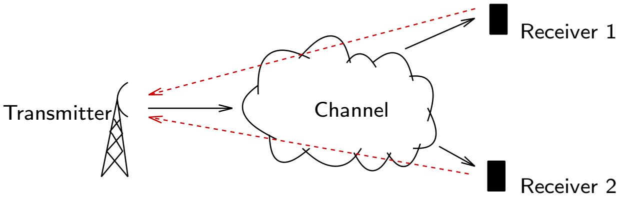
Schemes and Achievable Rates for Broadcast Channels with Rate-Limited Feedback



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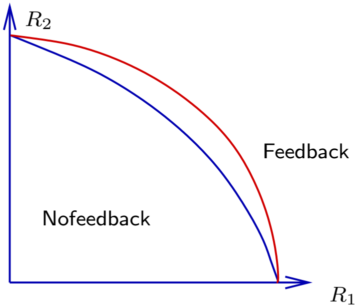
joint work with Youlong Wu and Thomas Laich

ETH Zurich, 22 August 2013

In a Nutshell

Ex.: Asymmetric binary symmetric BC

No matter how small $R_{\mathsf{fb}} > 0$, feedback increases *entire capacity region!*



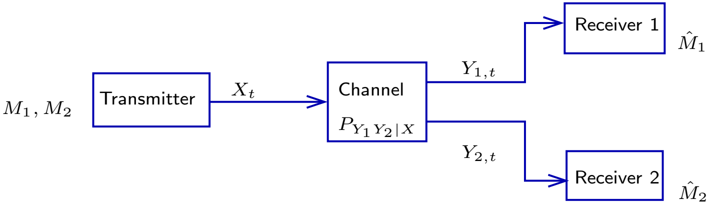
- Rate-limited or noisy feedback of capacity R_{fb} from weaker receiver

Outline

- Review on BCs *without* feedback (incl. a BC with side information)
- BC *with one- or two-sided rate-limited feedback*
- A related source coding setup

Broadcast Channels without Feedback

Discrete Memoryless BC without Feedback

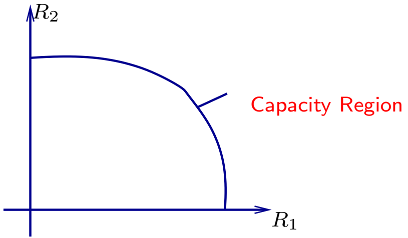


- Rx i wants to learn $M_i \in \{1, \ldots, 2^{nR_i}\}$
- Inputs $X_t = f_t(M_1, M_2)$
- Finite input and output alphabets $\mathcal{X}, \mathcal{Y}_1, \mathcal{Y}_2$
- Memoryless channel $P_{Y_1 Y_2|X}$

Capacity Region

- Rates of communication $R_1, R_2 \geq 0$

- Capacity region: Pairs (R_1, R_2) s.t. $p(\text{error})$ arbitrarily small



Marton's Achievable Region

(R_1, R_2) achievable if:

$$R_1 \leq I(U_0, U_1; Y_1)$$

$$R_2 \leq I(U_0, U_2; Y_2)$$

$$R_1 + R_2 \leq I(U_1; Y_1 | U_0) + I(U_2; Y_2 | U_0) + \min_i I(U_0; Y_i) - I(U_1; U_2 | U_0)$$

for some (U_0, U_1, U_2) s.t. $(U_0, U_1, U_2) \multimap X \multimap (Y_1, Y_2)$

- In general capacity region unknown
- For some channels Marton's region equals capacity

Strictly Less-Noisy DMBCs

Less-Noisy DMBC $Y_2 \succeq Y_1$

For every auxiliary $U \multimap X \multimap (Y_1, Y_2)$:

$$I(U; Y_2) \geq I(U; Y_1)$$

- Binary Symmetric BC
- Binary Erasure BC
- Binary Symmetric/Erasure BC for certain parameters

Strictly Less-Noisy DMBCs

Strictly Less-Noisy DMBC $Y_2 \succ Y_1$

For every auxiliary $U -\!\!\circ\!\!- X -\!\!\circ\!\!- (Y_1, Y_2)$ with $I(U;Y_1) > 0$:

$$I(U;Y_2) > I(U;Y_1)$$

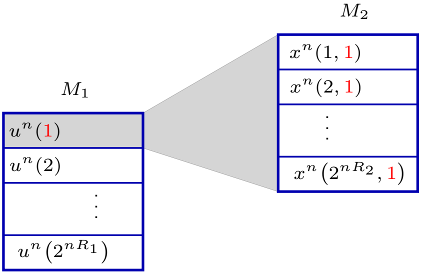
- Asymmetric Binary Symmetric BC
- Asymmetric Binary Erasure BC
- Binary Symmetric/Erasure BC for certain parameters

Capacity of Less-Noisy DMBCs, $Y_2 \succeq Y_1$

- Capacity: all rate pairs (R_1, R_2) where for some $U - X - (Y_1, Y_2)$

$$R_1 \leq I(U; Y_1)$$
$$R_2 \leq I(X; Y_2|U)$$

- Achieved by superposition coding (degenerate Marton coding)

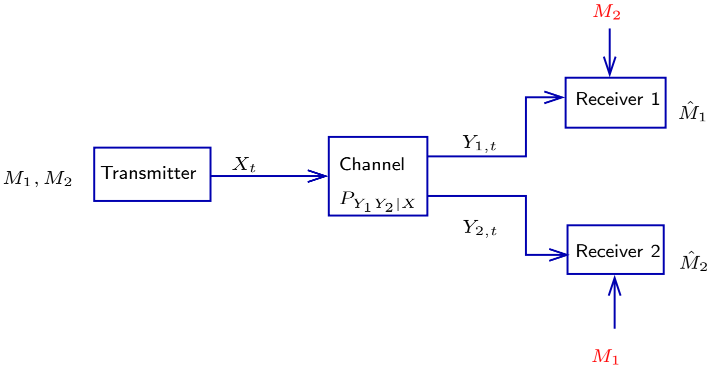


- Receiver 2 can decode M_1 because $I(U; Y_2) \geq I(U; Y_1)$

- If $I(U; Y_2) > I(U; Y_1)$, Rx 2 can potentially decode larger cloud center

A BC with Side-Info: Complementary Delivery

- Receiver 1 knows M_2 and Receiver 2 knows M_1



- Capacity: $R_1 \leq I(X;Y_1)$ and $R_2 \leq I(X;Y_2)$ for some P_X

- Given P_X: point-to-point performance to both receivers

Complementary Delivery: Capacity Achieving Scheme

- Product codebook for messages (M_1, M_2)

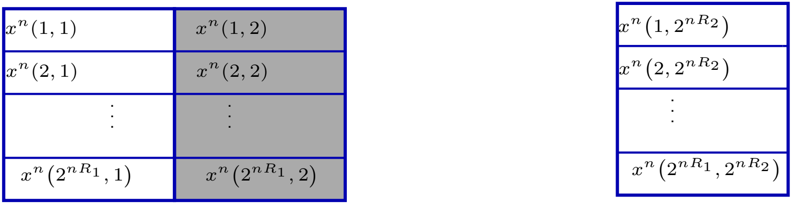
$x^n(1,1)$	$x^n(1,2)$
$x^n(2,1)$	$x^n(2,2)$
$\vdots$	$\vdots$
$x^n(2^{nR_1},1)$	$x^n(2^{nR_1},2)$

$x^n(1,2^{nR_2})$
$x^n(2,2^{nR_2})$
$\vdots$
$x^n(2^{nR_1},2^{nR_2})$

- vertical dimension encodes M_1
- horizontal dimension encodes M_2

Complementary Delivery: Capacity Achieving Scheme

- Product codebook for messages (M_1, M_2)



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Complementary Delivery: Capacity Achieving Scheme

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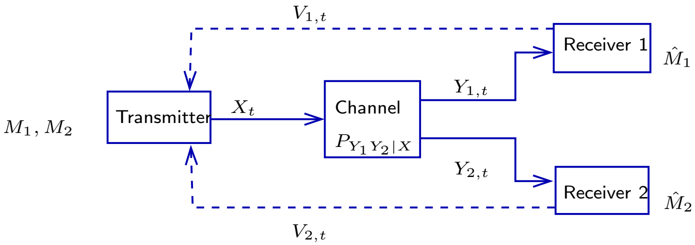
$x^n(2,1)$	$x^n(2,2)$	$x^n(2,2^{nR_2})$
$\vdots$	$\vdots$	$\vdots$
$x^n(2^{nR_1},1)$	$x^n(2^{nR_1},2)$	$x^n(2^{nR_1},2^{nR_2})$

- vertical dimension encodes M_1
- horizontal dimension encodes M_2

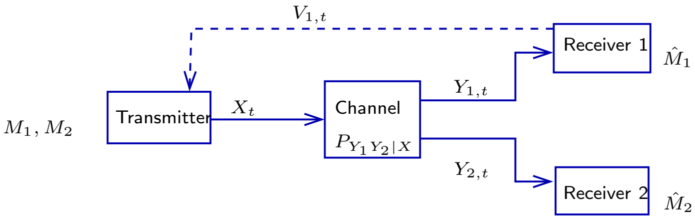
Now with Feedback

Noise-Free Rate-Limited Feedback



- $V_{i,t} = \phi_{i,t}(Y_i^t)$ where $|\mathcal{V}_{i,1}| \cdots |\mathcal{V}_{i,n}| \leq nR_{\text{fb}}, \quad i = 1, 2$
- Two-sided feedback: $X_t = f_t(M_1, M_2, V_1^{t-1}, V_2^{t-1})$
- One-sided feedback: $X_t = f_t(M_1, M_2, V_1^{t-1})$

Noise-Free Rate-Limited Feedback



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- One-sided feedback: $X_t = f_t(M_1, M_2, V_1^{t-1})$

Previous Results: Two-sided feedback, $R_{\mathsf{fb}} = \infty$

- Capacity region not known in general
- El Gamal'78: No feedback-gain for physically degraded BCs
- Dueck'80, Kramer'00: Feedback-gain for some discrete memoryless BCs
- Shayevitz-W'13 and Venkataramanan-Pradhan'13:
  single-letter achievable region for general memoryless BCs
- Outer bound: a genie reveals Y_1 to Receiver 2 or vice versa

For most BCs: don't know whether feedback helps

Feedback for the Gaussian BC, $R_{\text{fb}} = \infty$

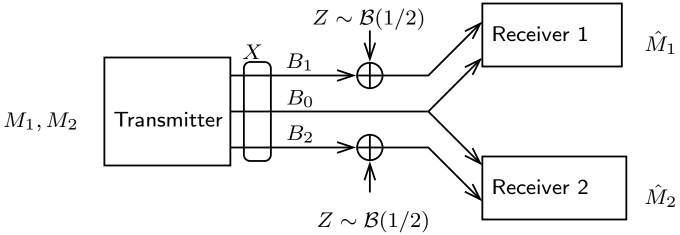
Two-sided feedback:

- Ozarow-Leung'83: Feedback can increase capacity
- Gastpar-Lapidoth-Steinberg-W'13: High-SNR sum-capacity asymptotes
- Ardestinazadeh-Minero-Franceschetti'11: Best achievable sum-rate for equal noise-variances

One-sided feedback:

- Bhaskaran'08: Feedback helps
- Lapidoth-Steinberg-W'11: improved rates

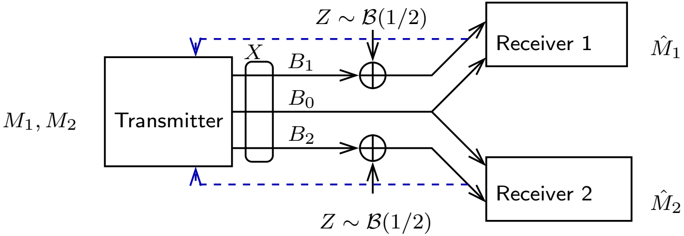
First Intuition how Feedback can Help: Dueck's Example



Without feedback:

- Top and bottom links useless
- Capacity: $0 \leq R_1 + R_2 \leq 1$

First Intuition how Feedback can Help: Dueck's Example



With feedback, $R_{\text{fb}} = 1$:

- Feedback: $V_{i,t} = Y_{i,t}$
- Transmitter sends $B_{0,t} = Z_{t-1}$
- Capacity: $0 \leq R_1 \leq 1$ and $0 \leq R_2 \leq 1$

$\rightarrow$ Tx can identify common update info Z_{t-1} useful to both Rxs

Generalization of this Idea ($R_{\text{fb}} = \infty$) (Shayevitz&Wigger'13)

- Send fresh data $M_{1,b}, M_{2,b}$ & update info using Marton's scheme
- Update info: quantize $U_{0,b}^N, U_{1,b-1}^N, U_{2,b-1}^N$ given SI $Y_{1,b-1}^N$ or $Y_{2,b-1}^N$
- Backward decoding

Generalization of this Idea ($R_{\text{fb}} = \infty$) (Shayevitz&Wigger'13)

- Send fresh data $M_{1,b}, M_{2,b}$ & update info using Marton's scheme
- Update info: quantize $U_{0,b}^N, U_{1,b-1}^N, U_{2,b-1}^N$ given SI $Y_{1,b-1}^N$ or $Y_{2,b-1}^N$
- Backward decoding

- Under SI $Y_{1,b-1}^n$ or $Y_{2,b-1}^n$, quantization of $U_{0,b-1}^N, U_{1,b-1}^N, U_{2,b-1}^N$ has common part useful to both rxs
- Tradeoff: update-info sent at expense of fresh data!

Shayevitz-W. Region for Two-Sided Feedback, $R_{\mathsf{fb}} = \infty$

Achievable Region

(R_1, R_2) achievable, if for some $P_{U_0U_1U_2}P_{X|U_0U_1U_2}P_{Y_1Y_2|X}P_{V_0V_1V_2|U_0U_1U_2}$:

$$
\begin{aligned}
R_1 &\leq I(U_0, U_1; Y_1, V_1) - I(U_0, U_1, U_2, Y_2; V_0, V_1|Y_1) \\
R_2 &\leq I(U_0, U_2; Y_2, V_2) - I(U_0, U_1, U_2, Y_1; V_0, V_2|Y_2) \\
R_1 + R_2 &\leq I(U_1; Y_1, V_1|U_0) + I(U_2; Y_2, V_2|U_0) + \min_{i\in\{1,2\}} I(U_0; Y_i, V_i) \\
&\quad - I(U_0, U_1, U_2, Y_2; V_1|V_0, Y_1) - I(U_0, U_1, U_2, Y_1; V_2|V_0, Y_2) \\
&\quad - I(U_1; U_2|U_0) - \max_{i\in\{1,2\}} I(U_0, U_1, U_2, Y_1, Y_2; V_0|Y_i)
\end{aligned}
$$

New Scheme with One-Sided Feedback for $Y_2 \succ Y_1$

- Superposition coding
- Cloud center $U_b^N(M_{1,b}, \tilde{M}_{\text{fb},b-1})$ and satellite $X_b^N(M_{2,b}|M_{1,b}, \tilde{M}_{\text{fb},b-1})$

- $\tilde{M}_{\text{fb},b} = V_b^N$ feedback bits sent by Rx 1 — $R_{\text{fb}} > \tilde{R}$
- $\tilde{M}_{\text{fb},b}$ quantizes $Y_{1,b}^N$ for given SI $Y_{2,b}^N$ and U_b^N — $\tilde{R} > I(Y_1; \hat{Y}_1|UY_2)$

- Rx 1 decodes $M_{1,b}$ — $R_1 < I(U; Y_1)$
- Rx 2 decodes $(M_{1,b}, \tilde{M}_{\text{fb},b-1})$ — $R_1 + \tilde{R} < I(U; Y_2)$
- Rx 2 reconstructs $\hat{Y}_{1,b}^N$ and decodes $M_{2,b}$ — $R_2 < I(X; Y_2\hat{Y}_1|U)$

A New Achievable Region

New achievable region

(R_1, R_2) achievable, if for some $P_U P_{X|U} P_{Y_1Y_2|X} P_{\hat{Y}_1|UY_1}$:

$$
\begin{aligned}
R_1 &\le I(U;Y_1) \\
R_1 &\le I(U;Y_2) - I(\hat{Y}_1;Y_1|UY_2) \\
R_2 &\le I(X;\hat{Y}_1 Y_2|U)
\end{aligned}
$$

and $\quad I(\hat{Y}_1;Y_1|UY_2) \le R_{\mathsf{FB}}$.

- Always includes nofeedback capacity for $Y_2 \succeq Y_1$

An Easier (Potentially Smaller) Achievable Region

Corollary I

(R_1, R_2) achievable, if for some $P_U P_{X|U} P_{Y_1 Y_2|X} P_{\hat{Y}_1|UY_1}$:

$$R_1 \leq I(U; Y_1)$$
$$R_2 \leq I(X; \hat{Y}_1 Y_2|U) = I(X; Y_2|U) + I(X; \hat{Y}_1|UY_2)$$

and $$I(\hat{Y}_1; Y_1|UY_2) \leq \min\{R_{\mathsf{FB}}, I(U; Y_2) - I(U; Y_1)\}.$$

An Easier (Potentially Smaller) Achievable Region

Corollary I

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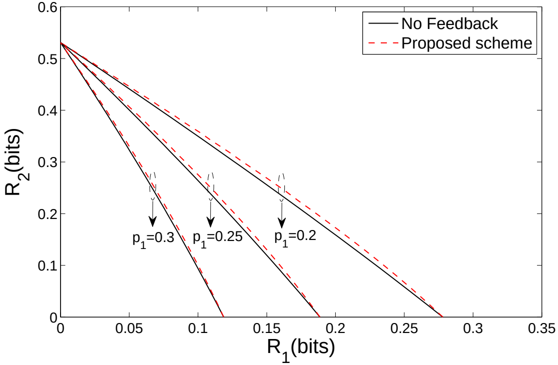
$$R_2 \leq I(X; \hat{Y}_1 Y_2|U) = I(X; Y_2|U) + I(X; \hat{Y}_1|UY_2)$$

and $\quad I(\hat{Y}_1; Y_1|UY_2) \leq \min\{R_{\mathsf{FB}}, I(U; Y_2) - I(U; Y_1)\}.$

- Sending $\hat{Y}_1$ is purely beneficial: not bothering Rx 1 and helping Rx 2
- Rate-gain is $I(X; \hat{Y}_1|UY_2)$ with $R_{\mathsf{FB}} = I(\hat{Y}_1; Y_1|UY_2)$ feedback bits
- Scheme good when $R_{\mathsf{fb}} \approx (I(U; Y_2) - I(U; Y_1))$

Example: Asymmetric Binary Symmetric BC

- $Y_i = X \oplus Z_i$ with independent Z_i ~Bern(p_i) and $0 < p_2 < p_1 < 1/2$



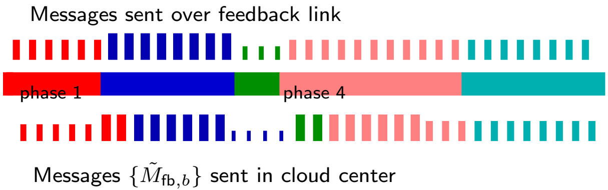
$p_2 = 0.1$

$R_{\text{fb}} = 0.8$

- U ~Bern(q) and $X = U \oplus W_1$ with W_1 ~Bern(r)
- $\hat{Y}_1 = U \oplus Y_1 \oplus W_2$ with W_2 ~Bern(s)

Extension I: Postponing $\tilde{M}_{\text{fb},b}$ to subsequent phases

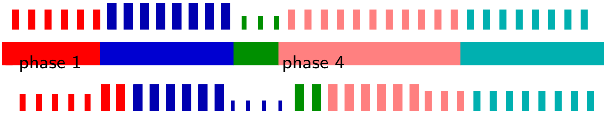
- Some U, X and $\hat{Y}_1$ allow for large $\tilde{R}$, for others $\hat{Y}_1$ very beneficial at Rx 2



- In each phase, we use different parameters
- Delay the transmission of $\{\tilde{M}_{\text{fb},b}\}$ to subsequent phases
- Receiver 2 waits with decoding $M_{2,b}$ until it can reconstruct $\tilde{M}_{\text{fb},b}$

Extension I: Postponing $\tilde{M}_{\mathsf{fb},b}$ to subsequent phases

- Some U, X and $\hat{Y}_1$ allow for large $\tilde{R}$, for others $\hat{Y}_1$ very beneficial at Rx 2



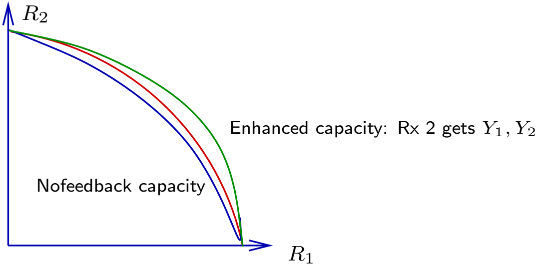
Achievable Region

(R_1, R_2) achievable, if for some $P_Q P_{U|Q} P_{X|UQ} P_{Y_1 Y_2|X} P_{\hat{Y}_1|UY_1Q}$:

$$R_1 \leq I(U; Y_1|Q)$$
$$R_1 \leq I(U; Y_2|Q) - I(\hat{Y}_1; Y_1|UY_2Q)$$
$$R_2 \leq I(X; \hat{Y}_1 Y_2|UQ)$$

and $\quad I(\hat{Y}_1; Y_1|UY_2Q) \leq R_{\mathsf{FB}}.$

If $R_{\text{fb}} > 0$, feedback increases entire capacity region



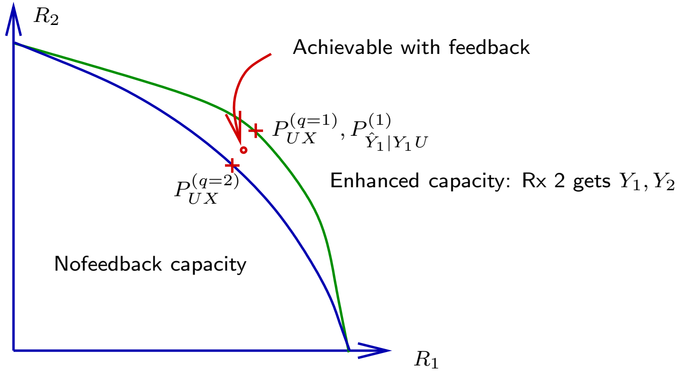
For Any $R_{\text{fb}} > 0$ and Strictly Less-Noisy BCs:

Feedback improves all $(R_1 > 0, R_2 > 0)$ of the nofeedback capacity,

unless (R_1, R_2) lies on boundary of capacity of enhanced channel

- For most strictly less-noisy BCs: all $(R_1 > 0, R_2 > 0)$ increased!
- Examples: Binary Symmetric, Binary Erasure BC, Gaussian BC

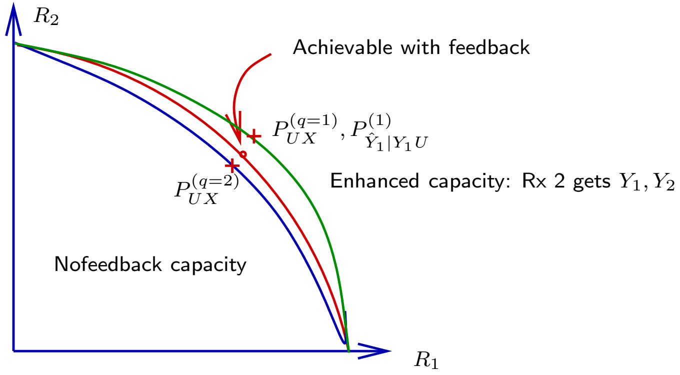
Proof that feedback increases entire capacity region



- Phase $q = 1$: Use $P_{UX}^{(q=1)}$ and $\hat{Y}_1^{(1)} = Y_1^{(1)}$
- Phase $q = 2$: Use $P_{UX}^{(q=2)}$, send remaining feedback bits from phase 1!

  $\rightarrow$ phase 2 needs to be sufficiently long!

Proof that feedback increases entire capacity region



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- Phase $q = 2$: Use $P_{UX}^{(q=2)}$, send remaining feedback bits from phase 1!

  $\rightarrow$ phase 2 needs to be sufficiently long!

Extension II: Receiver 2 performs backward decoding

- Receiver 2 does backward decoding
  - Decodes $(M_{1,b}, \tilde{M}_{\text{fb},b-1}, M_{2,b})$ based on $(Y_{2,b}^N, \hat{Y}_{1,b}^N)$!
  - Wyner-Ziv scheme cannot be superpositioned on U_b^n

(Potentially) Larger Achievable Region

(R_1, R_2) achievable, if for some $P_U P_{X|U} P_{Y_1 Y_2|X} P_{\hat{Y}_1|UY_1}$:

$$
\begin{aligned}
R_1 &\leq I(U;Y_1)\\
R_1 &\leq I(X;Y_2) + I(X;\hat{Y}_1|UY_2) - I(\hat{Y};Y_1|UY_2)\\
R_2 &\leq I(X;Y_2\hat{Y}_1|U)\\
R_1 + R_2 &\leq I(X;Y_2) + I(XY_2;\hat{Y}_1|U) + I(X;\hat{Y}_1|UY_2) - I(\hat{Y}_1;Y_1|UY_2)
\end{aligned}
$$

and $\quad I(\hat{Y}_1;Y_1|UY_2) \leq R_{\text{FB}}.$

Noisy Feedback Channel (instead of a Feedback Pipe)

- Feedback in block b: use a capacity $R_{\textsf{fb}}$ achieving code to send $\tilde{M}_{\textsf{fb},b-1}$

- Error probability for feedback code in a block $\rightarrow \epsilon_{\textsf{fb}}$
- Probability of feedback error in *any of the* blocks: $\epsilon_{\textsf{fb}} B$
- Overall probability of error at most $(\epsilon + \epsilon_{\textsf{fb}})B$

Our rates remain achievable for noisy feedback channels of capacity $R_{\textsf{fb}}$

Coding Scheme for Two-Sided Feedback

- Marton-coding in each block
- Cloud center $U_0^N(M_{1,c}, M_{2,c}, \tilde{M}_{\mathsf{fb},1}, \tilde{M}_{\mathsf{fb},2})$
- Satellites $U_1^N(M_{1,p}, M_1')$ and $U_2^N(M_{2,p}, M_2')$
- $\tilde{M}_{\mathsf{fb},2}$ quantizes $\hat{Y}_1^N$ given SI U_0^N and Y_2^N

- Rx 1 knows $\tilde{M}_{\mathsf{fb},2}$ and decodes $\tilde{M}_{1,c}, M_{2,c}, \tilde{M}_{\mathsf{fb},1}$
- Rx 1 reconstructs $\hat{Y}_2^N$ and decodes $M_{1,p}$ based on $(Y_1^N, \hat{Y}_2^N)$

Achievable Region for Two-Sided Feedback

Achievable Region for Two-Sided Feedback

(R_1, R_2) achievable, if for some
$P_Q P_{U_0 U_1 U_2|Q} P_{Y_1 Y_2|X} P_{\hat{Y}_1|U_0 Y_1 Q} P_{\hat{Y}_2|U_0 Y_2 Q}$ and $f: \mathcal{U}_0 \times \mathcal{U}_1 \times \mathcal{U}_2 \to \mathcal{X}$:

$$
\begin{aligned}
R_1 &\leq \Gamma + I(U_1; Y_1 \hat{Y}_2 | U_0 Q) \\
R_2 &\leq \Gamma + I(U_2; Y_2 \hat{Y}_1 | U_0 Q) \\
R_1 + R_2 &\leq \Gamma + I(U_1; Y_1 \hat{Y}_2 | U_0 Q) + I(U_2; Y_2 \hat{Y}_1 | U_0 Q) - I(U_1; U_2 | U_0 Q)
\end{aligned}
$$

and $\max\{I(\hat{Y}_1; Y_1 | U Y_2 Q), I(\hat{Y}_2; Y_2 | U Y_2 Q)\} \leq R_{\text{fb}}$

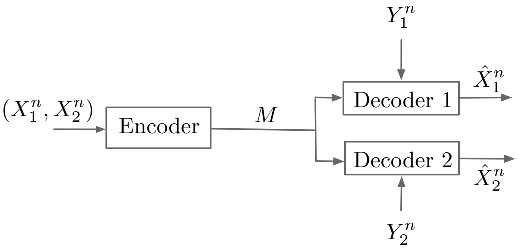
where
$\Gamma := \min\{I(U_0; Y_1|Q) - I(\hat{Y}_2; Y_2 | U_0 Y_1 Q), I(U_0; Y_2|Q) - I(\hat{Y}_1; Y_1 | U_0 Y_2 Q)\}$

Outlook: Combine our Scheme with Shayevitz-W. Scheme

How we got the Idea:

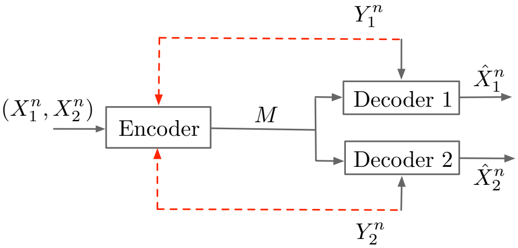
A related source coding problem

The Lossless Kaspi/Heegard-Berger Problem



- $(X_1^n, X_2^n, Y_1^n, Y_2^n)$ IID $\sim P_{X_1 X_2 Y_1 Y_2}$
- Compression index $M = f^{(n)}(X_1^n, X_2^n)$ takes value in $\{1, \ldots, \lfloor 2^{nR} \rfloor\}$
- Lossless reconstructions: $\Pr\left[(\hat{X}_1^n, \hat{X}_2^n) \neq (X_1^n, X_2^n)\right] \to 0 \quad$ as $n \to \infty$
- Minimum description rate $R^{\star}_{\text{ign}}$

Lossless Kaspi/Heegard-Berger with Encoder-SI (Laich&W'13)



- $(X_1^n, X_2^n, Y_1^n, Y_2^n)$ IID $\sim P_{X_1 X_2 Y_1 Y_2}$
- Compression index $M = f_{\text{EncSI}}^{(n)}(X_1^n, X_2^n, Y_1^n, Y_2^n)$ in $\{1, \ldots, \lfloor 2^{nR} \rfloor\}$
- Lossless reconstructions: $\Pr\left[(\hat{X}_1^n, \hat{X}_2^n) \neq (X_1^n, X_2^n)\right] \to 0$ as $n \to \infty$
- Minimum description rate $R^\star_{\text{cogn}}$

One Way to Exploit Encoder-SI

- $H(X_2|Y_2) \geq H(X_2Y_2|Y_1) \geq H(X_2|Y_1)$
- $(Y_1 \succeq Y_2|X_2)$: $\quad I(U;Y_1|X_2) \geq I(U;Y_2|X_2) \quad \forall U \to X_1, X_2 \to Y_1, Y_2$

Without Encoder-SI [Timo-Oechtering-W'12]

1. Describe X_2^n to both decoders
2. Describe X_1^n to Decoder 1 which knows X_2^n, Y_1^n

$$R^\star_{\text{ign}} = H(X_2|Y_2) + H(X_1|X_2Y_1)$$

One Way to Exploit Encoder-SI

- $H(X_2|Y_2) \geq H(X_2 Y_2|Y_1) \geq H(X_2|Y_1)$
- $(Y_1 \succeq Y_2|X_2)$: $\quad I(U;Y_1|X_2) \geq I(U;Y_2|X_2) \quad \forall U \to X_1, X_2 \to Y_1, Y_2$

Scheme With Encoder-SI

1. Describe (X_2^n, Y_2^n) to both decoders
2. Describe X_1^n to Dec. 1 which knows X_2^n, Y_1^n, Y_2^n

$$R^\star_{\text{cogn}} = H(X_2|Y_2) + H(X_1|X_2 Y_1 Y_2) > R^\star_{\text{ign}}$$

One Way to Exploit Encoder-SI

- $H(X_2|Y_2) \geq H(X_2 Y_2|Y_1) \geq H(X_2|Y_1)$
- $(Y_1 \succeq Y_2|X_2)$: $\quad I(U;Y_1|X_2) \geq I(U;Y_2|X_2) \quad \forall U \to X_1, X_2 \to Y_1, Y_2$

Scheme With Encoder-SI

1. Describe (X_2^n, Y_2^n) to both decoders
2. Describe X_1^n to Dec. 1 which knows X_2^n, Y_1^n, Y_2^n

$$R^\star_{\mathsf{cogn}} = H(X_2|Y_2) + H(X_1|X_2 Y_1 Y_2) > R^\star_{\mathsf{ign}}$$

$\to$ Sending Y_2^n purely beneficial: helps Dec 2 without additional rate

Ex. of Sources and SI where Encoder-SI strictly helps

- Source $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim \mathsf{DSBS}(p)$
- $E \sim \mathbb{B}(1/3)$ independent of source
- SI $Y_1 = \begin{cases} X_1, X_2 & \text{if } E = 1 \\ ? & \text{if } E = 0 \end{cases}$ and $Y_2 = \begin{cases} ? & \text{if } E = 1 \\ X_1, X_2 & \text{if } E = 0 \end{cases}$

$$R^\star_{\mathsf{ign}} \;=\; 2/3 + 1/3 H_b(p) > R^\star_{\mathsf{cogn}} = 2/3$$

Summary/Outlook

- New coding schemes for DMBC with rate-limited or noisy feedback

Feedback allows to identify information useful for Rx 2, known at Rx 1

- Feedback increases entire capacity for strictly less-noisy BCs
- Scheme should be combined with previous schemes
- Inspired by lossless Heegard-Berger scheme showing: SI at encoder helps!