\mathsf{e} + \frac{n_\mathsf{e}}{2}\ln\sigma^2 - \frac{n_\mathsf{U}k}{2}\ln\sigma_2^2\Bigg] \tag{138}$$
$$\leq \mathbb{P}\Bigg[\frac{\sigma^2-1}{2\sigma^2}\sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b||^2 + \frac{\sqrt{n_\mathsf{U}\mathsf{P}}}{\sigma^2}\sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b|| + \tau\sum_{b\in B_{\text{dt}}}||\boldsymbol{Z}_b|| + \frac{\sigma^2-\sigma_2^2}{2\sigma^2\sigma_2^2}\sum_{b\in B_{\text{dt}}}||\boldsymbol{Z}_b||^2 > \mu\Bigg] \tag{139}$$
$$\overset{(a)}{=} \mathbb{P}\Bigg[\frac{\sigma^2-1}{2\sigma^2}\tilde{Z}_1 + \frac{\sqrt{n_\mathsf{U}\mathsf{P}}}{\sigma^2}\sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b|| + \tau\sum_{b\in B_{\text{dt}}}||\boldsymbol{Z}_b|| + \frac{\sigma^2-\sigma_2^2}{2\sigma^2\sigma_2^2}\tilde{Z}_2 > \mu\Bigg] \tag{140}$$
$$\overset{(b)}{\leq} \frac{\mathbb{E}\left[\frac{\sigma^2-1}{2\sigma^2}\tilde{Z}_1 + \frac{\sqrt{n_\mathsf{U}\mathsf{P}}}{\sigma^2}\sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b||\right]}{\mu} + \frac{\mathbb{E}\left[\tau\sum_{b\in B_{\text{dt}}}||\boldsymbol{Z}_b|| + \frac{\sigma^2-\sigma_2^2}{2\sigma^2\sigma_2^2}\tilde{Z}_2\right]}{\mu} \tag{141}$$
$$= \frac{(n_\mathsf{e}-kn_\mathsf{U})(\sigma^2-1)}{2\sigma^2\mu} + \frac{(\eta+1-k)\sqrt{n_\mathsf{U}\mathsf{P}}}{\sigma^2\mu}\frac{\sqrt{2}\Gamma\left(\frac{n_\mathsf{U}+1}{2}\right)}{\Gamma\left(\frac{n_\mathsf{U}}{2}\right)} + \frac{k\tau}{\mu}\frac{\sqrt{2}\Gamma\left(\frac{n_\mathsf{U}+1}{2}\right)}{\Gamma\left(\frac{n_\mathsf{U}}{2}\right)} + \frac{kn_\mathsf{U}(\sigma^2-\sigma_2^2)}{2\sigma^2\sigma_2^2\mu} \tag{142}$$

where

$$\tau := \frac{\sqrt{n_\mathsf{U}\mathsf{P}}\left(\sqrt{\beta_\mathsf{v}}(\sigma^2+\sigma_2^2)+(1-\alpha)\sqrt{\beta_\mathsf{e}}\sigma_2^2\right)}{\sigma^2\sigma_2^2}$$
$$\mu := -\gamma^{(\mathsf{e})} + \ln J_\mathsf{e} + \frac{n_\mathsf{e}}{2}\ln\sigma^2 - \frac{kn_\mathsf{U}}{2}\ln\sigma_2^2 - \frac{\eta+1-k}{2\sigma^2}n_\mathsf{U}\mathsf{P} + \frac{k}{2\sigma_2^2}\beta_\mathsf{v}n_\mathsf{U}\mathsf{P} - \frac{k}{2\sigma^2}\left(\sqrt{\beta_\mathsf{v}}+(1-\alpha)\sqrt{\beta_\mathsf{e}}\right)^2 n_\mathsf{U}\mathsf{P}$$

In step (a), we define

$$\tilde{Z}_1 := \sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b||^2 \sim \mathcal{X}^2(n_\mathsf{e}-kn_\mathsf{U}) \tag{143}$$
$$\tilde{Z}_2 := \sum_{b\in B_{\text{dt}}}||\boldsymbol{Z}_b||^2 \sim \mathcal{X}^2(kn_\mathsf{U}) \tag{144}$$

where $\mathcal{X}^2(n)$ represents chi-squared distribution of degree n. In step (b), we use the following Markov's inequality:

$$\mathbb{P}[X > a] \leq \frac{\mathbb{E}[X]}{a}. \tag{145}$$

In step (c):

$$\mathbb{E}[\tilde{Z}_1] = n_\mathsf{e} - kn_\mathsf{U}, \tag{146}$$

$$\mathbb{E}[\tilde{Z}_2] = kn_{\mathsf{U}}, \tag{147}$$

$$\mathbb{E}[||\boldsymbol{Z}_b||] = \frac{\sqrt{2}\Gamma\left(\frac{n_{\mathsf{U}}+1}{2}\right)}{\Gamma\left(\frac{n_{\mathsf{U}}}{2}\right)}. \tag{148}$$

Appendix F
Proof of Lemma 7

Define $Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)$ as in (88), $Q_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{v}_b)$ as in (89) and $Q_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},2)})$ as in (133).

Introduce

$$\begin{aligned}&\tilde{i}_{\text{SIC}}^{(\mathsf{e})}\Big(\{\boldsymbol{x}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \boldsymbol{y}^{n_{\mathsf{e}}}|B_{\text{dt}}, B_{\text{dc}}, \{\boldsymbol{V}_b\}_{b\in B_{\text{dc}}}\Big)\\ &:= \ln\prod_{b\notin B_{\text{dt}}}\frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},1)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},1)})}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)} + \ln\prod_{b\in B_{\text{dt}}\setminus B_{\text{dc}}}\frac{Q_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},2)})}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)}\\ &+\ln\prod_{b\in B_{\text{dc}}}\frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)},\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},2)},\boldsymbol{v}_b)}{Q_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{v}_b)}\end{aligned} \tag{149}$$

Lemma 12: We can prove that

$$\begin{aligned}&i_{\text{SIC}}^{(\mathsf{e})}\Big(\{\boldsymbol{x}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \boldsymbol{y}^{n_{\mathsf{e}}}|B_{\text{dt}}, B_{\text{dc}}, \{\boldsymbol{v}_b\}_{b\in B_{\text{dc}}}\Big)\\ &\geq \tilde{i}_{\text{SIC}}^{(\mathsf{e})}\Big(\{\boldsymbol{x}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{x}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}}; \boldsymbol{y}^{n_{\mathsf{e}}}|B_{\text{dt}}, B_{\text{dc}}, \{\boldsymbol{v}_b\}_{b\in B_{\text{dc}}}\Big)\\ &\quad+\ln\tilde{J}_{\mathsf{e}},\end{aligned} \tag{150}$$

where

$$\begin{aligned}\tilde{J}_{\mathsf{e}} := &\left(\frac{\pi 2^{\frac{n_{\mathsf{U}}+1}{2}} e^{\frac{-h^2\beta_{\mathsf{v}}\mathsf{P}n_{\mathsf{U}}}{2}}\sqrt{\beta_{\mathsf{v}}\beta_{\mathsf{e}}}}{9h^2(1-\alpha)^{n_{\mathsf{U}}-1}(\beta_{\mathsf{v}}+(1-\alpha)^2\beta_{\mathsf{e}})}\right)^{k-\tilde{k}}\\ &\cdot\left(\frac{\sqrt{8(1+2h^2\mathsf{P})}}{27\sqrt{\pi}(1+h^2\mathsf{P})}\right)^{\eta-k}\\ &\cdot\left(\frac{\sqrt{8(1+2h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P})}}{27\sqrt{\pi}(1+h^2(1-\alpha)^2\beta_{\mathsf{e}}\mathsf{P})}\right)^{\tilde{k}}\end{aligned} \tag{151}$$

Proof: similar to the proof of Lemmas 9 and 11. ∎

As a result, we have

$$\begin{aligned}&\mathbb{P}\Big[i_{\text{SIC}}^{(\mathsf{e})}\Big(\{\boldsymbol{X}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{X}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}};\\ &\qquad \boldsymbol{Y}^{n_{\mathsf{e}}}|B_{\text{dt}}, B_{\text{dc}}, \{\boldsymbol{V}_b\}_{b\in B_{\text{dc}}}\Big) \leq \tilde{\gamma}^{(\mathsf{e})}\Big]\end{aligned} \tag{152}$$

$$\begin{aligned}&\leq \mathbb{P}\Big[\tilde{i}_{\text{SIC}}^{(\mathsf{e})}\Big(\{\boldsymbol{X}_b^{(\mathsf{e},1)}\}_{b\notin B_{\text{dt}}}, \{\boldsymbol{X}_b^{(\mathsf{e},2)}\}_{b\in B_{\text{dt}}};\\ &\qquad \boldsymbol{Y}^{n_{\mathsf{e}}}|B_{\text{dt}}, B_{\text{dc}}, \{\boldsymbol{V}_b\}_{b\in B_{\text{dc}}}\Big) < \tilde{\gamma}^{(\mathsf{e})} - \ln\tilde{J}_{\mathsf{e}}\Big]\end{aligned} \tag{153}$$

$$\begin{aligned}&= \mathbb{P}\Bigg[\ln\prod_{b\notin B_{\text{dt}}}\frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},1)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},1)})}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)}\\ &\quad+\ln\prod_{b\in B_{\text{dt}}\setminus B_{\text{dc}}}\frac{Q_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)}}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},2)})}{Q_{\boldsymbol{Y}_b}(\boldsymbol{y}_b)}\\ &\quad+\ln\prod_{b\in B_{\text{dc}}}\frac{f_{\boldsymbol{Y}_b|\boldsymbol{X}_b^{(\mathsf{e},2)},\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{x}_b^{(\mathsf{e},2)},\boldsymbol{v}_b)}{Q_{\boldsymbol{Y}_b|\boldsymbol{V}_b}(\boldsymbol{y}_b|\boldsymbol{y}_b)} < \tilde{\gamma}^{(\mathsf{e})} - \ln\tilde{J}_{\mathsf{e}}\Bigg]\end{aligned} \tag{154}$$

$$\begin{aligned}&= \mathbb{P}\Bigg[\ln\prod_{b\notin B_{\text{dt}}\setminus\eta+1}\frac{\frac{1}{(\sqrt{2\pi})^{n_{\mathsf{U}}}}e^{-\frac{||\boldsymbol{Z}_b||^2}{2}}}{\frac{1}{(\sqrt{2\pi\sigma^2})^{n_{\mathsf{U}}}}e^{-\frac{||\boldsymbol{X}_b^{(\mathsf{e},1)}+\boldsymbol{Z}_b||^2}{2\sigma^2}}}\\ &\quad+\ln\prod_{b\in B_{\text{dt}}\setminus B_{\text{dc}}}\frac{\frac{1}{(\sqrt{2\pi\sigma_2^2})^{n_{\mathsf{U}}}}e^{-\frac{||\boldsymbol{V}_b+\boldsymbol{Z}_b||^2}{2\sigma_2^2}}}{\frac{1}{(\sqrt{2\pi\sigma^2})^{n_{\mathsf{U}}}}e^{-\frac{||\boldsymbol{X}_b^{(\mathsf{U})}+\boldsymbol{X}_b^{(\mathsf{e},2)}+\boldsymbol{Z}_b||^2}{2\sigma^2}}}\\ &\quad+\ln\prod_{b\in B_{\text{dc}}}\frac{\frac{1}{(\sqrt{2\pi})^{n_{\mathsf{U}}}}e^{-\frac{||\boldsymbol{Z}_b||^2}{2}}}{\frac{1}{(\sqrt{2\pi\sigma_3^2})^{n_{\mathsf{U}}}}e^{-\frac{||(1-\alpha)\boldsymbol{X}_b^{(\mathsf{e},2)}+\boldsymbol{Z}_b||^2}{2\sigma_3^2}}}\\ &\quad+\ln\frac{\frac{1}{(\sqrt{2\pi})^{n_{\mathsf{e}}-\eta n_{\mathsf{U}}}}e^{-\frac{||\boldsymbol{Z}_{\eta+1}||^2}{2}}}{\frac{1}{(\sqrt{2\pi\sigma^2})^{n_{\mathsf{e}}-\eta n_{\mathsf{U}}}}e^{-\frac{||\boldsymbol{X}_{\eta+1,1}^{(\mathsf{e})}+\boldsymbol{Z}_{\eta+1}||^2}{2\sigma^2}}} < \tilde{\gamma}^{(\mathsf{e})} - \ln\tilde{J}_{\mathsf{e}}\Bigg]\end{aligned} \tag{155}$$

$$\begin{aligned}&= \mathbb{P}\Bigg[\frac{1}{2}\sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b||^2 - \frac{1}{2\sigma^2}||\boldsymbol{X}_b^{(\mathsf{e},1)}+\boldsymbol{Z}_b||^2\\ &\quad+\sum_{b\in B_{\text{dt}}\setminus B_{\text{dc}}}\left(\frac{||\boldsymbol{V}_b+\boldsymbol{Z}_b||^2}{2\sigma_2^2}\right.\\ &\qquad\qquad\left.-\frac{||\boldsymbol{V}_b+(1-\alpha)\boldsymbol{X}_b^{(\mathsf{e},2)}+\boldsymbol{Z}_b||^2}{2\sigma^2}\right)\\ &\quad+\sum_{b\in B_{\text{dc}}}\frac{||\boldsymbol{Z}_b||^2}{2} - \frac{||(1-\alpha)\boldsymbol{X}_b^{(\mathsf{e},2)}+\boldsymbol{Z}_b||^2}{2\sigma_3^2}\\ &\quad> -\tilde{\gamma}^{(\mathsf{e})} + \ln\tilde{J}_{\mathsf{e}} + \frac{n_{\mathsf{e}}-kn_{\mathsf{U}}}{2}\ln\sigma^2\\ &\quad+\frac{(k-\tilde{k})n_{\mathsf{U}}}{2}\ln\frac{\sigma^2}{\sigma_2^2} + \frac{n_{\mathsf{U}}\tilde{k}}{2}\ln\sigma_3^2\Bigg]\end{aligned} \tag{156}$$

$$\begin{aligned}&\overset{(a)}{\leq} \mathbb{P}\Bigg[\frac{\sigma^2-1}{2\sigma^2}\sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b||^2 + \frac{\sqrt{n_{\mathsf{U}}\mathsf{P}}}{\sigma^2}\sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b||\\ &\quad+\tau\sum_{b\in B_{\text{dt}}\setminus B_{\text{dc}}}||\boldsymbol{Z}_b||\\ &\qquad+\frac{\sigma^2-\sigma_2^2}{2\sigma^2\sigma_2^2}\sum_{b\in B_{\text{dt}}\setminus B_{\text{dc}}}||\boldsymbol{Z}_b||^2\\ &\qquad+\frac{(1-\alpha)\sqrt{n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}}}{\sigma_3^2}\sum_{b\in B_{\text{dc}}}||\boldsymbol{Z}_b||\\ &\qquad+\frac{\sigma_3^2-1}{2\sigma_3^2}\sum_{b\in B_{\text{dc}}}||\boldsymbol{Z}_b||^2 > \tilde{\mu}\Bigg]\end{aligned} \tag{157}$$

$$\begin{aligned}&\leq \frac{\mathbb{E}\left[\frac{\sigma^2-1}{2\sigma^2}\sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b||^2 + \frac{\sqrt{n_{\mathsf{U}}\mathsf{P}}}{\sigma^2}\sum_{b\notin B_{\text{dt}}}||\boldsymbol{Z}_b||\right]}{\tilde{\mu}}\\ &\quad+\frac{\tau\mathbb{E}\left[\sum_{b\in B_{\text{dt}}\setminus B_{\text{dc}}}||\boldsymbol{Z}_b||\right]}{\tilde{\mu}}\\ &\quad+\frac{\mathbb{E}\left[\frac{\sigma^2-\sigma_2^2}{2\sigma^2\sigma_2^2}\sum_{b\in B_{\text{dt}}\setminus B_{\text{dc}}}||\boldsymbol{Z}_b||^2\right]}{\tilde{\mu}}\end{aligned}$$

$$+\frac{\mathbb{E}\left[\frac{(1-\alpha)\sqrt{n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}}}{\sigma_3^2}\sum_{b\in B_{\text{dc}}}||\boldsymbol{Z}_b||\right]}{\tilde{\mu}}$$

$$+\frac{\mathbb{E}\left[\frac{\sigma_3^2-1}{2\sigma_3^2}\sum_{b\in B_{\text{dc}}}||\boldsymbol{Z}_b||^2\right]}{\tilde{\mu}}$$

$$= \frac{(n_{\mathsf{e}}-kn_{\mathsf{U}})(\sigma^2-1)}{2\sigma^2\tilde{\mu}} + \frac{(\eta+1-k)\sqrt{n_{\mathsf{U}}\mathsf{P}}}{\sigma^2\tilde{\mu}}\frac{\sqrt{2}\Gamma\left(\frac{n_{\mathsf{U}}+1}{2}\right)}{\Gamma\left(\frac{n_{\mathsf{U}}}{2}\right)}$$

$$+\frac{k\tau}{\tilde{\mu}}\frac{\sqrt{2}\Gamma\left(\frac{n_{\mathsf{U}}+1}{2}\right)}{\Gamma\left(\frac{n_{\mathsf{U}}}{2}\right)} + \frac{kn_{\mathsf{U}}(\sigma^2-\sigma_2^2)}{2\sigma^2\sigma_2^2\tilde{\mu}}$$

$$-\frac{\tilde{k}}{\tilde{\mu}}\frac{\sqrt{2}\Gamma\left(\frac{n_{\mathsf{U}}+1}{2}\right)}{\Gamma\left(\frac{n_{\mathsf{U}}}{2}\right)}\left(\tau-\frac{(1-\alpha)\sqrt{n_{\mathsf{U}}\beta_{\mathsf{e}}\mathsf{P}}}{\sigma_3^2}\right)$$

$$-\frac{n_{\mathsf{U}}\tilde{k}}{\tilde{\mu}}\left(\frac{\sigma^2-\sigma_2^2}{2\sigma^2\sigma_2^2}-\frac{\sigma_3^2-1}{2\sigma_3^2}\right) \tag{158}$$

where

$$\tilde{\mu} := \frac{n_{\mathsf{e}}-kn_{\mathsf{U}}}{2}\ln\sigma^2 + \frac{(k-\tilde{k})n_{\mathsf{U}}}{2}\ln\frac{\sigma^2}{\sigma_2^2} + \frac{\tilde{k}n_{\mathsf{U}}}{2}\ln\sigma_3^2$$

$$-\frac{\eta-k}{2\sigma^2}n_{\mathsf{U}}\mathsf{P} + \frac{k-\tilde{k}}{2\sigma_2^2}\beta_{\mathsf{v}}n_{\mathsf{U}}\mathsf{P} - \frac{\tilde{k}(1-\alpha)^2 n_{\mathsf{U}}\mathsf{P}\beta_{\mathsf{e}}}{2\sigma_3^2}$$

$$-\frac{k-\tilde{k}}{2\sigma^2}\left(\sqrt{\beta_{\mathsf{v}}}+(1-\alpha)\sqrt{\beta_{\mathsf{e}}}\right)^2 n_{\mathsf{U}}\mathsf{P} - \tilde{\gamma}^{(\mathsf{e})} + \ln\tilde{J}_{\mathsf{e}}.$$

This concludes the proof.

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