Cache Allocations, Coding Schemes, and Converses for Cellular Networks with Caching

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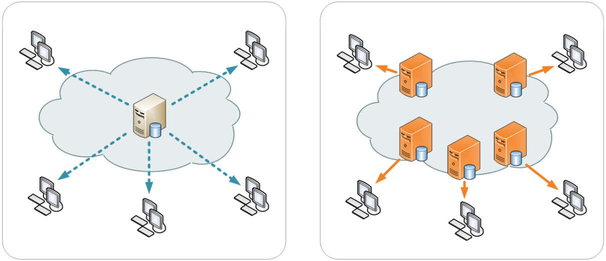
Joint work with Shirin Saeedi Bidokhti, Shlomo Shamai (Shitz), and Roy Timo, Aylin Yener

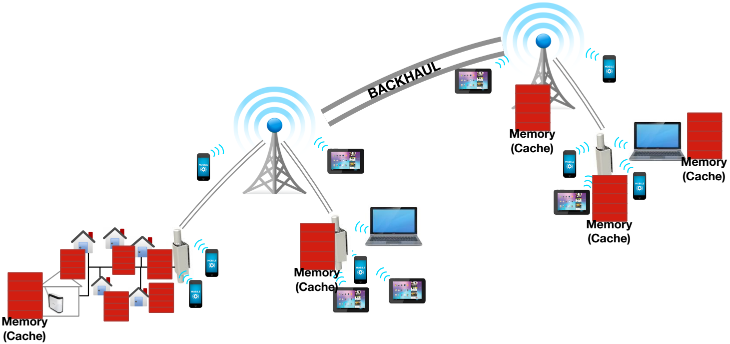
Ben Gurion University, Beer Sheva, Israel, 14 November 2016

Content Delivery Networks



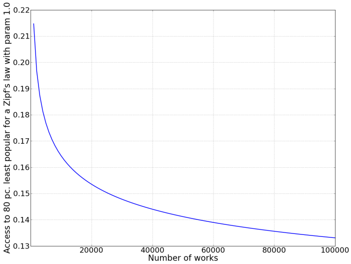
- Store contents in caches before file demands even known
- Reduce network load and latency during high-congestion periods
- Idea useful if certain files very popular and known in advance

Distributed Caches: Promising Solution for Cellular Networks



- Can cache at main BSs, picoBSs, femtoBSs, or directly at end users

File Popularities

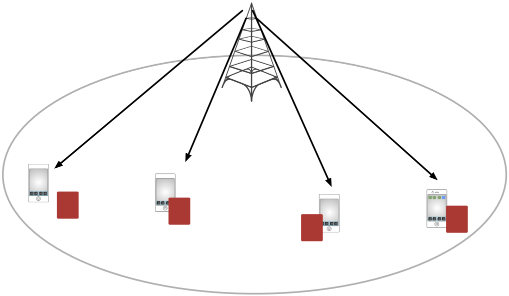


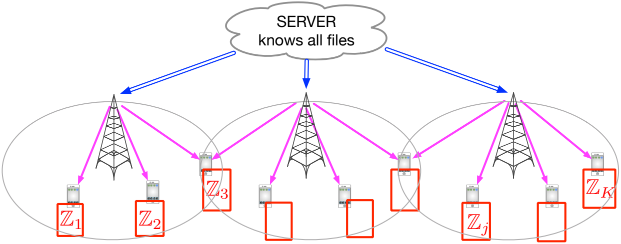
- Static file popularity follows a Zipf distribution $P(x) = Cx^{-\alpha}$

- Evolution of file popularities (youtube videos) can also be predicted

> Use pro-active caching to improve cellular systems!

Cellular Scenarios



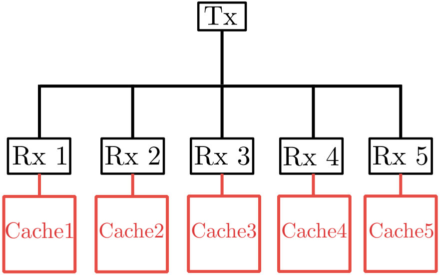


Other Assumptions

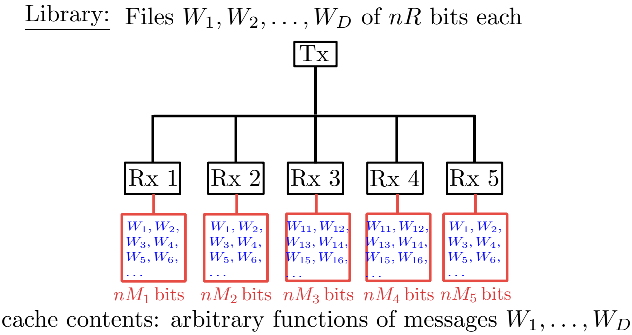
- All files equally popular $\rightarrow$ interested in worst-case performance
- Centralized protocol on how to fill caches
- Caches filled during nights when demands not yet known

Maddah-Ali & Niesen Source Coding Setup

Library: Files $W_1, W_2, \ldots, W_D$ of nR bits each (no popularities)



Maddah-Ali & Niesen Source Coding Setup



Communication in two phases:

- Caching phase: Tx fills caches without knowing demands $d_1, \ldots, d_5$

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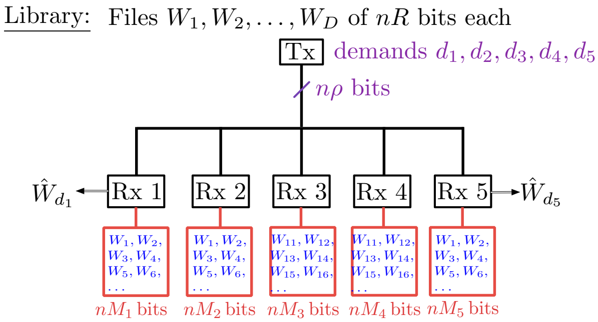
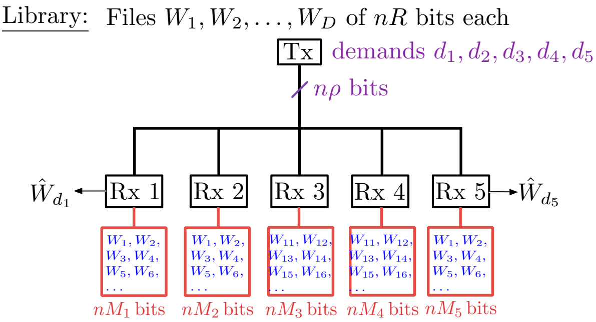


Communication in two phases:

- Caching phase: Tx fills caches without knowing demands $d_1, \ldots, d_5$
- Delivery phase: Tx describes $W_{d_1}, \ldots, W_{d_5}$ to Rxs $1, \ldots, 5$, respectively, through $n\rho$ common bits

Maddah-Ali & Niesen Source Coding Setup

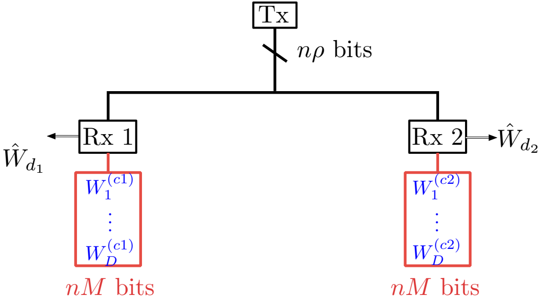


Rates-Memories Tradeoff

For which $(\rho, R, M_1, \ldots, M_K)$ is error-free data transmission possible?

Coded caching for $K = 2$ Receivers [Maddah-Ali&Niesen 2013]

Library: Files $W_1, W_2, \ldots, W_D$ of nR bits each

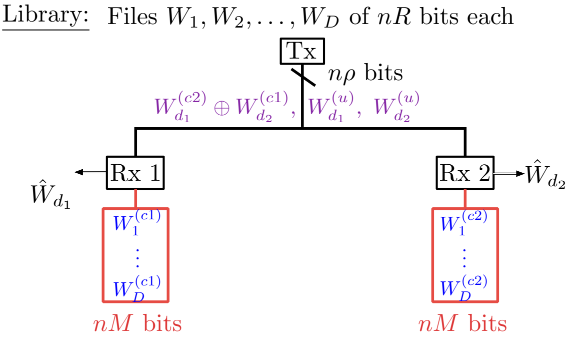


- Split $W_d = \left(W_d^{(c1)}, W_d^{(c2)}, W_d^{(u)}\right)$ of rates $\frac{M}{D}$, $\frac{M}{D}$, and $R - 2\frac{M}{D}$

Rates-Memory Trade-Off for moderate M:

Reconstruction possible, if $R \leq \frac{1}{2}\rho + \frac{3}{2}\frac{M}{D}$

Coded caching for $K = 2$ Receivers [Maddah-Ali&Niesen 2013]



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Rates-Memory Trade-Off for moderate M:

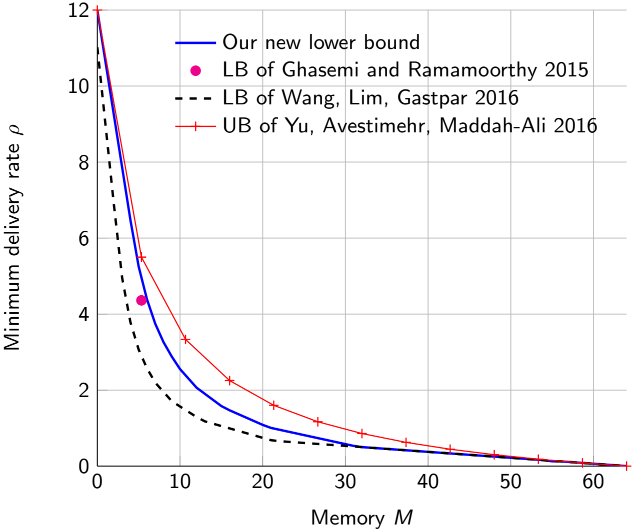
Reconstruction possible, if $R \leq \frac{1}{2}\rho + \frac{3}{2}\frac{M}{D}$

Reconstruction not possible, if $R \geq \frac{1}{2}\rho + \frac{M}{D}\left(1 + \frac{D+1}{2(D-1)}\right)$

(Saeedi,Wigger,Yener-2016)

Current Bounds on Minimum Delivery Rate ρ when $R = 1$

- $K = 12$ users, $D = 64$ files



Extensions

- Decentralized caching

  [M. A. Maddah-Ali, U. Niesen, “Decentralized coded caching attains order-optimal memory-rate tradeoff”]

- Nonuniform or random demands

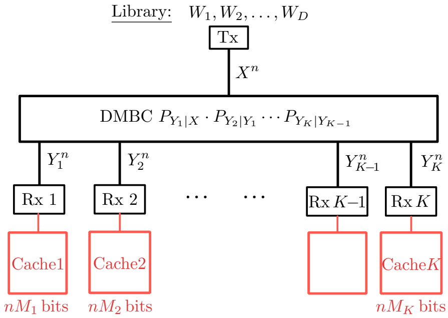
  [U. Niesen and M. A. Maddah-Ali, “Coded caching with nonuniform demands”]
  [Ji, Tulino, Llorca, and Caire, “Order-optimal rate of caching and coded multicasting with random demands”]

- Online caching phase

  [R. Pedarsani, M. A. Maddah-Ali and U. Niesen, “Online coded caching”]

Delivery over Noisy Broadcast Channel (BC)

[Saeedi, Timo, Wigger 2016, Saeedi, Wigger, Yener, 2016]



Capacity-Memory Tradeoff

$C(M_1, \ldots, M_K)$: supremum of achievable message rate R

- New achievability: joint cache-channel schemes based on piggyback coding
- New converse for degraded BCs

Converse for Degraded BCs with Arbitrary Caches

Theorem (Saeedi, Timo, Wigger 16)

$$C(M_1, \ldots, M_K) \leq \min_{\mathcal{S} \subseteq \{1,\ldots,K\}} \left(R_{\text{sym},\mathcal{S}}(M_1, \ldots, M_K) + \sum_{j=1}^{|\mathcal{S}|} \frac{M_{s_j}}{D} \right),$$

$R_{\text{sym},\mathcal{S}}$ and $M_{\mathcal{S}}$: symmetric capacity and total cache at receivers in $\mathcal{S}$

Theorem (Saeedi, Wigger, Yener 16)

$$C(M_1, \ldots, M_K) \leq I(U_1; Y_1) + \frac{M_1}{D},$$

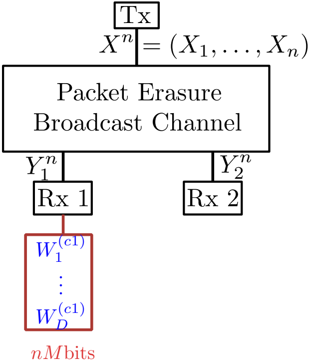
$$C(M_1, \ldots, M_K) \leq I(U_k; Y_k | U_{k-1}) + \frac{\sum_{i=1}^{k} M_i}{D - k + 1}, \qquad \forall k \in \{2, \ldots, K\}$$

where $U_1 - U_2 - \ldots - U_K - X - Y_K - \ldots - Y_1$.

- Suggests that cache-memory M_1 more useful than cache memory M_K
  $\rightarrow$ Each M_k serves for users $k, \ldots, K$

A Simple Example

Library: Files $W_1, W_2, \ldots, W_D$ of nR bits each



- $W_d = (W_d^{(c1)},\ W_d^{(u)})$ of rates $(\frac{M}{D},\ R - \frac{M}{D})$

A Simple Example

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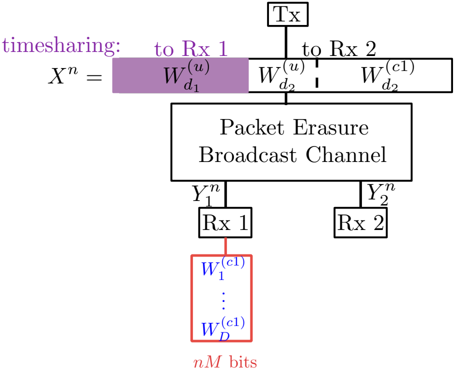


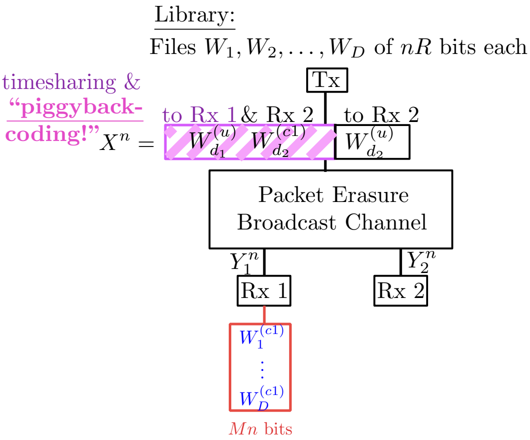
- $W_d = (W_d^{(c1)}, W_d^{(u)})$ of rates $(\frac{M}{D}, R - \frac{M}{D})$

Separate Cache-Channel Coding → No Global Caching Gain

$p(\text{error}) \to 0$ if: $$\frac{R - \frac{M}{D}}{F(1-\delta_1)} + \frac{R}{F(1-\delta_2)} \le 1$$

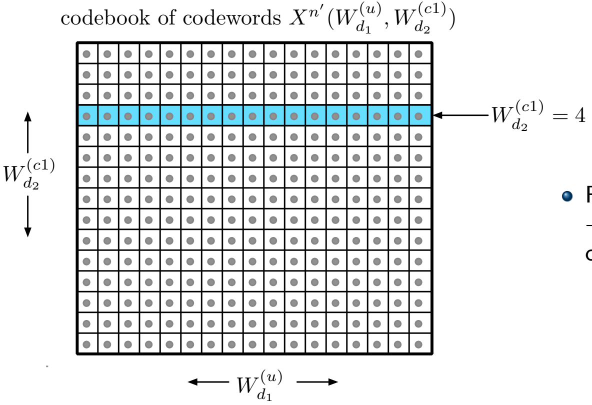
Standard Erasure BC: $p(error)$ if : $$\frac{R_1}{F(1-\delta_1)} + \frac{R_2}{F(1-\delta_2)} \le 1$$

Our Joint Cache-Channel Scheme for this Example



- $W_d = (W_d^{(c1)},\ W_d^{(u)})$ of sub-rates $(\frac{\mathsf{M}}{D},\ \rho - \frac{\mathsf{M}}{D})$

Piggyback Coding to Send $(W_{d_1}^{(u)}, W_{d_2}^{(c1)})$ to Both Rxs

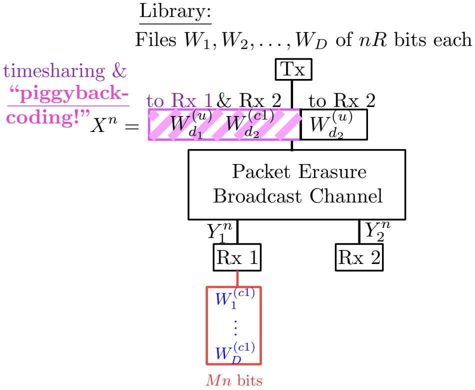


- Rx 1 knows $W_{d_2}^{(c1)}$ $\rightarrow$ restrict decoding to corresponding row

Transmission of $W_{d_2}^{(c1)}$ not affecting Rx 1 at all!

$$p(error) \rightarrow 0 \text{ as } n \rightarrow \infty : \qquad \max\left\{ \frac{R - \frac{M}{D}}{F(1-\delta_1)}, \frac{R}{F(1-\delta_2)} \right\} \leq \frac{n'}{n}$$

Performance of Joint Cache-Channel Scheme for Example

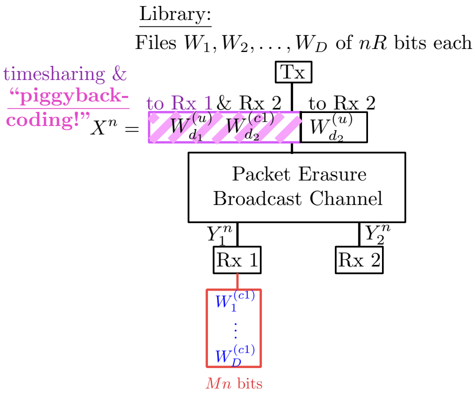


- $W_d = (W_d^{(c1)}, \ W_d^{(u)})$ of sub-rates $(\frac{M}{D}, \ R - \frac{M}{D})$

Joint Cache-Channel Coding → Global Caching Gain!

$$p(error) \to 0 \quad \text{if :} \qquad \underbrace{\max\left\{\frac{R-\frac{M}{D}}{F(1-\delta_1)}, \ \frac{R}{F(1-\delta_2)}\right\}}_{\text{piggyback coding}} + \frac{R-\frac{M}{D}}{F(1-\delta_2)} \leq 1$$

Performance of Joint Cache-Channel Scheme for Example



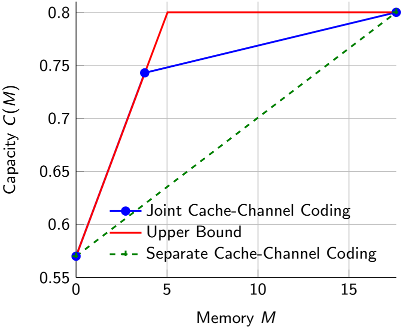
- $W_d = (W_d^{(c1)},\ W_d^{(u)})$ of sub-rates $(\frac{M}{D},\ R - \frac{M}{D})$

Joint Cache-Channel Coding → Global Caching Gain!

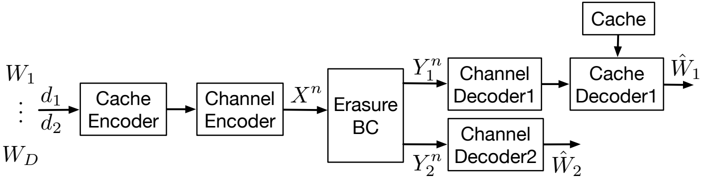
$$p(error) \to 0 \quad \text{if} : R \leq \min \left\{ \underbrace{\frac{F(1-\delta_1)(1-\delta_2)}{(1-\delta_1)+(1-\delta_2)}}_{C_{\text{Sym}}} + \frac{M}{D},\quad \frac{1}{2}F(1-\delta_2) + \frac{M}{2D} \right\}$$

K-User Erasure BC with Cache only at Weakest Receiver

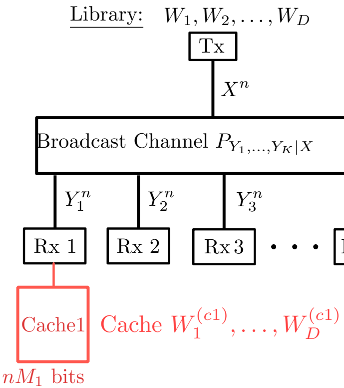
$\delta_1 = 0.8,$
$\delta_2 = \ldots = \delta_K = 0.2,$
$K = 11, \quad D = 22,$
$F = 10$



- Separate Cache-Channel Architecture



Single Cache and $K-1$ stronger receivers



$$W_d = (W_d^{(c1)},\ W_d^{(u)})$$

of rates $\left(\frac{M_1}{D},\ R - \frac{M_1}{D}\right)$

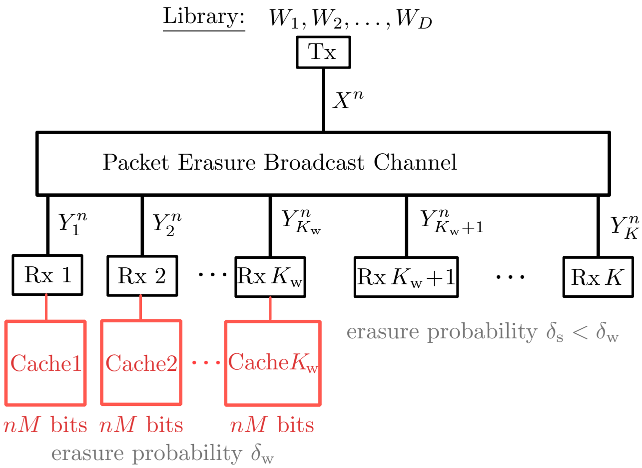
Optimal Coding Scheme for Single Cache and low Cache Memory

- Superposition code achieving C_{Sym} with piggyback cloud center
- Piggyback cloud center encodes $W_{d_1}^{(u)}, W_{d_2}^{(c_1)}, \ldots, W_{d_K}^{(c_1)}$
- Receiver 1 performs restricted decoding
- Receivers $2 \ldots, K$ can decode if $(K-1)\frac{M_1}{D} \leq I(U_1^\star; Y_2) - I(U_1^\star; Y_1)$

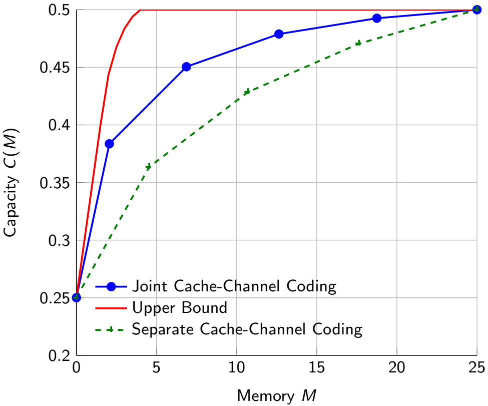
Performance

$$C(M_1, M_2 = 0, \ldots, M_K = 0) = C_{\text{Sym}} + \frac{M_1}{D}, \qquad M_1 \leq D \cdot \frac{I(U_1^\star; Y_2) - I(U_1^\star; Y_1)}{K-1}$$

K_w Weak Receivers with Caches and K_s Strong Receivers Without

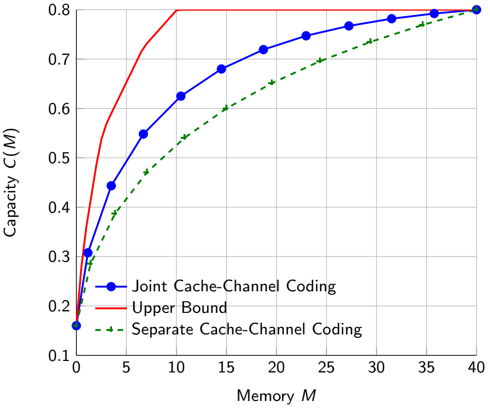


Weak and Strong Receivers: Example I



$$\delta_w = 0.8, \ \ \delta_s = 0.2, \ \ K_w = 4, \ K_s = 16, \ \ D = 50, \ \ F = 10$$

Weak and Strong Receivers: Example II



$$\delta_{\mathrm{w}} = 0.8, \;\; \delta_{\mathrm{s}} = 0.2, \;\; K_{\mathrm{w}} = 10, \; K_{\mathrm{s}} = 10, \;\; D = 50, \;\; F = 10$$

Most Significant Gains for Low Cache Sizes $M/D \leq \Gamma_1$

For $M/D \leq F \frac{(1-\delta_\mathrm{s})}{K_\mathrm{s}} \frac{(\delta_w - \delta_\mathrm{s})}{(K_\mathrm{s}(1-\delta_w)+(1-\delta_\mathrm{s}))}$:

$$C(M) \geq C_{\mathrm{Sym}} + \frac{M}{D} \cdot \gamma_{\mathrm{local}} \cdot \gamma_{\mathrm{global,sep}} \cdot \gamma_{\mathrm{global,joint}},$$

with

$$\gamma_{\mathrm{local}} := \frac{K_\mathrm{w}(1-\delta_\mathrm{s})}{K_\mathrm{w}(1-\delta_\mathrm{s}) + K_\mathrm{s}(1-\delta_\mathrm{w})},$$
$$\gamma_{\mathrm{global,sep}} := \frac{1+K_\mathrm{w}}{2},$$
$$\gamma_{\mathrm{global,joint}} := 1 + \frac{2K_\mathrm{w}}{1+K_\mathrm{w}} \cdot \frac{K_\mathrm{s}(1-\delta_\mathrm{w})}{K_\mathrm{w}(1-\delta_\mathrm{s})}.$$

All Receivers with Cache Memories

- $W_d = (W_d^{(1)}, \ldots, W_d^{(K)})$ or rates $I(X; Y_1), \ldots, I(X; Y_K)$
- Rate $R = \sum_{k=1}^{K} I(X; Y_k)$
- Receiver k caches $W_d^{(1)}, \ldots, W_d^{(k-1)}, W_d^{(k+1)}, \ldots, W_d^{(K)}$

  for all $d = 1, \ldots, D$
- For transmission, use a K-dimensional product codebook

  $\rightarrow$ Receiver k knows $K-1$ dimensions; can decode last dimension of rate $I(X; Y_k)$
- Total cache $(K-1) \sum_{k=1}^{K} I(X; Y_k)$

Capacity-Memory Tradeoff under Global Memory Constraint

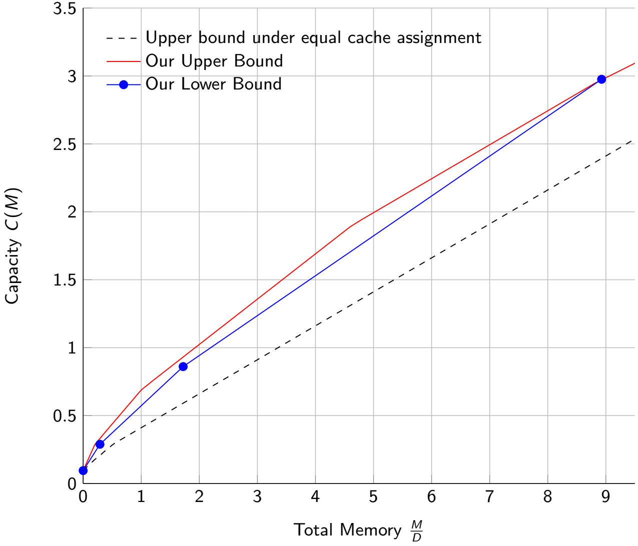
$$C^{\star}(M) \leq \max C(M_1, \ldots, M_K),$$

where max over $(M_1, \ldots, M_K)$: $\sum_{k=1}^{K} M_k \leq M$

‘

- Have upper and lower bounds from previous our results
- They are tight for small and large cache memories M
- Equa cache assignment highly suboptimal for non-symmetric BCs

Global Capacity-Memory Tradeoff for a Gaussian-BC Example



Insights and Intuition

- Important to consider noisy communication channel:
  - Joint cache-channel coding (piggyback coding)
  - Caching gains combine with feedback gains

    [A. Ghorbel, M. Kobayashi, S. Yang, “Cache-enabled broadcast packet erasure channels with state feedback”]
  - Interplay between caching gains and CSI gains

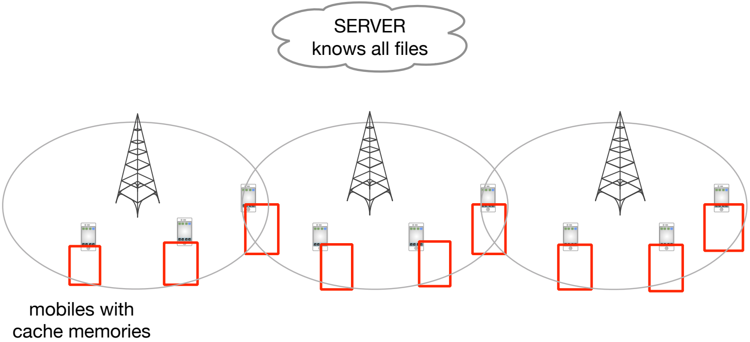
    [J. Zhang and P. Elia, “Fundamental limits of cache-aided wireless BC: interplay of coded-caching and CSIT feedback”]

- Gains with cache allocation $\rightarrow$ larger caches to weak receivers

- Cache allocation gains boosted through joint cache-channel coding

- Piggyback coding useful whenever info for strong Rx in cache of weak Rx!

Multi-Cell Model with 3 Communication Phases



1. Caching Phase: Server fills caches while ignoring demands $d_1, \ldots, d_K$
2. Download from Server: BSs download messages of connected rxs
3. Delivery to Users: BSs communicate messages to rxs over network

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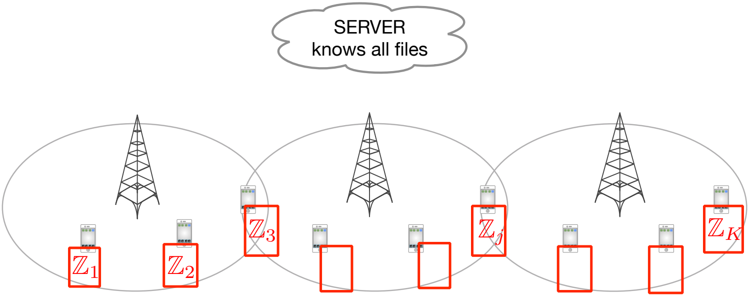

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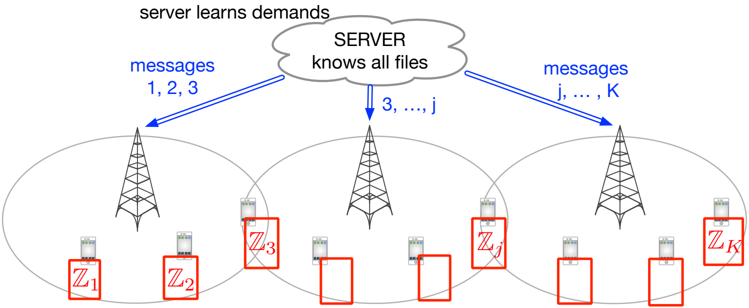

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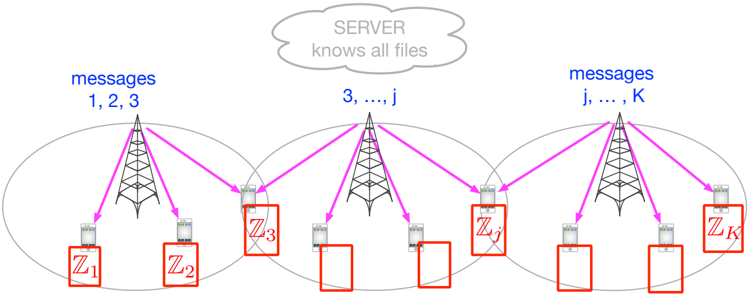

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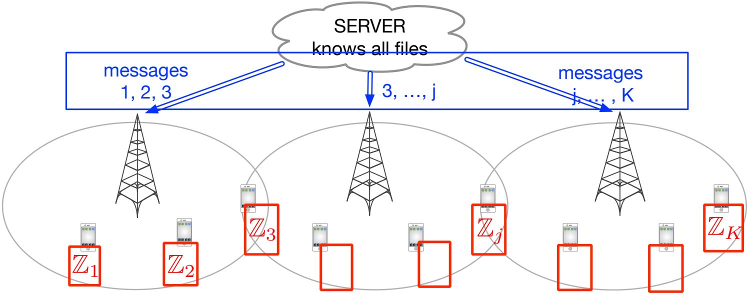
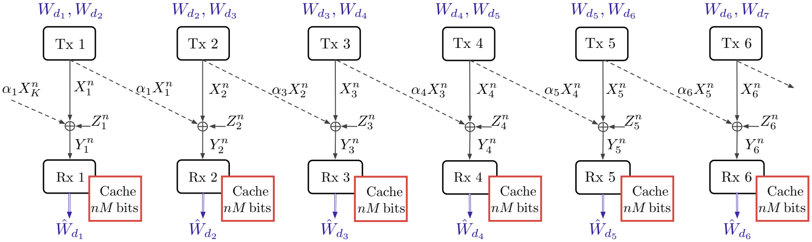


- All files revealed to all BSs → BC
- Caches also at BSs & rate-limited pipes from servers to BSs: Karamchandani, Niesen, Maddah-Ali, Diggavi-2014.
- BSs download files only in caching phase (tx&rx caches): Maddah Ali&Niesen-2015; Pooya-Abolfazl-Hossein-2015; Naderializadeh-Maddah Ali-Avestimehr-2016; Hachem-Niesen-Diggavi-2016

Caching in Wyner's Cellular Networks

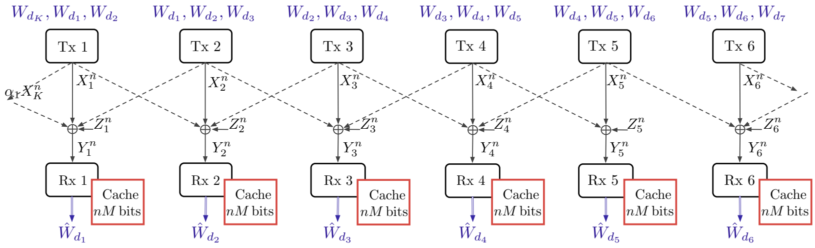
- Delivery over *Wyner's Asymmetric Network*



- Transmitters constrained to power P

Caching in Wyner's Cellular Networks

- Delivery over *Wyner's Symmetric Network*



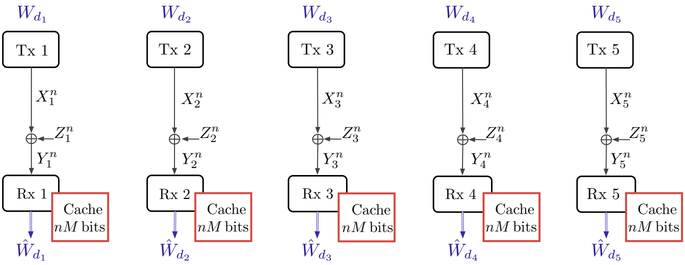
- Transmitters constrained to power P

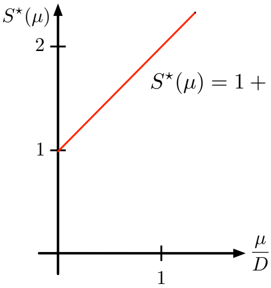
DoF Rate-Memory Tradeoff

- Power $P \gg 1$
- Rate $R = S \cdot \frac{1}{2}\log(1+P)$
- Memory $M = \mu \cdot \frac{1}{2}\log(1+P)$

$$S^\star(\mu) := \max\left\{S : \quad (S,\mu) \text{ achievable}\right\}$$

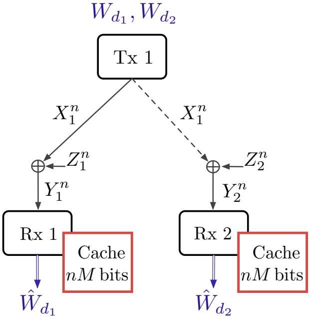
DoF Rate-Memory Tradeoff for Network with Parallel Links

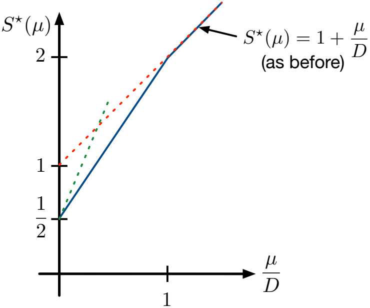




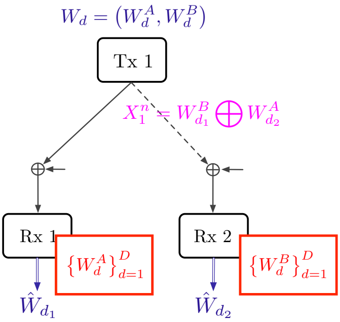
$$S^\star(\mu) = 1 + \frac{\mu}{D} : \quad \text{local caching gain!}$$

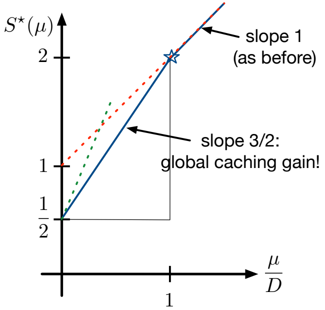
DoF Rate-Memory Tradeoff of Two-User BC





DoF Rate-Memory Tradeoff of Two-User BC

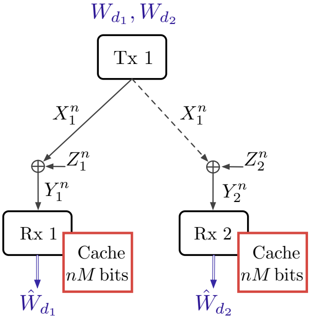


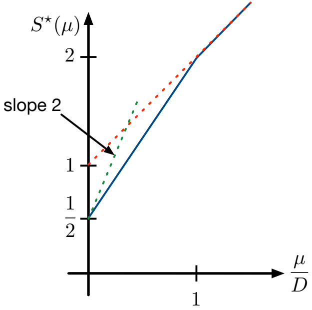


Maddah-Ali&Niesen Coded Caching:

- Cache half of each message at Rx1 and the other at Rx2 $\rightarrow \mu = 1$
- Send XOR of missing halves! $\rightarrow S = 2\mu/D = 2$

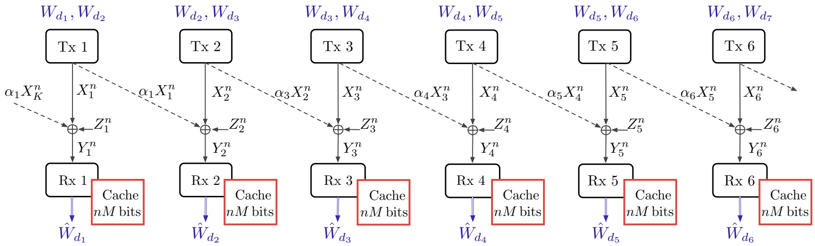
DoF Rate-Memory Tradeoff of Two-User BC





- Slope 2: each receiver can fully profit from both caches (Upper bound from [Saeedi-W-Timo 2016])

Back to Asymmetric Circular Wyner Model

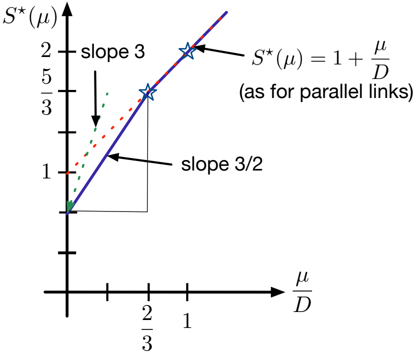


- Without cache memories: $S^\star(\mu = 0) = 2/3$

Our Bounds on DoF Rate-Memory Tradeoff for Asymmetric Model

Theorem

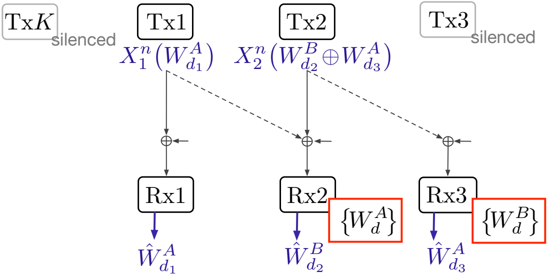
- *If* $\frac{\mu}{D} \leq \frac{2}{3}$: $\quad \frac{2}{3} + \frac{3}{2}\frac{\mu}{D} \leq S^\star \leq \min\left\{\frac{2}{3} + 3\frac{\mu}{D}\ ,\ 1 + \frac{\mu}{D}\right\}$
- *If* $\frac{\mu}{D} \geq \frac{2}{3}$: $\quad S^\star = 1 + \frac{\mu}{D}$



Scheme achieving $\mu/D = 2/3$ and $S = 5/3$ for Asymmetric Network

- Split $W_d = (W_d^A, W_d^B, W_d^C, W_d^D, W_d^E)$ each of DoF 1
- 3 delivery phases: in each we silence every third transmitter $\rightarrow$ subnets

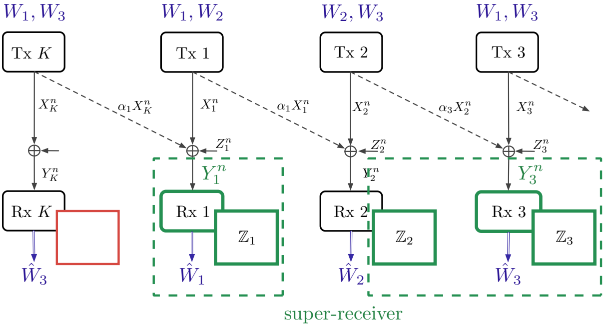
Phase 1 Subnet:



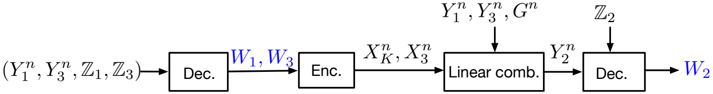
- Rx 3: Decodes $W_{d_2}^B \bigoplus W_{d_3}^A$; gets $W_{d_3}^A$;
- Rx 2: Forms $Y_2^n - X_1^n(W_{d_1}^A)$; decodes $W_{d_2}^B \bigoplus W_{d_3}^A$; gets $W_{d_2}^B$

Converse for Asymmetric Network

- Choose a demand vector (worst-case error)

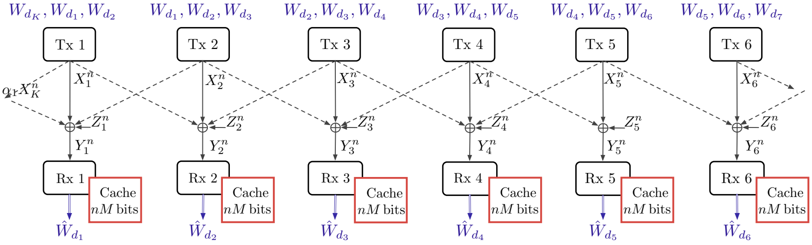


- MAC- bound: If Rx 1–3 decode correctly, super receiver can do the same:



- Average over demand vectors! $\rightarrow$ factor $1/D$

Symmetric Circular Wyner Model

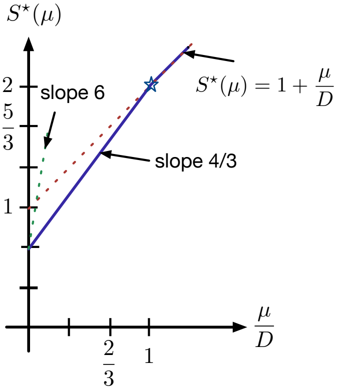


- Without cache memories: $S^{\star}(\mu = 0) = 2/3$

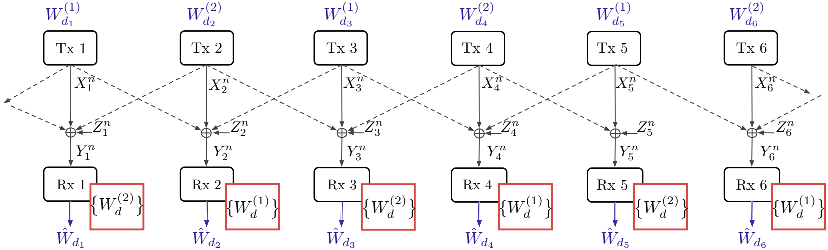
Our Bounds on DoF Rate-Memory Tradeoff for Symmetric Model

Theorem

- *If* $\frac{\mu}{D} \leq 1$: $\quad \frac{2}{3} + \frac{4}{3}\frac{\mu}{D} \leq S^\star \leq \min\left\{\frac{2}{3} + 6\frac{\mu}{D}\ ,\ 1 + \frac{\mu}{D}\right\}$
- *If* $\frac{\mu}{D} \geq 1$: $\quad S^\star = 1 + \frac{\mu}{D}$



Coding Scheme for Symmetric Network



- Split $W_d = (W_d^A, W_d^B)$ each of DoF 1
- Cache contents allow receivers to pre-cancel interference

Summary

- Noisy channels $\rightarrow$ joint cache-channel coding (piggyback coding)

- Cache allocation: more caches to weaker receivers

  $\rightarrow$ joint cache-channel codes boost cache allocation gains

- Caching allows to completely cancel interference in cellular networks
  $\rightarrow$ smart code design significantly reduces required cache sizes

- Outer bounds for degraded BCs and sparse interference networks with receiver caching