Error Exponent Bounds and Short-Length Coding Schemes for Distributed Hypothesis Testing (DHT)

Ismaila Salihou Adamou[1], Elsa Dupraz[1], and Tad Matsumoto[1,2]
[1] IMT Atlantique, CNRS UMR 6285, Lab-STICC, Brest, France
[2] JAIST and University of Oulu (Emeritus)

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Outline

Testing and Information Theory

- Hypothesis Testing is a standard problem in Statistics :
  The probability distribution of a sequence $\mathbf{x}^n = (x_1, x_2, ..., x_n)$ is given by

$$\mathcal{H}_0 : \mathbf{X}^n \sim P_{0\mathbf{X}}$$
$$\mathcal{H}_1 : \mathbf{X}^n \sim P_{1\mathbf{X}}$$

- **Statistician :** how to optimally decide between $\mathcal{H}_0$ and $\mathcal{H}_1$, by fully observing the data $\mathbf{x}^n$?

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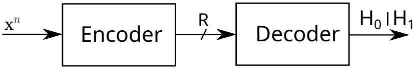

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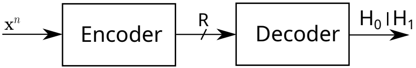
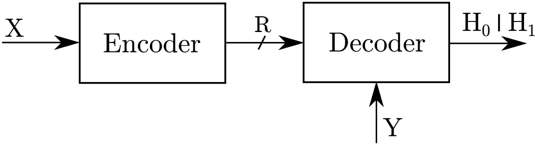


- **Information theorist :** How to design the Encoder such that to optimally decide between $\mathcal{H}_0$ and $\mathcal{H}_1$?

Distributed Hypothesis Testing (DHT)



DHT Formulation

$$\mathcal{H}_0 : (\mathbf{X}, \mathbf{Y}) \sim P_{0\mathbf{XY}}$$
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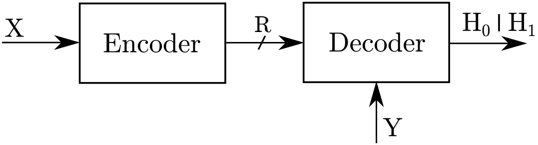

DHT Formulation

$$\mathcal{H}_0 : (\mathbf{X}, \mathbf{Y}) \sim P_{0\mathbf{XY}}$$
$$\mathcal{H}_1 : (\mathbf{X}, \mathbf{Y}) \sim P_{1\mathbf{XY}}$$

- Encoder : $f^{(n)} : \mathcal{X}^n \longrightarrow \mathcal{M}_n = \{1, \ldots \mathsf{M}\}$
- Rate-constraint : $\frac{\log \mathsf{M}}{n} \leq \mathsf{R}$,
- Decoder : $g^{(n)} : \mathcal{M}_n \times \mathcal{Y}^n \longrightarrow \mathcal{H} = \{H_0, H_1\}$

Performance criterion

- Type-I error probability

$$\alpha_n = \mathbb{P}\left[g^{(n)}\left(f^{(n)}\left(\mathbf{X}^n\right), \mathbf{Y}^n\right) = H_1 \mid H_0 \text{ is true}\right],$$

- Type-II error probability

$$\beta_n = \mathbb{P}\left[g^{(n)}\left(f^{(n)}\left(\mathbf{X}^n\right), \mathbf{Y}^n\right) = H_0 \mid H_1 \text{ is true}\right]$$

Performance criterion

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- Type-II error probability

$$\beta_n = \mathbb{P}\left[g^{(n)}\left(f^{(n)}\left(\mathbf{X}^n\right), \mathbf{Y}^n\right) = H_0 \mid H_1 \text{ is true}\right]$$

- Objective : For given $\alpha_n \leq \epsilon$, find the Type-II error exponent θ such that

$$\limsup_{n\to\infty} \frac{1}{n} \log \frac{1}{\beta_n} \geq \theta \qquad (1)$$

Achievable error exponent bounds for i.i.d. sources model

Quantization scheme [Ahlswede86] [HAN87]

- Ahlswede and al derived optimal error exponent for testing against independence.
- HAN improved it with joint typicality check at the encoder.

Quantize-binning scheme [SHA 94]

- Shimokowa et. al combined quantization with random binning (similar to Wyner-Ziv and Slepian-Wolf).
- It allows to exploit the correlation between the sources to reduce the compression rate
- It optimality was considered in [Rahman 2012], [Katz 2015], [Watanabe 2022]
- It was extended to various more complex setups [Salehkalaibar 18], [Sreekuma 19], [Escamilla2020]
- It has been recently improved by [Kochman2023]

However, it was limited only to i.i.d. sources model
Objective : extend it to more general sources models, not necessarily i.i.d.

Outline

1. Hypothesis Testing in Information Theory
2. Considered source model
3. Error exponent bounds for general sources
4. Examples of source models
5. Practical short-length coding schemes for DHT
6. Simulation results
7. Conclusion and Perspectives

General source model

We consider a more generic sources model [Han1998]

$$\mathbf{X} = \{X^n = (X_1, X_2, \cdots, X_n)\}_{n=1}^{\infty} \text{ and } \mathbf{Y} = \{Y^n = (Y_1, Y_2, \cdots, Y_n)\}_{n=1}^{\infty}$$

- The components of $\mathbf{X}$ and $\mathbf{Y}$ are not necessarily i.i.d
- Includes the previous i.i.d. models as particular instances.

- **The objective is to derive more generic exponent error bounds**
- We rely on Information-Spectrum approach [Han1998]

Information-Spectrum tools [Han1998]

For a sequence $\{Z_n\}_{n=1}^{\infty}$

- $\mathrm{p}-\limsup_{n\to\infty} Z_n = \inf\left\{\alpha \mid \lim_{n\to+\infty} \mathbb{P}\left(Z_n > \alpha\right) = 0\right\}$,
- $\mathrm{p}-\liminf_{n\to\infty} Z_n = \sup\left\{\alpha \mid \lim_{n\to+\infty} \mathbb{P}\left(Z_n < \alpha\right) = 0\right\}$

Examples

For a pair $(\mathbf{U}^n, \mathbf{X}^n)$, sup and inf spectral mutual information :

- $\bar{I}(\mathbf{X};\mathbf{U}) = \mathrm{p}-\limsup_{n\to\infty} \frac{1}{n}\log\frac{P_{\mathbf{U}^n|\mathbf{X}^n}(\mathbf{U}^n|\mathbf{X}^n)}{P_{\mathbf{U}^n}(\mathbf{U}^n)}$
- $\underline{I}(\mathbf{X};\mathbf{U}) = \mathrm{p}-\liminf_{n\to\infty} \frac{1}{n}\log\frac{P_{\mathbf{U}^n|\mathbf{X}^n}(\mathbf{U}^n|\mathbf{X}^n)}{P_{\mathbf{U}^n}(\mathbf{U}^n)}$

When $\mathbf{U}$ and $\mathbf{X}$ are i.i.d., we can show that

- $\bar{I}(\mathbf{X};\mathbf{U}) = \underline{I}(\mathbf{X};\mathbf{U}) = I(X;U)$

Outline

Error exponent bounds for general sources [1]

Theorem

$$\theta \leq \sup_{P_{\mathbf{U}|\mathbf{X}}} \left\{ \min\left\{\theta_{test}, \theta_{bin}\right\} \right\}, \quad (2)$$

- $\theta_{bin} = r - \left(\overline{I}(\mathbf{X};\mathbf{U}) - \underline{I}(\mathbf{U};\mathbf{Y})\right)$, $r \leq$ R
- $\theta_{test} = \underline{D}\left(P_{0\mathbf{UY}} \| P_{1\mathbf{UY}}\right) + \left(\underline{I}(\mathbf{X};\mathbf{U}) - \overline{I}(\mathbf{X};\mathbf{U})\right)$

The error exponent is a trade-off between a binning error and a testing error.

- $\bar{I}(\mathbf{X};\mathbf{U}) = \mathrm{p}-\limsup_{n\to\infty} \frac{1}{n} \log \frac{P_{0\mathbf{U}^n|\mathbf{X}^n}(\mathbf{U}^n|\mathbf{X}^n)}{P_{0\mathbf{U}^n}(\mathbf{U}^n)}$
- $\underline{D}\left(P_{0\mathbf{UY}} \| P_{1\mathbf{UY}}\right) = \mathrm{p}-\liminf_{n\to\infty} \frac{1}{n} \log \frac{P_{0\mathbf{U}^n\mathbf{Y}^n}(\mathbf{U}^n,\mathbf{Y}^n)}{P_{1\mathbf{U}^n\mathbf{Y}^n}(\mathbf{U}^n,\mathbf{Y}^n)}$.

1. Adamou, I. S., Dupraz, E., Matsumoto, T. "An Information-Spectrum Approach to Distributed Hypothesis Testing for General Sources", *International Zurich Seminar (IZS) on Information and Communication, Zurich 2024*

Achievable coding scheme

Codebook generation

- Generate $e^{n\mathsf{R}}$ sequences $\mathbf{u}^n$ according to a fixed distribution $P_{\mathbf{U}^n|\mathbf{X}^n}$
- Distribute them uniformly in $e^{n\mathsf{r}}$ bins

Encoder

- Search a sequence $\mathbf{u}^n$ such that $(\mathbf{u}^n, \mathbf{x}^n) \in T_n^{(1)}$
- Send the index of the bin to which $\mathbf{u}^n$ belongs
- Otherwise, send a message error to the decoder to simply declare $\mathcal{H}_1$

$$T_n^{(1)} = \left\{ (\mathbf{x}^n, \mathbf{u}^n) \text{ s.t. } \frac{1}{n} \log \frac{P_{\mathbf{U}^n|\mathbf{X}^n}(\mathbf{u}^n \mid \mathbf{x}^n)}{P_{\mathbf{U}^n}(\mathbf{u}^n)} < r_0 - \varepsilon \right\}$$

Achievable coding scheme

Decoder

From the received bin index and side information $\mathbf{y}^n$:

- Pick $\hat{\mathbf{u}}^n$ such that $(\hat{\mathbf{u}}^n, \mathbf{y}^n) \in T_n^{(2)}$
- Decide $\mathcal{H}_0$ if $(\hat{\mathbf{u}}^n, \mathbf{y}^n)$ belongs to an acceptance region A_n
- Otherwise, decide $\mathcal{H}_1$ (also when a message error is received)

$$T_n^{(2)} = \left\{ (\mathbf{y}^n, \mathbf{u}^n) \text{ s.t. } \frac{1}{n} \log \frac{P_{\mathbf{U}^n|\mathbf{Y}^n}(\mathbf{u}^n \mid \mathbf{y}^n)}{P_{\mathbf{U}^n}(\mathbf{u}^n)} < r_0 - \varepsilon \right\}$$

$$\mathsf{A}_n = \left\{ (\mathbf{y}^n, \mathbf{u}^n) \text{ s.t.} \frac{P_{\mathbf{U}^n\mathbf{Y}^n}(\mathbf{u}^n, \mathbf{y}^n)}{P_{\overline{\mathbf{U}}^n\overline{\mathbf{Y}}^n}(\mathbf{u}^n, \mathbf{y}^n)} > S \right\}.$$

- We can not rely on the **method of types** as in i.i.d. case
- Our achievability proof is **information-spectrum** based.
- Here $T_n^{(2)}$ and A_n are different that typical set as known in i.i.d. case

Proof outlines : Type-I error α_n analysis

- $E_{11} = \left\{ \nexists \mathbf{u}^n \text{ s.t. } (\mathbf{X}^n, \mathbf{u}^n) \in T_n^{(1)}, (\mathbf{Y}^n, \mathbf{u}^n) \in \mathcal{A}_n \right\}$,
- $E_{12} = \left\{ \exists \mathbf{u}'^n \neq \mathbf{u}^n \text{ s.t. } \boldsymbol{i}(\mathbf{u}'^n) = \boldsymbol{i}(\mathbf{u}^n) \text{but } (\mathbf{u}'^n, \mathbf{Y}^n) \notin \mathcal{A}_n \right\}$.
- We show that : $\alpha_n \leq \mathbb{P}\left(E_{11}\right) + \mathbb{P}\left(E_{12}\right)$

Information-Spectrum approach

- For $r_0 = \overline{I}(\mathbf{X};\mathbf{U})$, and from the definition of $\overline{I}(\mathbf{X};\mathbf{U})$, we show

$$\lim_{n\to\infty} \mathbb{P}\left((\mathbf{X}^n, \mathbf{U}^n) \notin T_n^{(1)}\right) = 0.$$

- When $S = \underline{D}\left(P_{\mathbf{UY}} \| P_{\overline{\mathbf{UY}}}\right)$ and from the definition of $\underline{D}\left(P_{\mathbf{UY}} \| P_{\overline{\mathbf{UY}}}\right)$, we have

$$\lim_{n\to\infty} \mathbb{P}\left((\mathbf{Y}^n, \mathbf{U}^n) \notin \mathcal{A}_n\right) = 0.$$

- Thus, We show that : $\alpha_n \overset{n\to\infty}{\longrightarrow} 0.$

$T_n^{(1)} = \left\{ (\mathbf{x}^n, \mathbf{u}^n) \text{ s.t. } \frac{1}{n} \log \frac{P_{\mathbf{U}^n|\mathbf{X}^n}(\mathbf{u}^n|\mathbf{x}^n)}{P_{\mathbf{U}^n}(\mathbf{u}^n)} < r_0 - \varepsilon \right\}$
$A_n = \left\{ (\mathbf{y}^n, \mathbf{u}^n) \text{ s.t. } \frac{P_{\mathbf{U}^n\mathbf{Y}^n}(\mathbf{u}^n,\mathbf{y}^n)}{P_{\overline{\mathbf{U}}^n\overline{\mathbf{Y}}^n}(\mathbf{u}^n,\mathbf{y}^n)} > S \right\}$.

Proof outlines : Type-II error β_n analysis

- $E_{21} = \left\{ \exists \tilde{\mathbf{u}}^n \neq \mathbf{u}^n : \boldsymbol{i}(\tilde{\mathbf{u}}^n) = \boldsymbol{i}(\mathbf{u}^n), \text{but } \left(\overline{\mathbf{Y}}^n, \tilde{\mathbf{u}}^n\right) \in \mathcal{A}_n \right\}$,
- $E_{22} = \left\{ (\mathbf{u}^n, \overline{\mathbf{Y}}^n) \in T_n^{(2)}, (\mathbf{u}^n, \overline{\mathbf{Y}}^n) \in \mathcal{A}_n \right\}$.
- $\beta_n \leq \mathbb{P}\left(E_{21}\right) + \mathbb{P}\left(E_{22}\right)$

Information-Spectrum approach

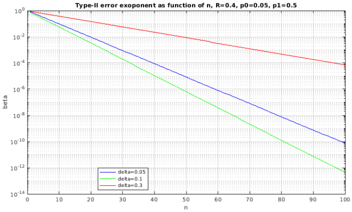
- When $r_0 = \overline{I}(\mathbf{X};\mathbf{U})$, $r^{'} = \underline{I}(\mathbf{Y};\mathbf{U})$, and $S = \underline{D}\left(P_{\mathbf{UY}} \| P_{\overline{\mathbf{UY}}}\right)$
- $\beta_n \leq e^{-n\left(r - \left(\overline{I}(\mathbf{X};\mathbf{U}) - \underline{I}(\mathbf{Y};\mathbf{U})\right) - \epsilon\right)} + e^{-n\left(\underline{I}(\mathbf{X};\mathbf{U}) - \overline{I}(\mathbf{X};\mathbf{U}) + \underline{D}\left(P_{\mathbf{UY}} \| P_{\overline{\mathbf{UY}}}\right) - 2\epsilon\right)}$.
- Since, $\limsup_{n\to\infty} \frac{1}{n} \log \frac{1}{\beta_n} \geq \theta$
- This shows that the error exponent θ in the theorem is achievable.

$$\mathsf{T}_n^{(2)} = \left\{ (\mathbf{y}^n, \mathbf{u}^n) \text{ s.t. } \frac{1}{n} \log \frac{P_{\mathbf{U}^n|\mathbf{Y}^n}(\mathbf{u}^n|\mathbf{y}^n)}{P_{\mathbf{U}^n}(\mathbf{u}^n)} > r^{'} - \varepsilon \right\}$$
$$A_n = \left\{ (\mathbf{y}^n, \mathbf{u}^n) \text{ s.t. } \frac{P_{\mathbf{U}^n\mathbf{Y}^n}(\mathbf{u}^n, \mathbf{y}^n)}{P_{\overline{\mathbf{U}}^n\overline{\mathbf{Y}}^n}(\mathbf{u}^n, \mathbf{y}^n)} > S \right\}.$$

Outline

Short-length nature of DHT

- For Binary i.i.d. sources, we compute $\beta_n = e^{-n\theta}$ as function of code length n



- For instance, for $n = 100$, $\beta_n = 10^{-12}$, for $n = 50$, $\beta_n = 10^{-6}$.
- This strongly suggests that practical schemes should focus on values of $n < 50$.
- We now introduce **two practical coding schemes** for such short sequence lengths
- So far, we only focus on **Binary sources**

Outline

DHT for Binary sources model

- Source model : (X, Y) are such that $Y = X + Z$, where X and Z are independent and $P(X = 1) = 0.2$ and and $P(Z = 1) = p$
- Hypothesis definition :

$$\mathcal{H}_0 : p = p_0,$$
$$\mathcal{H}_1 : p = p_1.$$

- Decoder : Decide between $\mathcal{H}_0$ and $\mathcal{H}_1$ from $\mathbf{Y}^n$ and a coded version of $\mathbf{X}^n$
- Objective : Design practical coding schemes for this setup

Implementation

- From the **Generator matrix** G of a **linear block code**, we calculate $\mathbf{z}_q^m$ as :

$$\mathbf{z}_q^m = \arg\min_{\mathbf{z}^m} d\left(G\mathbf{z}^m, \mathbf{x}^n\right)$$

- At the receiver, we obtain the **quantized vector** $\mathbf{x}_q^n = G\mathbf{z}_q^m$
- The receiver performs the NP test over $(\mathbf{x}_q^n, \mathbf{y}^n)$

Scheme 1 : Binary Quantization

Performance[2]

$$\alpha_n = 1 - \frac{1}{N_0^{(q)}} \sum_{\lambda=0}^{\lambda_q} \sum_{\gamma=0}^{d_{\max}^{(q)}} \sum_{j=0}^{n} E_\gamma^{(q)} \Gamma_{\lambda,j,\gamma} p_0^j (1-p_0)^{n-j}, \tag{3}$$

$$\beta = \frac{1}{N_0^{(q)}} \sum_{\lambda=0}^{\lambda_q} \sum_{\gamma=0}^{d_{\max}^{(q)}} \sum_{j=0}^{n} E_\gamma^{(q)} \Gamma_{\lambda,j,\gamma} p_1^j (1-p_1)^{n-j}, \tag{4}$$

where for $j = \gamma + \lambda - 2u$ and $0 \leq u \leq \min(\gamma, \lambda) \leq n$, $\Gamma_{\lambda,j,\gamma} = \binom{\gamma}{u}\binom{n-\gamma}{\lambda-u}$.

- $d_{\max}^{(q)}$ is the maximum hamming weight of words $\mathbf{x}^n$ of a decision region C_0
- $E_\gamma^{(q)}$ is the number of words $\mathbf{x}^n$ of Hamming weight γ, and $N_0^{(q)} = \sum_{\gamma=0}^{d_{\max}^{(q)}} E_\gamma^{(q)}$
- **One can optimize $E_\gamma^{(q)}$ to obtain a lower bound for practical DHT**

2. Dupraz, E., Adamou, I. S., Asvadi, R., Matsumoto, T. "APractical Short-Length Coding Schemes for Binary Distributed Hypothesis Testing", *International Symposium in Information Theory (ISIT) 2024*

Scheme 2 : Quantize-binning

Encoder

- Quantization : From the **Generator matrix** G of a **linear block code**, we calculate $\mathbf{z}_q^m$ as :

$$\mathbf{z}_q^m = \arg\min_{\mathbf{z}^m} d\left(G\mathbf{z}^m, \mathbf{x}^n\right)$$

- Binning : From a parity check matrix H of another linear block code, we compute

$$\mathbf{u}^k = H\mathbf{z}_q^m$$

- Send $\mathbf{u}^k$ at rate k/n

Decoder

- Identify the vector $\mathbf{z}_q^m$ such that

$$\hat{\mathbf{z}}^m = \arg\min_{\mathbf{z}_q^m} d\left(G\mathbf{z}_q^m, \mathbf{y}^n\right) \text{ s.t. } H\mathbf{z}^m = \mathbf{u}^k$$

- Compute $\hat{\mathbf{x}}^n = G\hat{\mathbf{z}}^m$
- Then, apply the NP test onto $(\hat{\mathbf{x}}^n, \mathbf{y}^n)$.

Scheme 2 : Quantize-Binning

Performance

$$\alpha_n = 1 - \mathbb{P}_B(p_0) - \mathbb{P}_{\bar{B}}(p_0), \quad (5)$$

$$\beta_n = \mathbb{P}_B(p_1) + \mathbb{P}_{\bar{B}}(p_1), \quad (6)$$

where

$$\mathbb{P}_B(\delta) = \sum_{\nu=0}^{\min(d_{\max}^{(qb)},\lambda_{qb})} \frac{E_\nu^{(qb)}}{\binom{n}{\nu}} \sum_{\gamma=0}^{d_{\max}^{(q)}} \frac{E_\gamma^{(q)}}{N_0^{(q)}} \sum_{j=0}^{n} \Gamma_{\nu,j,\gamma} \delta^j (1-\delta)^{n-j},$$

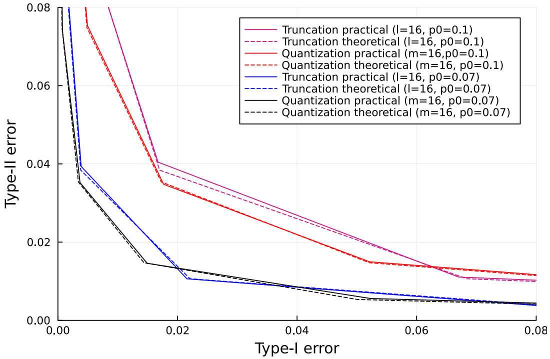
$$\mathbb{P}_{\bar{B}}(\delta) = \sum_{i=0}^{n} \left[\left(\sum_{\gamma=0}^{d_{\max}^{(q)}} \frac{E_\gamma^{(q)}}{N_0^{(q)}} \sum_{j=0}^{n} \Gamma_{i,j,w} \delta^j (1-\delta)^{n-j} \right) \left(\sum_{t=1}^{n} \sum_{\nu=0}^{\lambda_{qb}} \frac{E_\nu^{(qb)}}{\binom{n}{\nu}} \frac{A_t^{(qb)} \Gamma_{i,\nu,t}}{\binom{n}{i}} \right) \right].$$

- $E_\gamma^{(q)}$ is the number of words $\mathbf{x}^n$ of Hamming weight γ
- the set $\{A_t^{(qb)}\}_{t \in [\![0,n]\!]}$ is the code weight distribution of the concatenated code (G and H)
- **One can optimized $E_\gamma^{(q)}$ and $A_t^{(qb)}$ to obtain a lower bound for practical DHT**

Outline

Simulation results

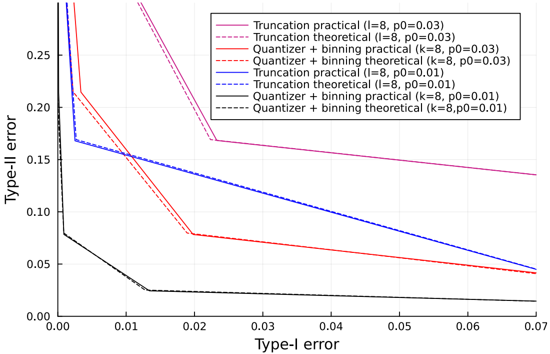
- Baseline Truncation Scheme : we send $l = 16$ non-coded bits
- Quantization Scheme : BCH $(31, 16)$-code with $d_{min} = 7$. As a result $m = 16$ coded bits are sent.



- The quantization scheme performs better that the Truncation scheme
- The theoretical expressions are consistent with the Monte-Carlo simulations

Simulation results

- Baseline Truncation scheme : we send $l = 8$ non-coded bits of $\mathbf{x}^n$
- Quantize-binning Scheme : BCH $(31, 16)$-code with $d_{min} = 7$ and Reed-Muller $(16, 5)$-code with $d_{min} = 8$. As a result $k = 8$ coded bits are sent



- The Quantize-binning scheme performs better that the Truncation scheme
- The theoretical expressions are consistent with the Monte-Carlo simulations

Outline

Conclusion and Perspectives

Conclusion

- We derived a general expression of the Type-II error exponent for **general sources that are not necessarily i.i.d.**
- Our approach is **information-spectrum based**.

- We then proposed practical **quantization scheme** and **quantize-binning scheme**
- Our proposed schemes perform better than the baseline non-coded scheme
- We provided exact **analytical expressions** of Type-I and Type-II errors for the proposed schemes.

Current Works

- Add an **empirical entropy check** in our quantize-binning as in [Kochman2023]
- Extend both theoretical and practical results to the results to the case where both $\mathbf{X}$ and $\mathbf{Y}$ are encoded

MERCI

References

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[Han 2000] Han, T. S. (2000). Hypothesis testing with the general source. arXiv preprint math/0004121.