Sensitivity of Distributed Computing to Placement

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Motivation

- Offloading computations across servers [1–5]
- Nonlinear encoding of distributed sources
- Efficient usage of scarce resources [1–5]
- Evaluating complex functions [1–5]
- Reducing the communication cost [2–5]

Related works

- Distributed source coding [6, 7]:
  Compression for function computation
- Coded computing [2–5]:
  Redundancy to tolerate stragglers, Storage-communication-computation tradeoffs

Q: What is missing in the literature?
A: The **placement-transmission tradeoff**.

Our novel approach:

- Captures any Boolean function
- Is sensitivity-based
- Adapts any placement (uncoded, acyclic)

References

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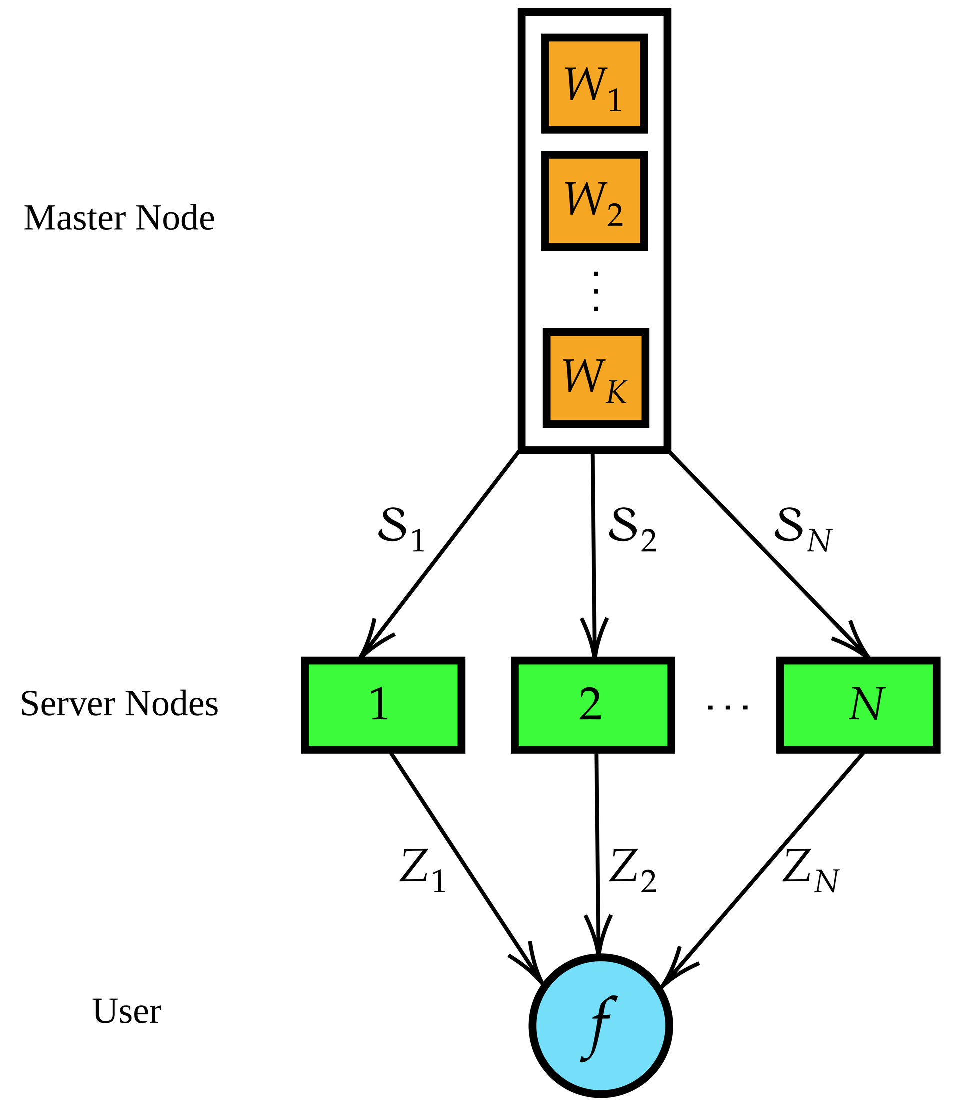
Sensitivity-based computation



Exploiting the polynomial representation:

$$f(\mathbf{W}) = \bigoplus_{\mathcal{P}\subseteq[K]} c_{\mathcal{P}} \prod_{k\in\mathcal{P}} W_k$$

for some subsets $\mathcal{P}$ of K datasets
and coefficients $c_{\mathcal{P}} \in \mathbb{F}_2$.

Dataset placement:

$$\rho_n : \mathbb{F}_2^K \longrightarrow \mathbb{F}_2^M \ , \ \forall n \in [N] \ ,$$
$$S_n = \rho_n(\mathbf{W}) \subseteq \{\mathbf{W}\} \ , \ |S_n| = M \ , \ \forall n \in [N] \ .$$

Encoding and Transmissions:

$$E_n^f : \mathbb{F}_2^M \longrightarrow \mathbb{F}_2^{|Z_n|} \ ,$$
$$Z_n = E_n^f(S_n) = \{z_{ni} \,|\, i \in [|Z_n|]\} \ , \ \forall n \in [N] \ .$$

Decoding:

$$D : \mathbb{F}_2^{|Z|} \longrightarrow \mathbb{F}_2 \ , \ Z = \{Z_n \,|\, n \in [N]\} \ .$$

The function f:

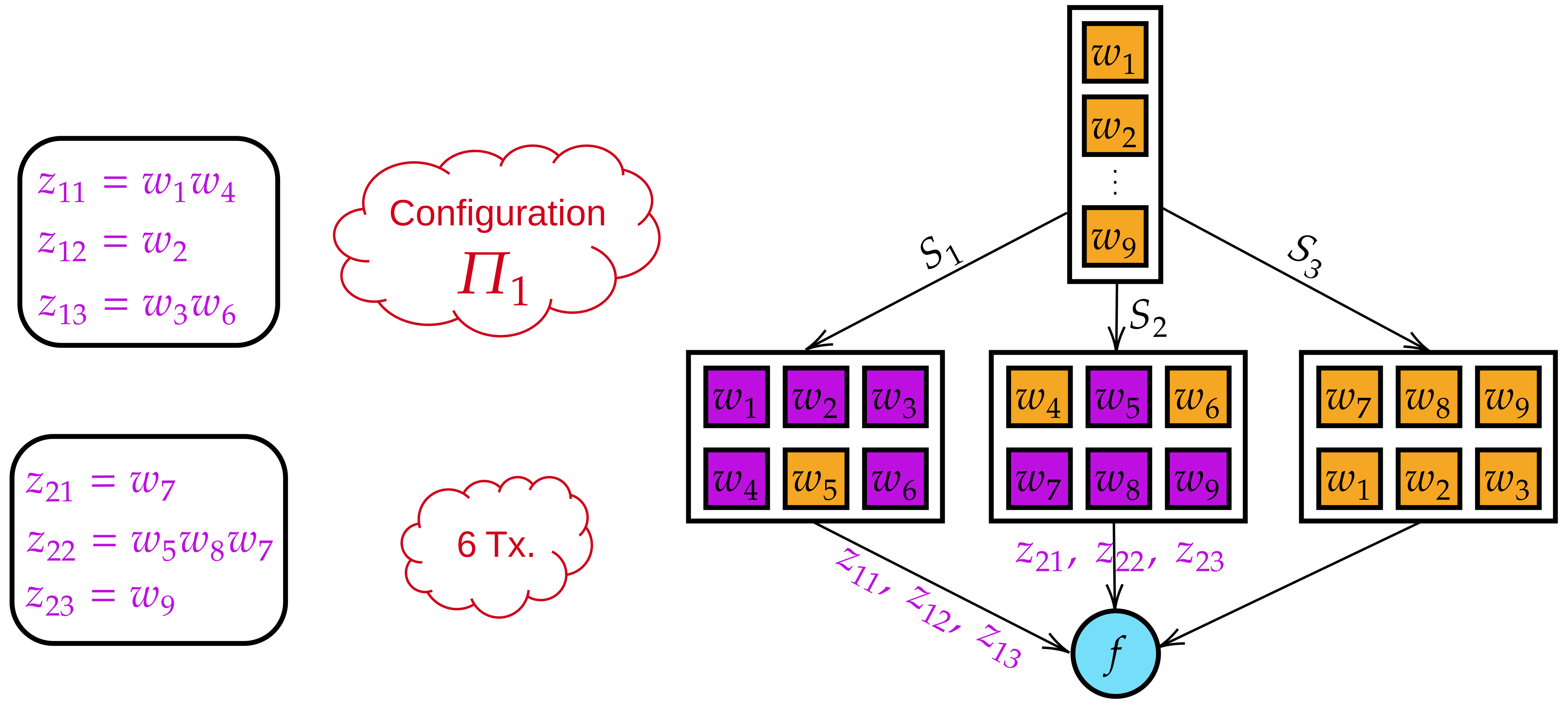
- **Nonlinear** (computationally complex)
- Any (low/high) **degree**
- **Placement-dependent** complexity

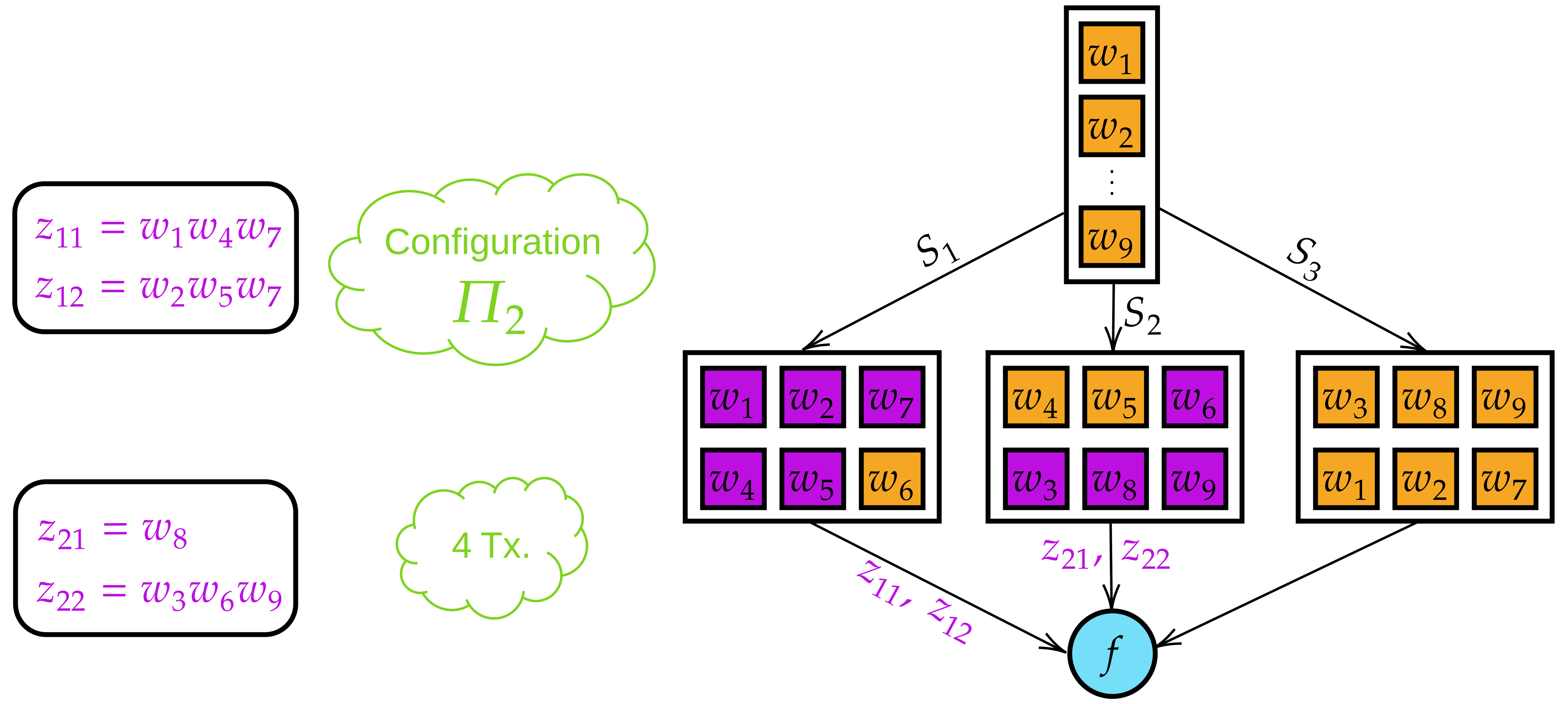
$$\Pi = \arg\min |Z| = \arg\min \sum_{n=1}^{N} |Z_n| \ ?$$

The interplay of placement and transmission: an example

Consider a $(K = 9,\ N = 3,\ M = 6,\ \Pi,\ f)$ distributed computing system, where

$$f(\mathbf{W}) = W_1W_4W_7 \oplus W_2W_5W_8W_7 \oplus W_3W_6W_9 \ .$$





Conclusions and future works

- The impact of placement on transmissions in distributed computing
- The communication/computation-optimal placement configurations for distributed computing
- Fundamental limits of distributed computing of nonlinear Boolean functions
- Fundamental limits of multi-user nonlinear distributed computing