Habilitation Exam

Communication, Compression, and Coordination over Networks: Benefits of Cooperation and Side-Information

Michèle Wigger

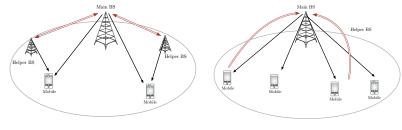
Telecom ParisTech

3 July 2015

Distributed compression (source coding) with side-information

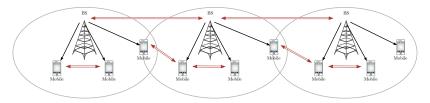


Communication networks with cooperating basestations or mobiles



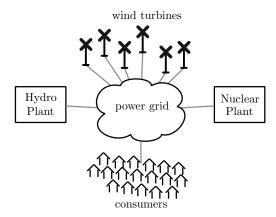
BS-BS cooperation inside a cell

Mobile-BS cooperation inside a cell



BS-BS or mobile-mobile cooperation across cells

Coordinating partially-informed agents



- Here: Coordinate hydro- and nuclear-plant to stabilize system and minimize productions
- General cyber-physical system for transportation, rescue, health care, etc.

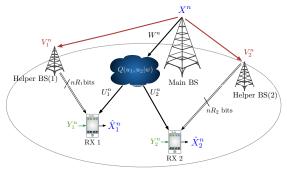
Information-theoretic setups

Asymptotically vanishing probability of error

- Distributed compression systems: how many bits to describe outcome of a source?
- Communication networks: how many bits can one transmit?
- Coordination of distributed agents: which actions-tuples are implementable?

Communication networks with cooperating users

BS-to-BS cooperation inside a cell: downlink



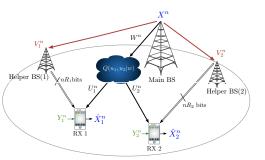
(Almost) lossless joint source-channel coding

Reliable communications for rates (R_1, R_2) possible, if \exists encodings and decodings s.t.

$$\mathbb{P}\Big[\big\{\hat{X}_1^n \neq X^n\big\} \cup \big\{\hat{X}_2^n \neq X^n\big\}\Big] \longrightarrow 0$$

[1] R. Timo and M. Wigger, "Slepian-Wolf coding for broadcasting with cooperative basestations," *IEEE Trans. Communications*, 2015.

Two scenarios for helper side-informations V_1^n and V_2^n



Scenario 1 : Scalar quantisation

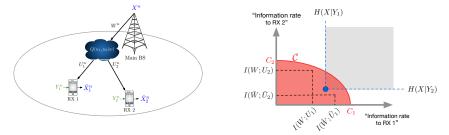
 $V_{k,t} = \phi_k(W_t)$

Scenario 2 : Correlated sources

 $X^n, Y_1^n, Y_2^n, V_1^n, V_2^n$ i.i.d. $\sim P_{XY_1Y_2V_1V_2}$

[2] D. Gunduz, E. Erkip, A. Goldsmith, and H. V. Poor, "Reliable joint source-channel cooperative transmission over relay networks," *IEEE Trans. Inform. Theory*, 2013.

Background: No helper basestations, $R_1 = R_2 = 0$



Theorem [3]:

Reliable communications possible iff $\exists W \sim P_W$ s.t.

 $H(X|Y_1) \leq I(W; U_1)$

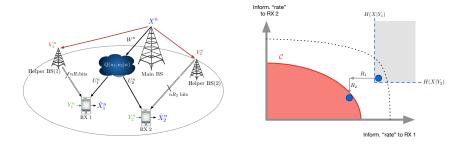
and

 $H(X|Y_2) \leq I(W;U_2)$

[3] E. Tuncel, "Slepian-Wolf coding over broadcast channels," *IEEE Trans. Inform. Theory*, 2006.

Wigger ---- HdR: Communication, Compression, and Coordination over Networks: Benefits of Cooperation and Side-Information

Result for Scenario 1: $V_{k,t} = \phi_k(W_t)$



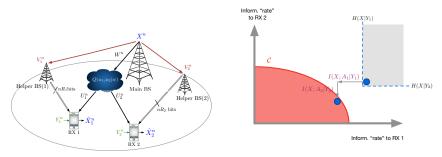
Theorem:

Reliable communications possible with helper rates (R_1, R_2) iff $\exists W \sim P_W$ s.t.

$$H(X|Y_1) \le I(W; U_1) + \min \left\{ R_1, \ I(W; V_1|U_1) \right\}$$
$$H(X|Y_2) \le I(W; U_2) + \min \left\{ R_2, \ I(W; V_2|U_2) \right\}$$

• Helper BS k randomly hashes V_k^n as in deterministic relay channels

Result for Scenario 2: $(X^n, Y_1^n, Y_2^n, V_1^n, V_2^n)$ i.i.d. $\sim P_{XY_1Y_2Y_1Y_2}$



Theorem:

Reliable communication possible with helper rates (R_1, R_2) iff $\exists W, A_1, A_2$ s.t.

 $\begin{aligned} &H(X|Y_1, A_1) \leq I(W; U_1) & R_1 \geq I(V_1; A_1|Y_1) \geq I(X; A_1|Y_1) \\ &H(X|Y_2, A_2) \leq I(W; U_2) & R_2 \geq I(V_2; A_2|Y_2) \geq I(X; A_2|Y_2) \end{aligned}$

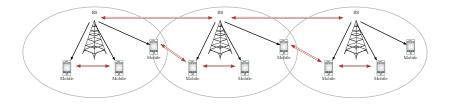
and $(X, Y_1) \rightarrow V_1 \rightarrow A_1$ and $(X, Y_2) \rightarrow V_2 \rightarrow A_2$

• Helper BS k uses Wyner's helper source-code to compress V_k^n into A_k^n

Summary on: BS-to-BS cooperation inside a cell

- Capacity for two-scenarios of BSs-cooperation models
- Modularity/Duality of optimal solutions
- same operations at main BS
- Helper BSs: use Wyner's helper source code or random hashing as for det. relay channels
- receiving mobiles: Tuncel's decoding with improved
 - channel outputs (Scenario 1); or
 - source side-information (Scenario 2)

BS-to-BS or mobile-to-mobile cooperation across cells

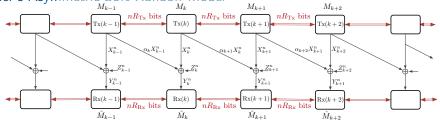


• Cooperation over digital links of given capacities

of conferencing rounds limited due to latency or complexity constraints

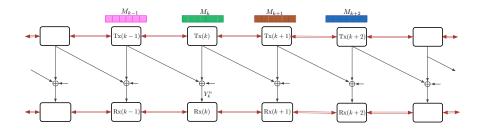
[4] R. Timo, S. Shamai, M. Wigger, "Conferencing in Wyner's asymmetric interference network: effect of number of rounds," in *Proc. of ITW*, 2015.

Wyner's Asymmetric Soft-Handoff Model



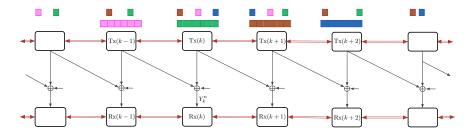
- K transmitter/receiver pairs
- Channel gains $\{\alpha_k\}$ fixed, constant, non-zero
- Memoryless Gaussian noises of variance σ^2 and equal power constraints P





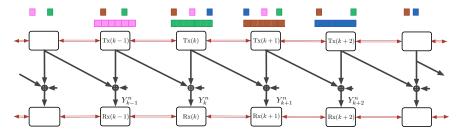
- \rightarrow Messages spread over κ transmitters to left & right
- \rightarrow Output signals spread over κ receivers to left & right

Phase 1: Transmitter-conferencing



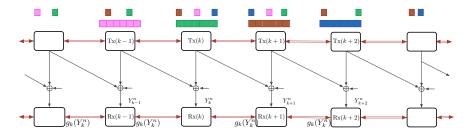
- \rightarrow Messages spread over κ transmitters to left & right
- \rightarrow Output signals spread over κ receivers to left & right

Phase 2: Cooperative communication over network



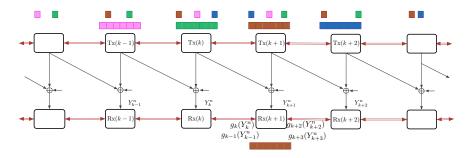
- \rightarrow Messages spread over κ transmitters to left & right
- \rightarrow Output signals spread over κ receivers to left & right

Phase 3: Receiver-conferencing



- \rightarrow Messages spread over κ transmitters to left & right
- \rightarrow Output signals spread over κ receivers to left & right

Phase 4: Clustered decoding



- \rightarrow Messages spread over κ transmitters to left & right
- \rightarrow Output signals spread over κ receivers to left & right

High-SNR Performance: Multiplexing-Gain Per User

• Sum-capacity: C_{Σ} maximum sum of rates $R_1 + R_2 + \cdots + R_K$ s.t. $p(error) \rightarrow 0$

• Asymptotic multiplexing gain per user S:

Sum-capacity:
$$C_{\Sigma} \approx S \cdot \frac{K}{2} \log(1 + P/\sigma^2), \qquad P\sigma^2 \gg 1$$

• Conferencing prelogs μ_{Tx} and μ_{Rx} :

$$R_{\mathrm{Tx}} = \mu_{\mathrm{Tx}} \cdot \frac{1}{2} \log(1 + P/\sigma^2)$$
 and $R_{\mathrm{Rx}} = \mu_{\mathrm{Rx}} \cdot \frac{1}{2} \log(1 + P/\sigma^2)$

Results:

Theorem (Achievability when number of conferencing rounds $\kappa < \infty$)

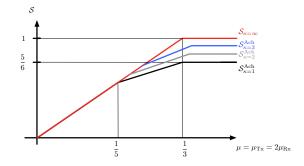
$$\mathcal{S} \geq \begin{cases} \frac{1+2\mu_{\mathsf{Tx}}+2\mu_{\mathsf{Rx}}}{2} & 2\mu_{\mathsf{Tx}}+2\mu_{\mathsf{Rx}}+\frac{2\max\{\mu_{\mathsf{Tx}},\mu_{\mathsf{Rx}}\}}{\kappa} < 1\\ \frac{1+2\kappa+\max\{2\mu_{\mathsf{Tx}},2\mu_{\mathsf{Rx}}\}}{2\kappa+2} & \text{otherwise} \\ \frac{4\kappa+1}{4\kappa+2} & \min\{\mu_{\mathsf{Tx}},\mu_{\mathsf{Rx}}\} > \frac{\kappa}{4\kappa+2} \end{cases}$$

Converse tight when $\kappa = 1$ and at the same time $\mu_{Tx} = 0$ or $\mu_{Rx} = 0$.

Theorem (Number of conferencing rounds $\kappa = \infty$)

$$\mathcal{S}_{\kappa=\infty} = \min\left\{1, rac{1+2\mu_{\mathsf{Tx}}+2\mu_{\mathsf{Rx}}}{2}
ight\}$$

Comparison

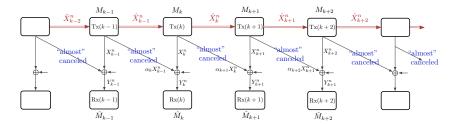


• For small conferencing prelogs $\kappa = 1$ suffices!

- For finite κ , multiplexing gain per user saturates below 1
- Duality between transmitter-cooperation and receiver-cooperation

Coding scheme for $\kappa = \infty$

•
$$\mu_{\mathsf{Tx}} = \mu_{\mathsf{Rx}} = 1/2 \implies \mathcal{S}_{\kappa=\infty} = 1$$



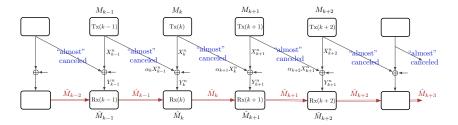
• Tx-conferencing: $\hat{X}_k = f_k(M_1, \dots, M_k)$

• Rx-conferencing: $\hat{M}_k = g_k(Y_1^n, \dots, Y_k^n)$

[5] V. Ntranos, M.A. Maddah-Ali and G. Caire, "Cellular interference alignment," IEEE Trans. Inform. Theory, Mar. 2015

Coding scheme for $\kappa = \infty$

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$$\mu_{\mathsf{Tx}} = \mu_{\mathsf{Rx}} = 1/2 \implies \mathcal{S}_{\kappa=\infty} = 1$$

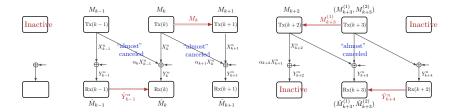


• Tx-conferencing: $\hat{X}_k = f_k(M_1, \dots, M_k)$

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$$\hat{M}_k = g_k(Y_1^n, \dots, Y_k^n)$$

[5] V. Ntranos, M.A. Maddah-Ali and G. Caire, "Cellular interference alignment," IEEE Trans. Inform. Theory, Mar. 2015 Coding Scheme for $\kappa = 1$

•
$$\mu_{\mathrm{Tx}} = \mu_{\mathrm{Rx}} = 1/2 \implies \mathcal{S}_{\kappa=\infty} = 5/6$$



• Tx-conferencing: M_k

• Rx-conferencing:
$$\hat{Y}_k^n = g_k(Y_k^n)$$

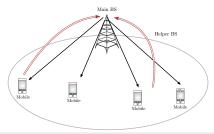
Generally, interference mitigation causes interference to propagate \rightarrow for finite κ need to switch off txs!

Summary and Outlook on: Cooperation across cells

- For small conferencing prelogs $\kappa = 1$ suffices!
- For finite κ , multiplexing gain per user saturates below 1
- Duality between transmitter-cooperation and receiver-cooperation

- In future: analyze different cooperation constraints
 - oblivious codebooks
 - more accurate latency and complexity constraints? (mobiles!)

Mobiles-to-BS cooperation in a cell using feedback



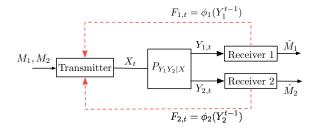
- How and how much does feedback help on a memoryless BC
- Duality to the MAC

[6] O. Shayevitz and M. Wigger, "On the capacity of the discrete memoryless broadcast channel with feedback," *IEEE Trans. Inform. Theory*, Mar. 2013

[7] Y. Wu and M. Wigger, "Coding schemes with rate-limited feedback that improve over the nofeedback capacity for a large class of broadcast channels," submitted to *IEEE Trans. on Inform. Theory*, July 2014

[8] S. Belhadj Amor, Y. Steinberg, and M. Wigger, "MAC-BC duality with linear-feedback schemes," submitted to *IEEE Trans. on Inform. Theory*, Apr. 2014

Rate-limited feedback on discrete memoryless BCs



• Feedback rate constraint: $|\mathcal{F}_{i,1}| \cdots |\mathcal{F}_{i,n}| \leq nR_{\mathsf{fb},i}, \quad i = 1, 2$

•
$$X_t = f_t(M_1, M_2, F_1^{t-1}, F_2^{t-1})$$

Dueck's example provides first intuition how feedback helps on the BC

$$X_{t} = \begin{pmatrix} B_{1,t} \\ B_{0,t} \\ B_{2,t} \end{pmatrix}, \quad Y_{1,t} = \begin{pmatrix} B_{1,t} + Z_{0,t} \\ B_{0,t} \end{pmatrix}, \quad Y_{2,t} = \begin{pmatrix} B_{2,t} + Z_{0,t} \\ B_{0,t} \end{pmatrix}, \quad Z_{0,t} \sim \mathcal{B}(1/2)$$

• Capacity without feedback: $0 \le R_1 + R_2 \le 1$

• Capacity with feedback: $0 \le R_1 \le 1$ and $0 \le R_2 \le 1$ \rightarrow send uncoded bits through $\{B_{1,t}\}$ and $\{B_{2,t}\}$ and send $B_{0,t} = Z_{0,t-1}$

Feedback allows identifying update inform. that will be useful to *both* receivers \rightarrow common update information allows to increase *both* private rates!

Subsequent schemes building on this idea

- Wang'09: \rightarrow Erasure BC
- Tassiulas&Georgiadis'10: \rightarrow Erasure BC
- \bullet Shayevitz&W'10: \rightarrow General channels, generalized feedback
- $\bullet\,$ Maddah-Ali&Tse'10: $\rightarrow\,$ Fading channels and deterministic models; state-feedback
- Chen&Elia'13, Yang/Kobayashi/Gesbert/Yi'13

Feedback-gain remains unknown for most BCs

(Strictly) Less-Noisy DMBCs

Strictly Less-Noisy DMBC $Y_2 \succ Y_1$

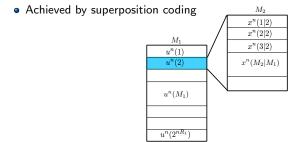
For every auxiliary $U \to X \to (Y_1, Y_2)$ with $I(U; Y_1) > 0$: $I(U; Y_2) > I(U; Y_1)$

- Asymmetric Binary Symmetric BC
- Asymmetric Binary Erasure BC
- Binary Symmetric/Erasure BC for certain parameters

Capacity of Less-Noisy DMBCs, $Y_2 \succeq Y_1$, i.e. $I(U; Y_2) \ge I(U; Y_1)$

• Capacity: all rate pairs (R_1,R_2) where for some $U o X o (Y_1,Y_2)$

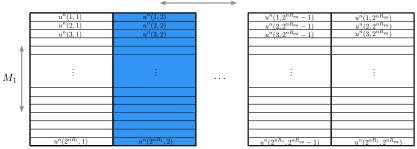
 $R_1 \leq I(U; Y_1)$ $R_2 \leq I(X; Y_2|U)$



- If I(U; Y₂) > I(U; Y₁), Rx 2 could even decode an extra message in cloud center
- Problem: Rx 1 cannot decode, unless it knows this extra message...

Piggyback-coding

- Receiver 1 knows extra message $M_{piggyback}$
- Product codebook for messages M_1 and $M_{piggyback}$



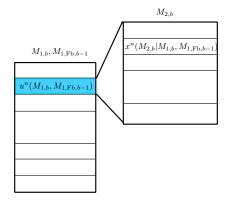
 $M_{\text{piggyback}}$

• Decoding possible if: $R_1 + R_{pg} < I(U; Y_2)$ and $R_1 < I(U; Y_1)$

 $M_{\rm piggyback}$ is not harming Receiver 1!

BC-scheme with feedback from the weaker Receiver 1

• Block-Markov coding with a piggyback superposition code in each block b:



• Choose $M_{1,{\sf Fb},b-1}$ as a Wyner-Ziv message to compress $Y_{1,b-1}^n o ilde Y_{1,b}^n$

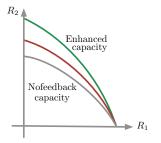
• Rx 2 decodes $M_{2,b}$ based on $(\tilde{Y}_{1,b}^n, Y_{2,b}^n)$

A Simpler Achievable Region

Theorem	
(<i>R</i> ₁ ,	R_2) achievable, if for some $P_U P_{X U} P_{\hat{Y}_1 UY_1}$:
	$R_1 \leq I(U; Y_1)$
	$R_2 \leq I(X; \tilde{Y}_1, Y_2 U) = I(X; Y_2 U) + I(X; \tilde{Y}_1 U, Y_2)$
and	$I(\tilde{Y}_1; Y_1 U, Y_2) \leq \min\{R_{FB,1}, I(U; Y_2) - I(U; Y_1)\}.$

 \bullet Sending \tilde{Y}_1 is purely beneficial: not bothering Rx 1 and helping Rx 2

If $R_{FB,1} > 0$, Feedback Increases Entire Capacity Region



Theorem: For Any DMBC $Y_2 \succ Y_1$, when $R_{FB,1} > 0$

Feedback improves all $(R_1 > 0, R_2 > 0)$ of the nofeedback capacity,

unless (R_1, R_2) lies on boundary of capacity of enhanced channel

• Ex.: Asymmetric Binary Symmetric, Binary Erasure, Gaussian BC

Extension to Two-Sided Feedback

- Marton-coding
- Send feedback messages $M_{FB,1,b-1}$ and $M_{FB,2,b-1}$ in cloud center of block b using double piggyback-coding
- Feedback messages compress outputs $Y_{1,b}^n$ or $Y_{2,b}^n$

 $M_{\text{FB},1,b-1}$ "transparent" for Receiver 1, $M_{\text{FB},2,b-1}$ for Receiver 2 \rightarrow like "double-booking" resources in cloud-center

Duality between Gaussian MIMO MAC and BC with feedback

Gaussian MIMO MAC:

Gaussian MIMO BC:

 $\mathbf{Y}_k = H_k \mathbf{X} + \mathbf{Z}_k, \qquad k = 1, \dots, K$

 $\mathbf{Y}_k = \sum_{k=1}^K H_k^T \mathbf{X}_k + \mathbf{Z}$

sum-power const. $P_1 + P_2 + \ldots + P_K = P$

perfect feedback

Theorem

Rates achievable with linear-feedback schemes for dual BC and MAC coincide!

[9] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. on Inf. Theory*, 2003

[10] S. Belhadj Amor, Y. Steinberg, M. Wigger, "MIMO MAC-BC duality with linear-feedback coding schemes," submitted to *IEEE Trans. on Inf. Theory*, Apr. 2014

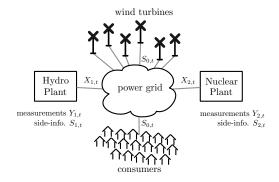
Summary and outlook on: mobiles-to-BS cooperation using feedback

- Proposed two ways of exploiting feedback on memoryless BCs
- Even low-rate feedback increases capacity of large class of memoryless BCs \rightarrow not only strictly less noisy!
- Memoryless Gaussian MAC-BC duality when restricting to linear-feedback schemes (perfect feedback)
- Explore MAC-BC duality with feedback for discrete memoryless case

 $[\]rightarrow$ BC-dual to the Cover-Leung scheme for MAC with feedback?

Coordinating partially-informed agents

Coordination over state-dependent networks: motivation



• Wish to coordinate inputs and state:

$$\frac{1}{T}\sum_{t=1}^{T} P_{S_{0,t}X_{1,t},X_{2,t}} \to \bar{Q} \quad \text{ as } T \to \infty$$

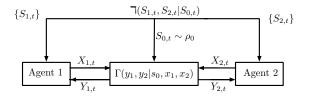
[11] B. Larousse, S. Lasaulce, and M. Wigger, "Coordinating partially-informed agents over state-dependent networks," in *Proc. of ITW 2015*

Game-Theoretic Motivation: Infinitively Repeated Games

- Agents' actions $X_{1,t}$ and $X_{2,t}$
- Payoff-functions $\omega_1(s_{0,t}, x_{1,t}, x_{2,t})$ and $\omega_2(s_{0,t}, x_{1,t}, x_{2,t})$
- Average expected payoff:

$$\overline{\omega}_{k} = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E} \left[\omega_{k}(S_{0,t}, X_{1,t}, X_{2,t}) \right]$$
$$= \sum_{(s_{0}, x_{1}, x_{2})} \omega_{k}(s_{0}, x_{1}, x_{2}) \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} P_{S_{0,t}X_{1,t}X_{2,t}}(s_{0}, x_{1}, x_{2}).$$

Setup and implementable distributions



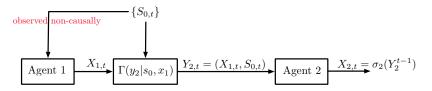
• Causal or non-causal SI:

$$X_{k,t} = \sigma_{k,t}^{(\mathbf{c})}(S_k^t, Y_k^{t-1}) \quad \text{or} \quad X_{k,t} = \sigma_{k,t}^{(\mathbf{nc})}(S_k^T, Y_k^{t-1})$$

Implementable distributions \overline{Q}

 $\forall \epsilon > 0 \text{ there exist } T \text{ and encodings, s.t.} \\ \left| \frac{1}{T} \sum_{t=1}^{T} P_{S_{0,t}X_{1,t}X_{2,t}}(s_0, x_1, x_2) - \overline{Q}(s_0, x_1, x_2) \right| \leq \epsilon.$

Early related model and result: Gossner et al. 2006



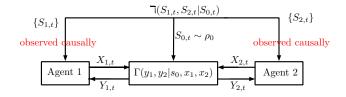
• $\{X_{1,t}\}$ communicates $\{S_{0,t}\}$ to Agent 2

Theorem[12] $\overline{Q}(s_0, x_1, x_2)$ implementable iff $I(S_0; X_2) \leq H(X_1|S_0, X_2)$

[12] O. Gossner, P. Hernandez, and A. Neyman, "Optimal use of communication resources," Econometrica 2006.

[13] G. Kramer and S. Savari, "Communicating probability distributions," *IEEE Trans.* on *Inf. Theory*, 2007. (ISIT-version 2002)

New results under causal SI at both agents: $X_{k,t} = \sigma_{k,t}^{(c)}(S_k^t, Y_k^{t-1})$



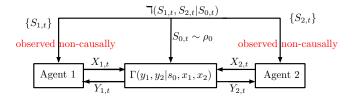
Theorem

 \overline{Q} implementable iff it factorizes as

$$\overline{Q}(s_0, x_1, x_2) = \sum_{u, s_1, s_2} \left[\rho_0(s_0) \exists (s_1, s_2 | s_0) P_U(u) \prod_{k=1}^2 P_{X_k | US_k}(x_k | u, s_k) \right]$$

- No coding/communication required
- Extends to K agents

Non-causal state-info at both agents



Theorem: If $S_{2,t} = f(S_{1,t})$ or $\Gamma(y_1, y_2 | s_0, x_1, x_2) = \tilde{\Gamma}(y_1, y_2 | s_0, x_1)$

$$\begin{split} & \overline{Q}(s_0, x_1, x_2) \text{ implementable iff it is marginal of some} \\ & Q(s_0, s_1, s_2, u, v, x_1, x_2, y_2) \\ & = \rho_0(s_0) \exists (s_1, s_2|s_0) P_{UVX_1|S_1}(u, v, x_1|s_1) P_{X_2|US_2}(x_2|u, s_2) \Gamma(y_1, y_2|s_0, x_1, x_2) \\ & \text{satisfying} \\ & I(S_1; U|S_2) \leq I(V; Y_2, S_2|U) - I(V; S_1|U) \end{split}$$

• Communication only in one direction: Agent 1 coordinates Agent 2

Summary and outlook on: coordinating partially-informed agents

- Proposed *K*-agent framework for coordination over state-dependent networks
- Only local coordination when state-information at agents local
- Implementable distributions under non-causal state-information when K = 2 and when communication only in one direction
- In future:
 - Distributed coordination, e.g., many-to-one
 - · Benefits of coding strategies for real cyber-physical networks

More research plans

- Cache-aided communication networks (noisy channels!)
- Video streaming
- Interplays between information theory and statistics:
 - distributed hypothesis testing
 - distributed clustering (information bottleneck method)

Curriculum

- Master of Sciences ETH Zurich, March 2003
- PhD ETH Zurich, October 2008
- PostDoc University of California San Diego, May-November 2009
- Assistant Professor (Maître de Conférences) Telecom ParisTech, December 2009
- Visiting Professor at the Technion—Israel Institute of Technology June 2011
- Visiting Professor at ETH Zurich July/August 2010, August 2013, July/August 2015

Teaching

- Introduction to digital communications, Telecom ParisTech
- Information theory, Telecom ParisTech
- Iterative decoding methods, Telecom ParisTech
- Multi-user information theory, ETH Zurich
- Coding and cellular automata, ETH Zurich

Supervision of students

- Post Docs:
 - Roy Timo, November 2013-April 2014
 - Sadaf Salehkalaibar, February/March 2015
- PhD students:
 - Youlong Wu, November 2011–October 2014
 - Selma Belhadj Amor, October 2011–March 2015
- Master Thesis students:
 - Andreas Malär, June–December 2010
 - Thomas Laich, June-December 2012

Service to the society

- Associate Editor of IEEE Communication Letters, since December 2012
- TPC member of:
 - ISIT 2011, 2012, 2013, 2014, 2015, 2016
 - ITW 2015
 - IZS 2012, 2014, 2016
 - ISWCS 2011
 - PIMRC 2013, 2014, 2015
- Organization of conferences, workshops, and invited sessions:
 - Publicity chair ITW 2016, Cambridge, UK
 - Co-organizer of GdR ISIS workshop "Recent advances in information theory", April 2012, Paris, France
 - $\bullet\,$ Organizer of invited sessions at ISWCS 2011 and IZS 2012

Awards and grants

- ETH Medal for excellent PhD Thesis (8%)
- ETH Medal for excellent Master Thesis (2.5%)
- Diploma of Master of Science with Distinction
- "Emergences" Starting-Grant from the city of Paris, October 2012–March 2015
- CEFIPRA Project *D2D*, starting May 2015. Collaborative project with Alcatel-Lucent, INRIA, and IIT Mumbay
- Technion scholarship for visiting professors, June 2011
- IDEA League scholarships for visiting professors Jul/Aug. 2010 and Aug. 2013
- Google scholarship for research visit of PhD student Youlong Wu at the Technion
- Swiss National Science Foundation fellowship for prospective researchers 2009

Acknowledgements to

My collaborators:

Selma Belhadj Amor, Shirin Bidokhti Saeedi, Annina Bracher, Michael Gastpar, Tara Javidi, Young-Han Kim, Thomas Laich, Amos Lapidoth, Benjamin Larousse, Samson Lasaulce, Andreas Malär, Paolo Minero, Mohammad Nagshvar, Tobias Oechtering, Shlomo Shamai, Ofer Shayevitz, Yossef Steinberg, Roy Timo, Youlong Wu

The jury members:

Giuseppe Caire, Elza Erkip, Inbar Fialkow, Michael Gastpar, David Gesbert, Philippe Loubaton, Luc Vandendorpe