Graphs for image processing, analysis and pattern recognition

Florence Tupin

Overview

1. Definitions and representation models

2. Single graph methods
   - Segmentation or labeling and graph-cuts
   - Graphs for pattern recognition

3. Graph matching
   - Graph or subgraph isomorphisms
   - Error tolerant graph-matching
   - Approximate algorithms (*inexact matching*)

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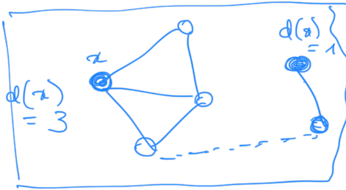
Why using graphs ?

- Interest: they give a compact, structured and complete representation, easy to handle
- Applications:
  - Image processing: segmentation, boundary detection
  - Pattern recognition: printed characters, objects (buildings 2D ou 3D, brain structures, ...), faces, ...
  - Image registration
  - Understanding of structured scenes
  - ...

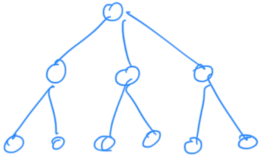
Definitions

Graph : $G = (X, E)$



- X set of nodes ($|X|$ order of the graph)
- E set of edges ($|E|$ size of the graph)

- complete graph (size $\frac{n(n-1)}{2}$)
- partial graph $G = (X, E')$ with E' part of E
- subgraph $F = (Y, E')$, $Y \subseteq X$ et $E' \subseteq E$
- degree of a node x : $d(x)$ = number of edges
- connected graph: for each pair of nodes you find a path linking them
- tree: connected graph without cycle
- clique: complete subgraph
- dual graph (face $\rightarrow$ node)
- segment graph (edge $\rightarrow$ node)
- hypergraph (n-ary relations)
- weighted graphs: weights on the edges



Notations

Graph : $G = (X, E)$

$$\begin{array}{c|cccc} & 0 & 1 & \cdots & n \\ \hline 0 & w_{00} & w_{01} & & w_{0n} \\ \vdots & & & & \\ n & & & & \end{array}$$

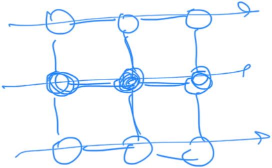
- weight of an edge linking i et j : w_{ij}
- adjacency matrix W of size $|X| \times |X|$ defined by

$$W_{ij} = \begin{cases} w_{ij} & \text{if} \quad e_{ij} \in E \\ 0 & \text{else} \end{cases}$$

$$\begin{array}{c|cccc} & 0 & \cdots & & n \\ \hline 0 & d(0) & & & \\ \vdots & & d(1) & & \\ & & & \ddots & \\ n & & & & d(n) \end{array}$$

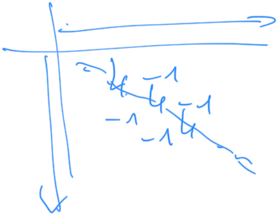
for undirected edges W is symetric

- Laplacian matrix of an undirected graph
  $d_i = \sum_{e_{ij} \in E} w_{ij}$



$$L_{ij} = \begin{cases} d_i & \text{if} \quad i = j \\ -w_{ij} & \text{if} \quad e_{ij} \in E \\ 0 & \text{else} \end{cases}$$

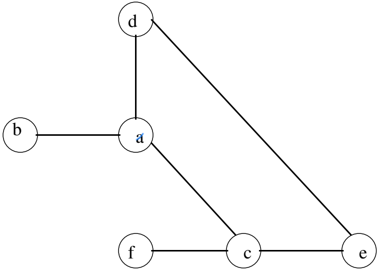
$$\boxed{L = D - W}$$



with $D_{ii} = d_i$ (D degree matrix)

Representation

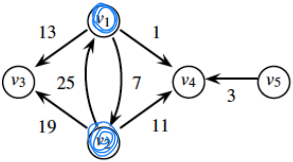
Adjacency matrix, adjacency lists

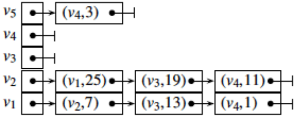


	a	b	c	d	e	f
a	0	1	1	1	0	0
b	1	0	0	0	0	0
c	1	0	0	0	1	1
d	1	0	0	0	1	0
e	0	0	1	1	0	0
f	0	0	1	0	0	0

Representation

Adjacency matrix, adjacency lists





	v_1	v_2	v_3	v_4	v_5
v_1	0	7	13	1	0
v_2	25	0	19	11	0
v_3	0	0	0	0	0
v_4	0	0	0	0	0
v_5	0	0	0	3	0

	e_{12}	e_{13}	e_{14}	e_{21}	e_{23}	e_{24}	e_{54}
v_1	+1	+1	+1	-1	0	0	0
v_2	-1	0	0	+1	+1	+1	0
v_3	0	-1	0	0	-1	0	0
v_4	0	0	-1	0	0	-1	-1
v_5	0	0	0	0	0	0	+1

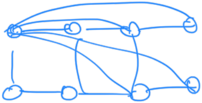
FIGURE 1.4
From top-left to bottom-right: a weighted directed graph, its adjacency list, its adjacency matrix, and its (transposed) incidence matrix representations.

(figure from "Image processing and analysis with graphs", Lezoray - Grady)

Which graphs for images ?

Graphe du pixels

- $\begin{cases} X = \text{ensbl. des pixels} \\ E = \text{4-conn ou 8-conn} \end{cases}$



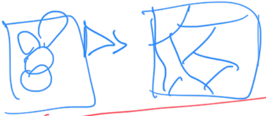
Attributs des noeuds ⟶ ng, RVB,

Attributs des arcs ⟶ $\begin{cases} w_{ij} = |ng_i - ng_j| \\ a_{ij} = \| P_i - P_j \| \end{cases}$

↳ radiométriques / colorimétriques

↳ géométriques (distances, relations spatiales...)

- Graphes des primitives:



Graphes des régions : ⟶ watershed, superpixels, K-means

noeuds ⟷ régions

arcs ⟶ régions adjacentes

RAG (GAR)

- moyenne des ng/couleurs
- μ, Σ
- histogramme?
- Textures

- taille
- position du centre de gravité
- moments géométriques

- w_{ij} ⟶ longueur de la frontière entre la région i à la région j

- gradient moyen sur ∂_{ij}
- $\| \mu_i - \mu_j \|$ . . . .

Examples of graphs

- Attributed graph : $G = (X, E, \mu, \nu)$
  - $\mu : X \to L_X$ nodes interpreter (L_X = attributes of nodes)
  - $\nu : E \to L_E$ edges interpreter (L_E = attributes of edges)

  Exemples :
  - graph of pixels
  - region adjacency graph (RAG)
  - Voronogions / Delaunay triangulation
  - graph of primitives with complex relationships
- Random graph : edges and nodes = random variables
- Fuzzy graph : $G = (X, E = X \times X, \mu_f, \nu_f)$
  - $\mu_f : X \to [0, 1]$
  - $\nu_f : E \to [0, 1]$
  - avec $\forall (u, v) \in X \times X \quad \nu_f(u, v) \leq \mu_f(u)\mu_f(v)$ or $\nu_f(u, v) \leq \min[\mu_f(u)\mu_f(v)]$

Examples of image graphs

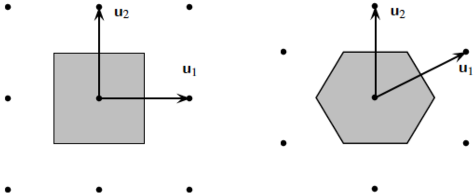
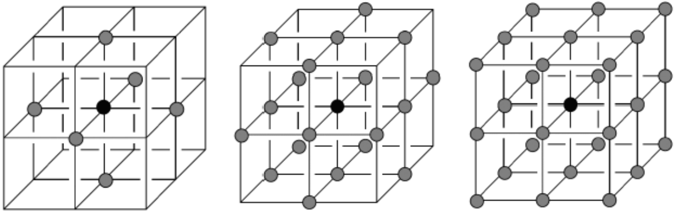


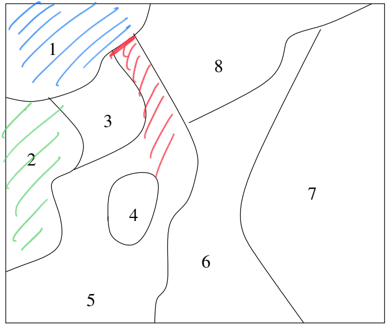
FIGURE 1.11
The rectangular (left) and hexagonal (right) lattices and their associated Voronoi cells.

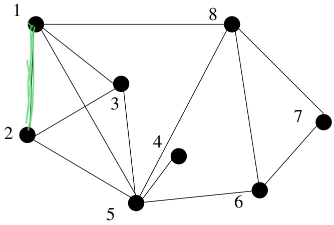


(a) 6-adjacency. (b) 18-adjacency. (c) 26-adjacency.

FIGURE 1.12

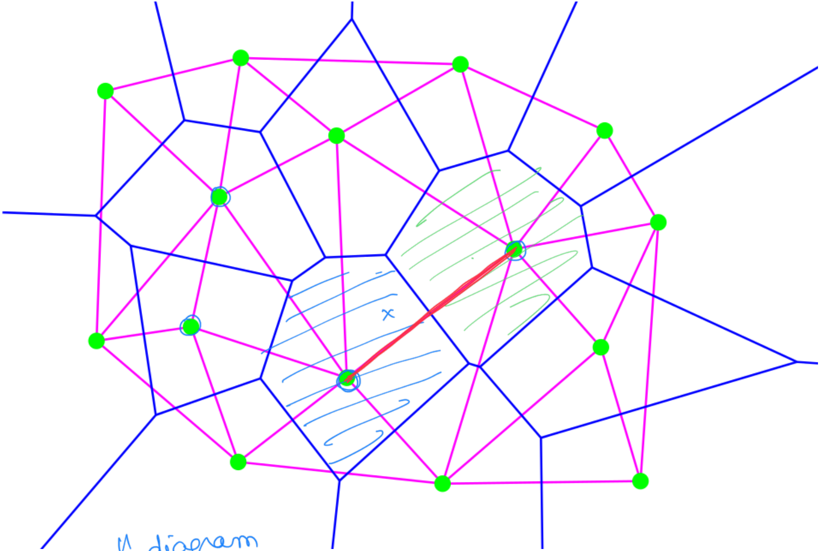
Examples of image graphs





RAG (Region Adjacency Graph)

Examples of image graphs



i diagram

Voronoagram (in blue) and Delaunay triangulation (pink)

Examples of image graphs

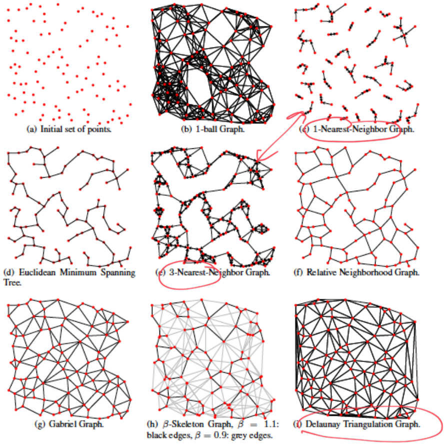


FIGURE 1.14
Examples of proximity graphs from a set of 100 points in $\mathbb{Z}^2$.

(figure from "Image processing and analysis with graphs", Lezoray - Grady)

Examples of graphs

- Graph of fuzzy attributes : attributed graph with fuzzy value for each attribute
- Hierarchical graph :
  multi-level graph and and bi-partite graph between 2 levels (multi-level approaches, object grouping, ...)

  Exemples :
  - quadtrees, octrees
  - hierarchical representation of the brain
- Graph for reasoning
  decision tree, matching graph

Graph example:

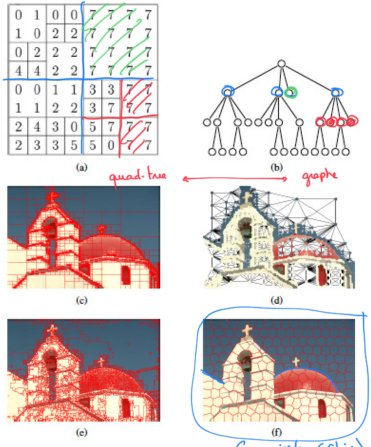


FIGURE 1.13
(a) An image with the quadtree tessellation, (b) the associated partition tree, (c) a real image with the quadtree tessellation, (d) the region adjacency graph associated to the quadtree partition, (e) and (f) two different irregular tessellations of an image using image-dependent superpixel segmentation methods: Watershed [23] and SLIC superpixels [24].

(figure from "Image processing and analysis with graphs", Lezoray - Grady)

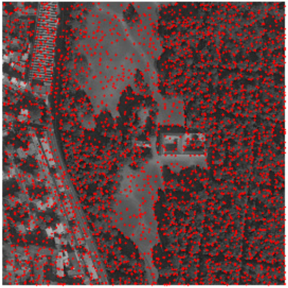
Graph examples

Figure 2 – Représentation de variété des points clés de $\mathcal{S}_{\omega}^{\max}(I)$ (en rouge) et $\mathcal{S}_{\omega}^{\min}(I)$ (en bleu
image Pléiades ayant des textures locales différentes.

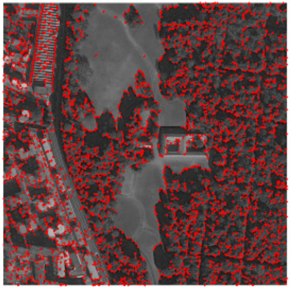
(figure from M.T. Pham PhD)

Graph examples

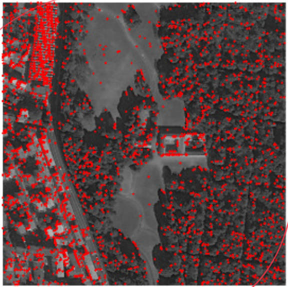
(a) Image initiale 512 × 512



(b) Extrema locaux

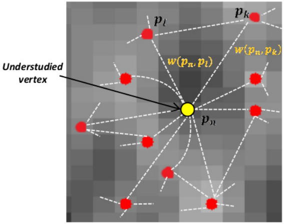


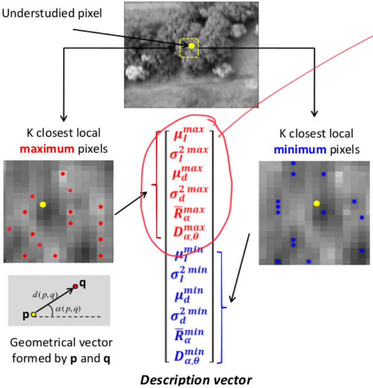
(c) Détecteur de Harris



(d) Détecteur SIFT

Graph examples





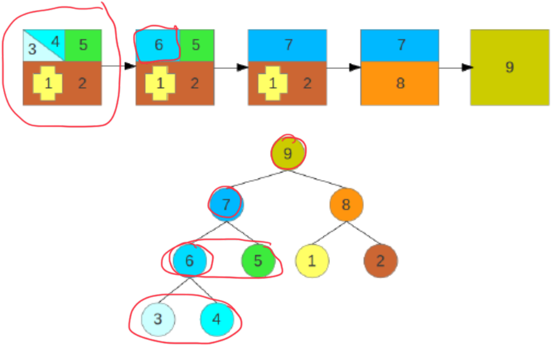
median/

[illegible]

Figure 5 – Vecteur de description proposé pour l'analyse ponctuelle de la texture.

(figure from M.T. Pham PhD)

Graph examples - BPT Binary Partition Tree



Which algorithms from graph theory ?

- Plus court chemin
- Min Cut / Max Flow
- Min Spanning Tree

Some classical algorithms

Search of the minimum spanning tree

- Kruskal algorithm $O(n^2 + mlog_2(m))$
- Prim algorithm $O(n^2)$

Shortest path problems

- positive weights: Dijkstra algorithm $O(n^2)$
- arbitrary weights but without cycle: Bellman algorithm $O(n^2)$

Max flow and Min cut

- $G = (X, E)$
- partitioning in two sets A et B ($A \cup B = X$, $A \cap B = \emptyset$)
- $cut(A, B) = \sum_{x \in A, y \in B} w(x, y)$
- Ford and Fulkerson algorithm

Search of maximal clique in a graph

- decision tree
- cut of already explored branches

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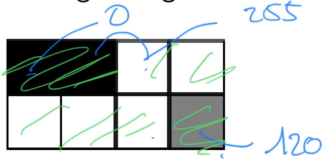
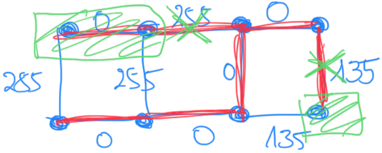
Segmentation by minimum spanning tree

$w_{ij} = |img_i - img_j|$

How can we segment this image using a minimum spanning tree ?





Segmentation by minimum spanning tree

Constantinides (1986)

- graph of pixels weighted by the gray levels (or colors) (weights = distances)
- search of the minimum spanning tree
- spanning tree $\Rightarrow$ partitioning by suppressing the most costly edges

image — graphe des pixels attribué — arbre couvrant de poids minimal — suppression des arêtes les plus coûteuses

Computation of the minimum spanning tree

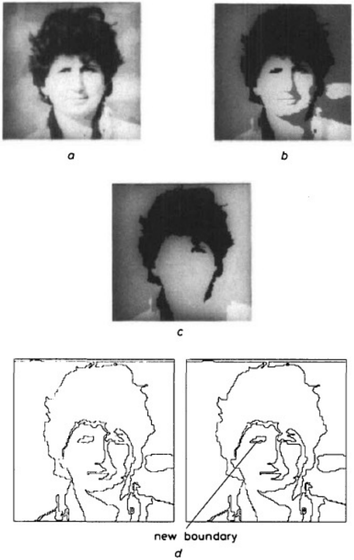
Kruskal algorithm

- Starting from a partial graph without any edge, iterate $(n-1)$ times : choose the edge of minimum weight creating no cycle in the graph with the previsouly chosen edges
- In practice:
  1. sorting of edges by increasing weights
  2. while the number of edges is less than $(n-1)$ do:
     - select the first edge not already examined
     - if cycle, reject
     - else, add the edge in the graph
- Complexity: $O(n^2 + mlog_2(m))$

Prim algorithm

- Extension from near to near of the current tree
- Complexity: $O(n^2)$

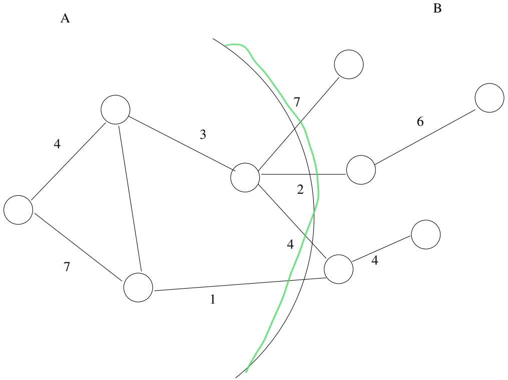
Constantinides (1986)



Segmentation by graph-cut

Graph-cut definition:

- graph $G = (X, E)$
- partitioning in 2 parts A et B ($A \cup B = X$, $A \cap B = \emptyset$)
- $cut(A, B) = \sum_{i \in A, j \in B} w_{ij}$



Segmentation by graph clustering

Clustering : partitioning of the graph in groups of nodes based on their similarities [Each cluster : a closely connected component]

The clustering corresponds to:

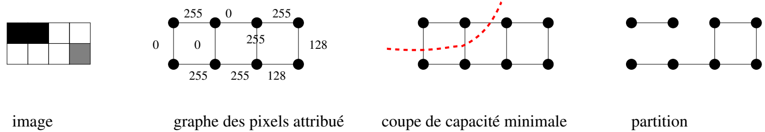
- edges between different groups have low weights (weak similarities)
- edges inside a group have high weights (high similarities)

Possible cost functions for the cut:

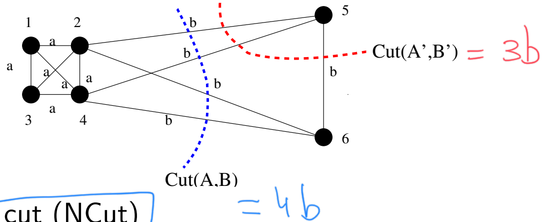
- minimum cut $Cut(A_1, ..., A_k) = \sum_{i=1}^{i=k} Cut(A_i, \overline{A_i})$
- minimum cut normalized by the size of each part (RatioCut) $RatioCut(A_1, ..., A_k) = \sum_{i=1}^{i=k} \frac{1}{|A_i|} Cut(A_i, \overline{A_i})$ ($|A_i|$ number of vertices in A_i)
- minimum cut normalized by the connectivity of each part (NCut) $NCut(A_1, ..., A_k) = \sum_{i=1}^{i=k} \frac{1}{vol(A_i)} Cut(A_i, \overline{A_i})$ ($vol(A_i) = \sum_{k \in A_i} d_k$ sum of the weight of all edges of vertices in A_i)

Toy example

Wu and Leavy (93): search for the MinCut



Influence of the number of edges: $Cut(A, B) = 4b$, $Cut(A', B') = 3b$



$\Rightarrow$ normalized cut (NCut)

Normalized cut

- Principle: graph clustering
- $+$ suppression of the influence of the number of edges: normalized cut

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,X)} + \frac{cut(A,B)}{assoc(B,X)}$$

$$assoc(A,X) = \sum_{a \in A, x \in X} w(a,x) = vol(A)$$

- Measuring the connectivity of a cluster:

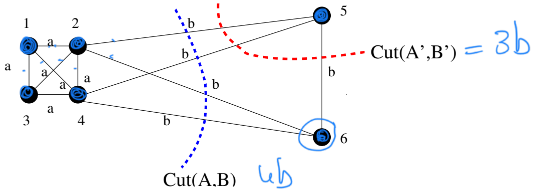
$$Nassoc(A,B) = \frac{assoc(A,A)}{assoc(A,X)} + \frac{assoc(B,B)}{assoc(B,X)}$$

$$Ncut(A,B) = 2 - Nassoc(A,B)$$

$$\boxed{\text{minimizing the cut} \Leftrightarrow \text{maximizing group connectivity}}$$

Toy example

Influence of the number of edges: $Cut(A,B) = 4b$, $Cut(A', B') = 3b$



$\Rightarrow$ normalized cut (NCut)

$vol(A) = 3a + 3a + 2b + 3a + 3a + 2b = 12a + 4b$

$vol(B) = 3b + 3b = 6b.$

$NCut(A,B) = \dfrac{4b}{12a+4b} + \dfrac{4b}{6b} = \dfrac{4b}{12a+4b} + \dfrac{2}{3}$

$vol(A') = 12a + 4b + 3b = 12a + 7b$

$vol(B') = 3b$

$NCut(A', B') = \dfrac{3b}{12a+7b} + \dfrac{3b}{3b} = \dfrac{3b}{12a+7b} + 1$

Graph theory and cuts

MinCut by combinatorial optimization

- Stoer-Wagner algorithm
- Principle: iterative reducing of the graph by fusion of the nodes linked by the maximal weights

Min K-cut by combinatorial optimization

- Partitioning the (un-oriented graph) graph in many components
- Gomory-Hu algorithm

minCut in oriented graph by combinatorial optimization

- Ford-Fulkerson algorithm (oriented graph with two terminal nodes (sink / tank)
- Principle: MaxFlow search (MinCut equivalence) by search for an augmenting chain to increase the flow

Graph theory and cuts

Laplacian matrices

$D = diag(d_i)$ with $d_i = \sum_j w_{ij}$

$W = (w_{ij})$

- Graph Laplacian matrix

  $\boxed{L = D - W}$

clustering en k classes

- Normalized graph Laplacian matrix

  $L_n = D^{-\frac{1}{2}} L D^{-\frac{1}{2}} = I - D^{-\frac{1}{2}} W D^{-\frac{1}{2}}$

Spectral clustering algorithms and cuts

- Computation of the eigen-values and eigen-vectors of some matrix (L, L_n, or generalized eigen problems $Lu = \lambda Du$)
- selection of the k smallest eigen-values and associated k eigen-vectors u_k
- $\boxed{U = (u_1, \ldots, u_k)} \in R^{n \times k}$

  $n \left(u_1 \; \ldots\ldots \; u_k \right)$

  k means sur les n vecteurs
- let $y_i \in R^k$ be the ith row of U ($i = 1, \ldots, n$)
- cluster the points $(y_i)_{1 \leq i \leq n}$ with the k-means algorithm into clusters $C_1, \ldots, C_k$
- clusters $A_1, \ldots, A_k$ with $A_i = \{j | y_j \in C_i\}$

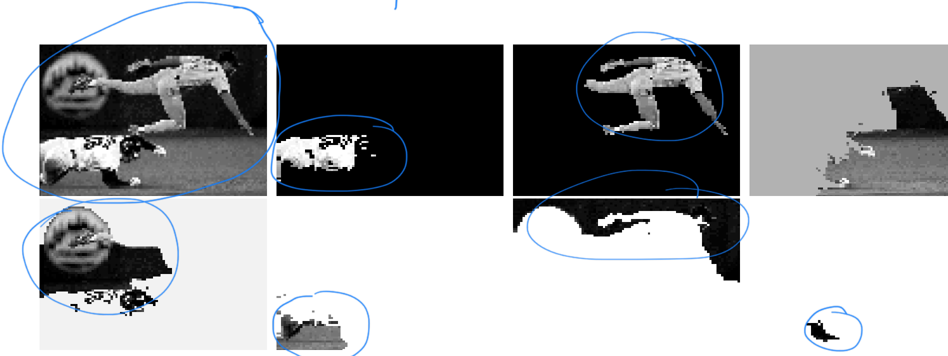
Examples (univ. Berkeley)

noeud i (pixel i) $\to P_i, C_i$

noeud j (pixel j) $\to P_j, C_j$

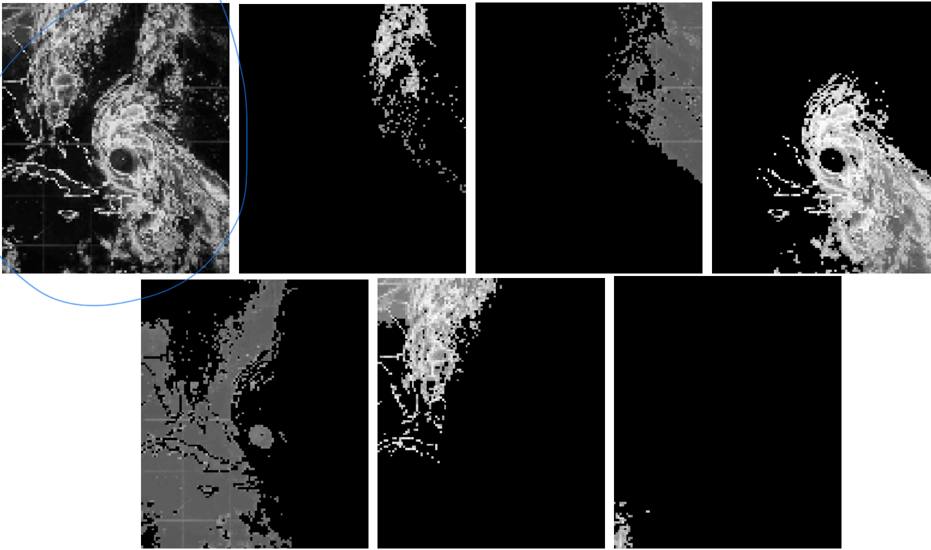
pixels similaires $\iff$ poids arc fort

$$w_{ij} = k\, e^{-\frac{dist(P_i, P_j)^2}{2\sigma^2}}\, e^{-\frac{\|C_i - C_j\|_2^2}{2\sigma'^2}}$$



http://www.cs.berkeley.edu/projects/vision/Grouping/

Examples (univ. Berkeley)



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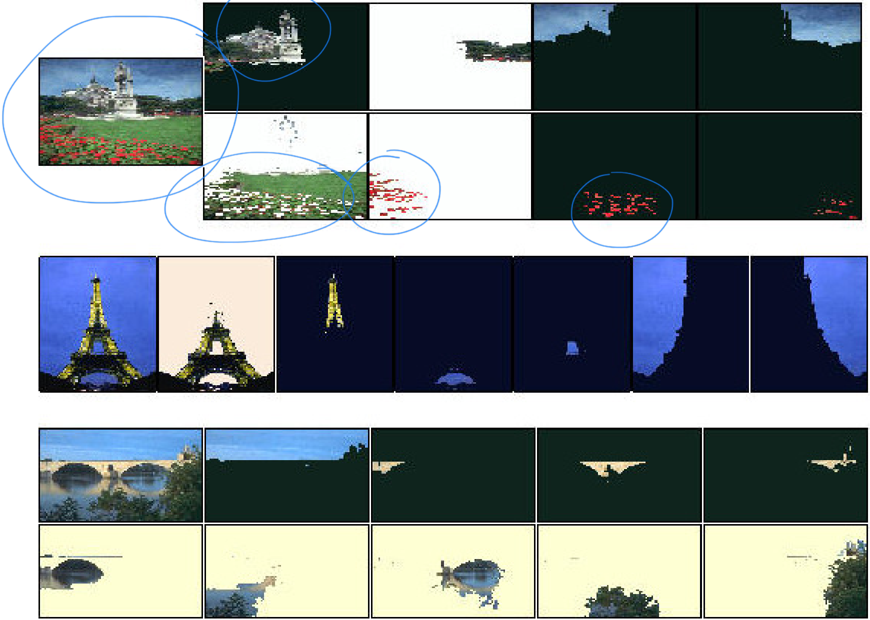
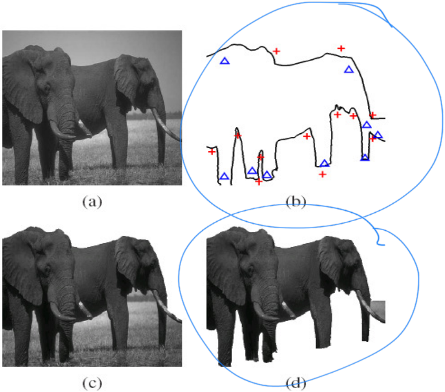
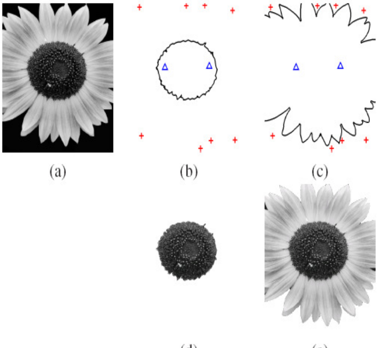


http://www.cs.berkeley.edu/projects/vision/Grouping/

Examples (univ. Alberta) with linear constraints



(a) (b) (c) (d)

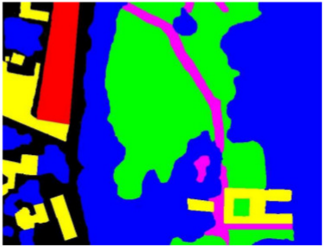


(a) (b) (c) (d) (e)

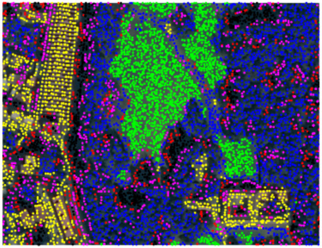
Examples (Mean Shift et Normalized Cut)

Examples (texture classification with point-wise graph)

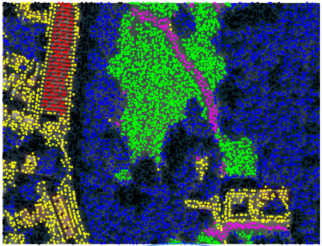
(a) Image originale



(b) Vérité terrain



(j) Vecteur de signature



(k) Classification spectrale des vecteurs de signature

Graph-cuts

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- *Efficient graph-based image segmentation*, Felzenszwalb, Huttenlocher, IJCV, 2004
- *A tutorial on spectral clustering*, U. von Luxburg, Statistics and Computing, 2007
- *Color Image segmentation Based on Mean-Shift and Normalized Cuts*, Tao, Zhang, IEEE Trans. on Systems, Man and Cybernetics, 2007
- *Pointwise approach for texture analysis and characterization*

Overview

1. Definitions and representation models

2. Single graph methods
   - Segmentation or labeling and graph-cuts
   - Graphs for pattern recognition

3. Graph matching
   - Graph or subgraph isomorphisms
   - Error tolerant graph-matching
   - Approximate algorithms (*inexact matching*)

Full scene labeling (scene parsing)

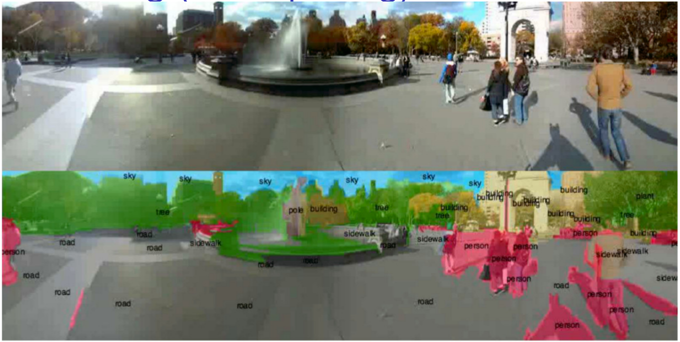


Figure from Farabet et al., PAMI

Tenenbaum and Barrow (1977)

- Segmentation in regions
- Building of the Region Adjacency Graph
- Labeling using a set of rules (expert system) :
  1. on objects (size, color, texture,...)
  2. on contextual relationships between objects (above, inside, near ...)

Generalization with fuzzy attributed graphs

Markovian labeling (random graphs)

i j

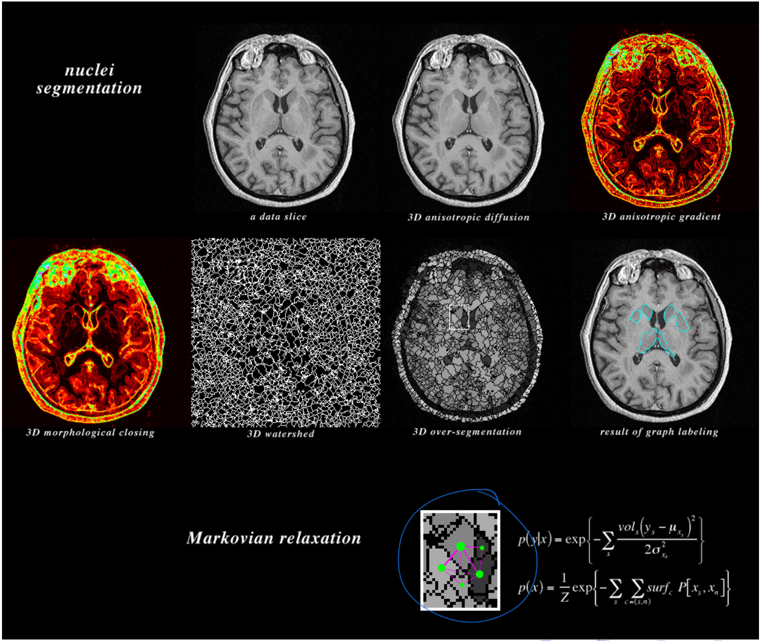
$$E(l) = \sum_i \Phi(d_i, l_i) + \beta \sum_{ij} \Psi(l_i, l_j)$$

d

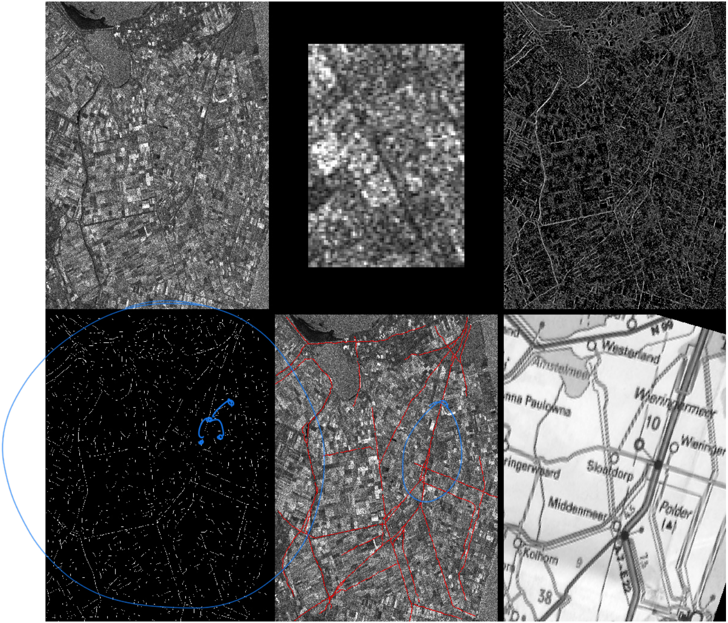
- Low-level applications:
  - pixel graphs
  - segmentation, classification, restoration
- High-level applications:
  - graph of super-pixels (SLIC, watershed, ...)
  - graph of primitives (edges, key-points, lines,...)
- CRF (Conditional Random Field) / MRF (Markov Random Field):
  - MRF: Ψ does not depend on d ("pure" prior)
  - CRF: Ψ depends on d (usually based on image gradient values)

$\Rightarrow$ pattern recognition, full scene labeling

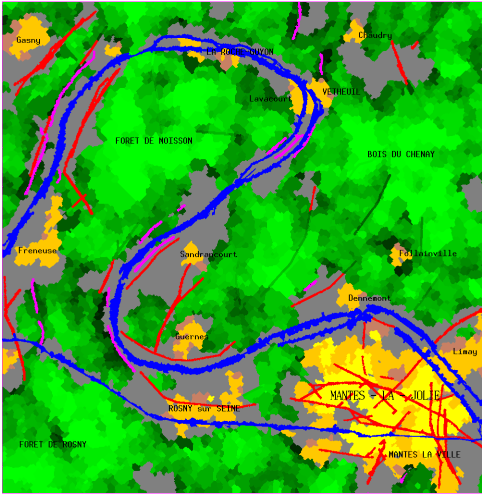
Example on a 3D RAG (T. Gud)



Example on a line graph



Example on a region adjacency graph



MRF and graph-cut optimization

Binary labeling (Greig et al. 89) :

$$\mathcal{E}(l) = \sum_i \Phi(d_i|l_i) + \sum_{(i,j)} \beta(l_i - l_j)^2$$

- efficient way of finding the global minimum by min-cut search

MRF and graph-cut optimization

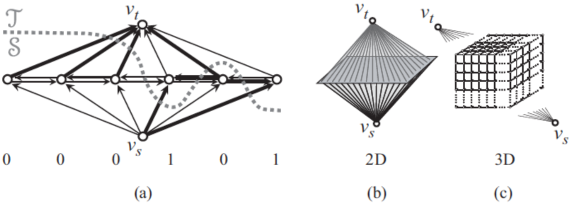


FIGURE 2.5
(a) The graph for binary MRF minimization. The edges in the cut are depicted as thick arrows. Each node other than v_s and v_t corresponds to a site. If a cut $(\mathcal{S}, \mathcal{T})$ places a node in $\mathcal{S}$, the corresponding site is labeled 0; if it is in $\mathcal{T}$, the site is labeled 1. The 0's and 1's at the bottom indicate the label each site is assigned. Here, the sites are arranged in 1D; but according to the neighborhood structure this can be any dimension as shown in (b) and (c).

(figure from "Image processing and analysis with graphs", Lezoray - Grady)

MRF/CRF and graph-cut optimization

Multi-level labeling (Boykov, Veksler) :

⇒ generalization of the binary labeling
Definition of two space moves (to go back to the binary labeling)

- α-expansion : source S and sink P correspond to label α and the current label $\overline{\alpha}$ (Ψ should be a metric)
- $\alpha - \beta$ swap: source S for α and sink P for β (Ψ should be a semi-metric)

Optimization by iterative mincut search:

- graph: nodes for super-pixels
- weights: depending on the current labeling
- good trade off time / efficiency compared to simulated annealing or ICM

But for multi-labeling no garantee on optimality of the solution

Interactive segmentation: "hard" constraints

Principle Background and object manually defined

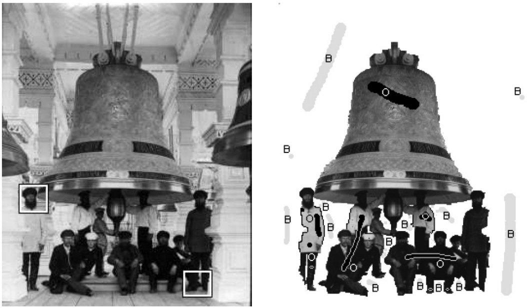
$\Rightarrow$ finding of a binary labeling minimizing an energy including "hard" constraints

Method Mincut search and edges with high weights (should not be cut)

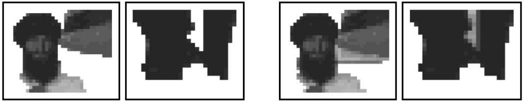
Advantages

- easy introduction of "hard" constraints
- the manually defined areas permit to do a fast learning
- iterative algorithm

Illustrations



(a) Original B&W photo (b) Segmentation results

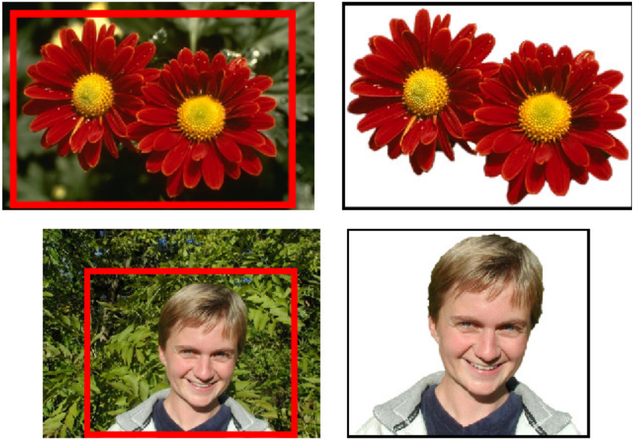


Interactive methods with mincut

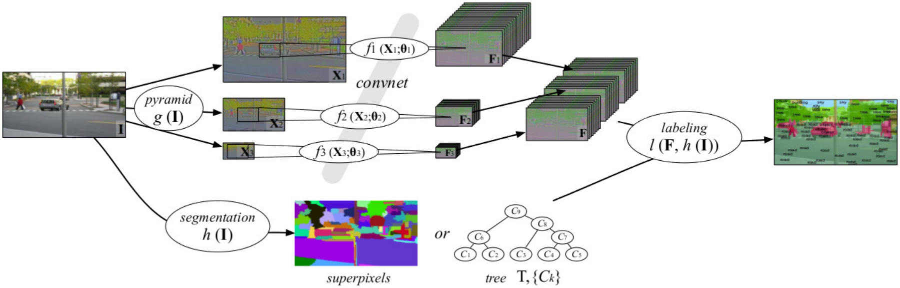
Grab-cut

- take into account color
- two labels (background and object but with a Gaussian Mixture Model)
- CRF (conditional random field): regularization term weighted by the image gradient
- iterative semi-supervised learning of the GMM parameters (after manual initialization and after each cut)

Illustrations -GrabCut-



Deep learning and graph labeling for full scene labeling

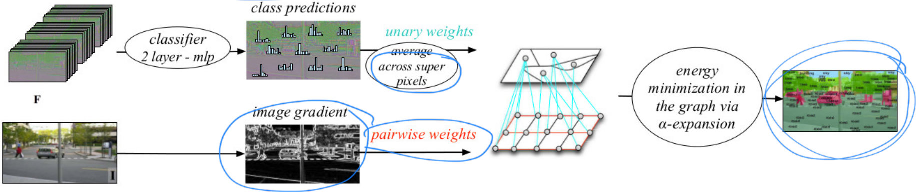


Deep learning and graph labeling for full scene labeling

appris par le CNN

$$\Phi(d_i, l_i) = \exp(-\alpha d_{i,a})1(l_i \neq a)$$

$$\Psi(l_i, l_j) = \exp(-\beta ||\nabla I||_i)1(l_i \neq l_j)$$



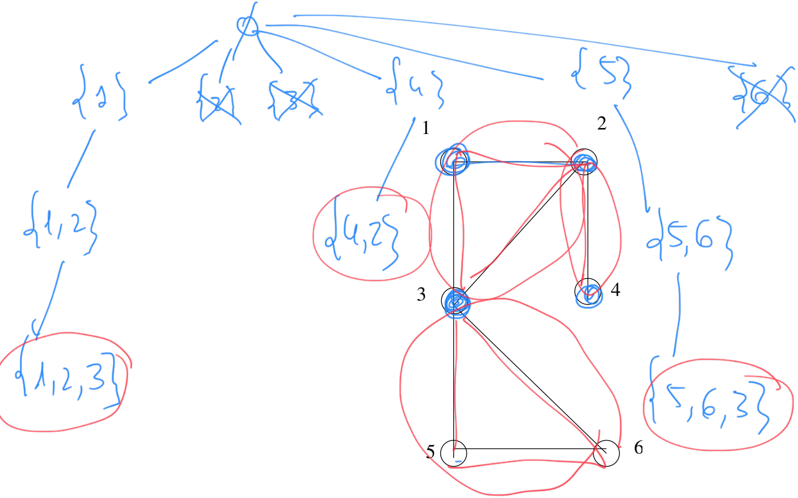
Pattern recognition

- Object: defined by a set of primitives (nodes of the graph)
- Binary relationship of compatibility between nodes (edges of the graph)
- Clique: sub-set of primitives all compatible between each other = possible object configuration
- recognition by maximal clique detection

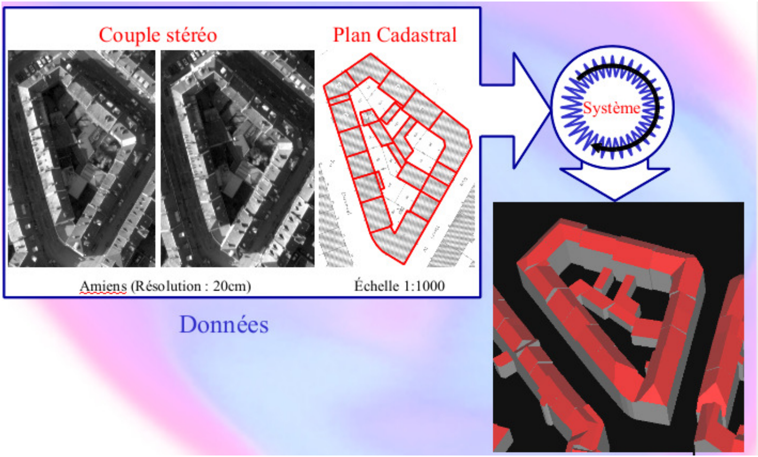
Search of maximal cliques :

- NP-hard problem
- Building of a decision tree: a node of the tree = 1 clique of the graph
- pruning of the tree to suppress already found cliques
- Theorem: let S be a node of the search tree T, and let x be the first unexplored child of S to be explored. If all the sub-trees of $S \cup \{x\}$ have been generated, only the sons S not adjacent to x have to be explored.

Example:maximal clique search



Example: buiding reconstruction by the maximal clique search (IGN)



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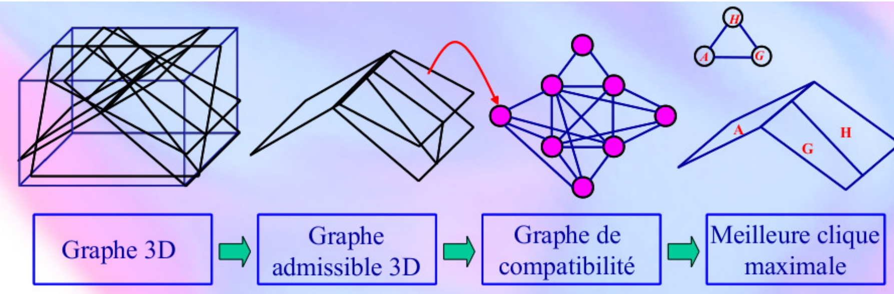

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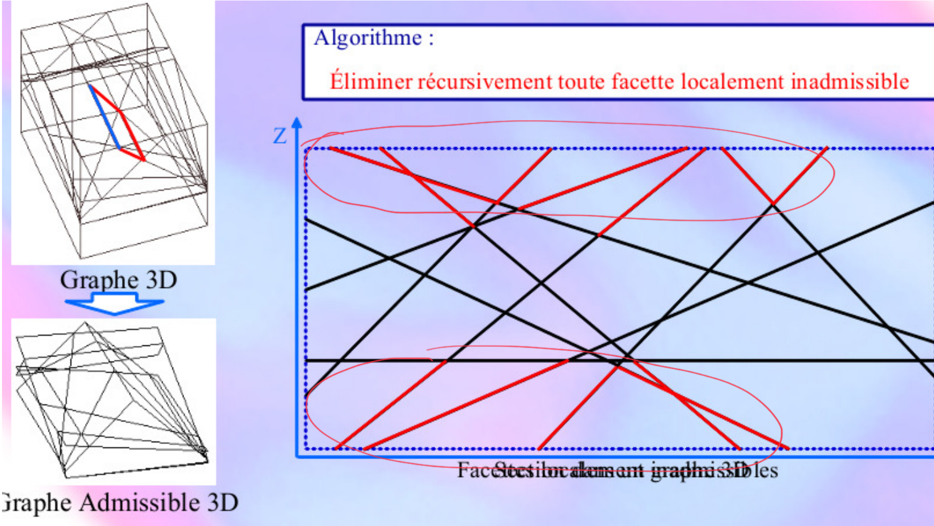
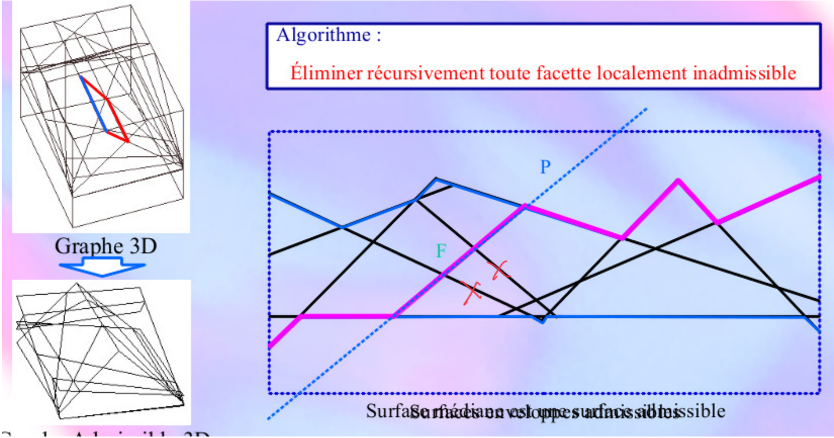


Example: buiding reconstruction by the maximal clique search (IGN)



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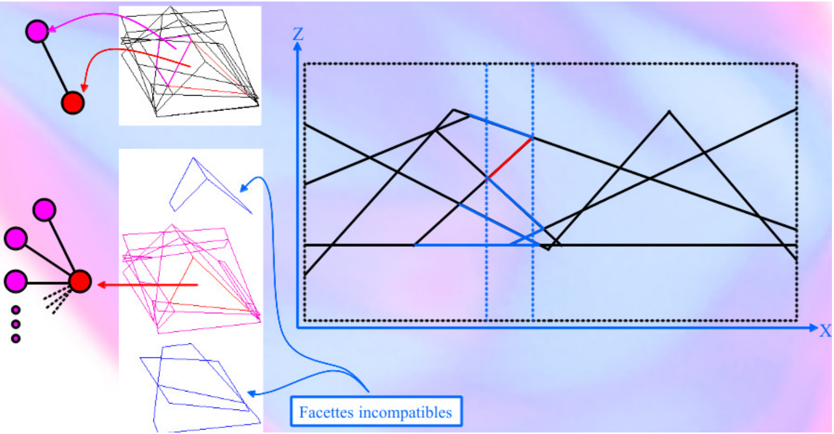

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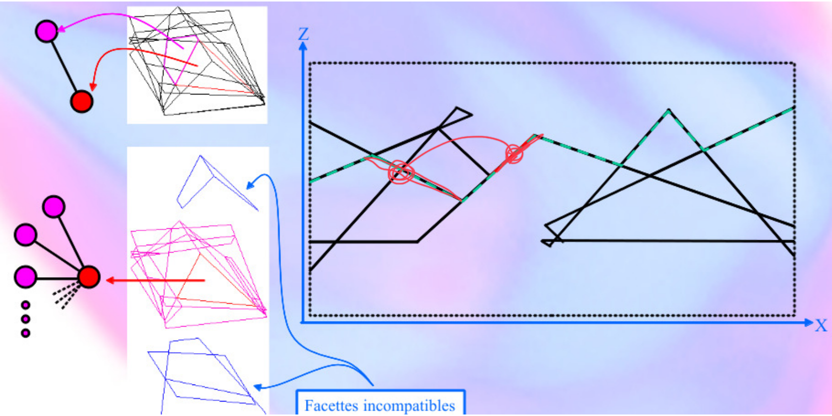

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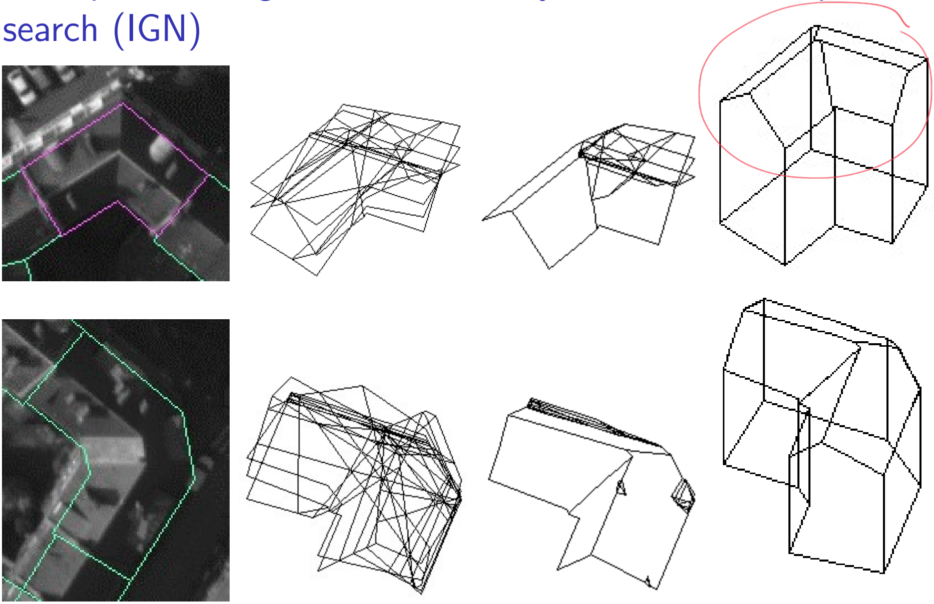


Overview

1. Definitions and representation models

2. Single graph methods
   - Segmentation or labeling and graph-cuts
   - Graphs for pattern recognition

3. Graph matching
   - Graph or subgraph isomorphisms
   - Error tolerant graph-matching
   - Approximate algorithms (*inexact matching*)

Graph matching

Correspondance problem:

- Graph(s) of the model (atlas, map, model of object)
- Graph built from the data ← bruité, ...
- Graph matching:

$$G = (X, E, \mu, \nu) \quad \to ? \quad G' = (X', E', \mu', \nu')$$

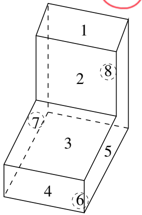
Graph isomorphism: bijective function $f \;:\; X \to X'$

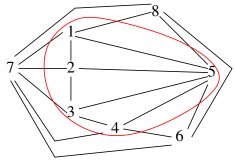
- $\mu(x) = \mu'(f(x))$
- $\forall e = (x_1, x_2),\ \exists e' = (f(x_1), f(x_2)) \;\;/\;\; \nu(e) = \nu'(e')$ and conversely

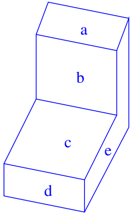
Too strict $\Rightarrow$ isomorphisms of sub-graphs

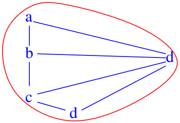
Sub-graph isomorphisms

- There exists a sub-graph S' of G' such that f is an isomorphism from G to S'







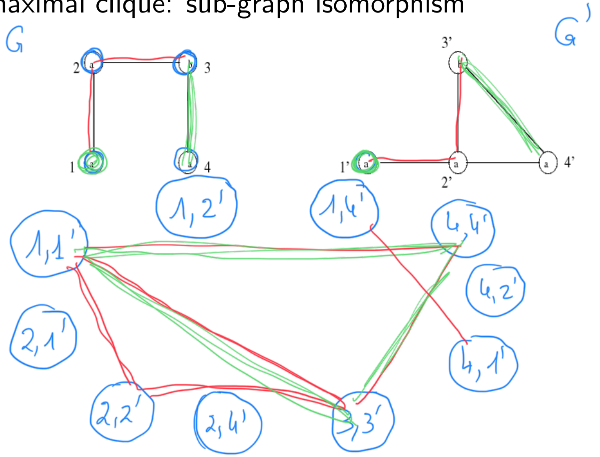


- There exists a sub-graph S of G and a sub-graph S' of G' such that f is an isomorphism from S to S'

Graph isomorphisms: searching the maximal clique

search of the maximal clique of the association graph

- principle: building of the association graph
- maximal clique: sub-graph isomorphism



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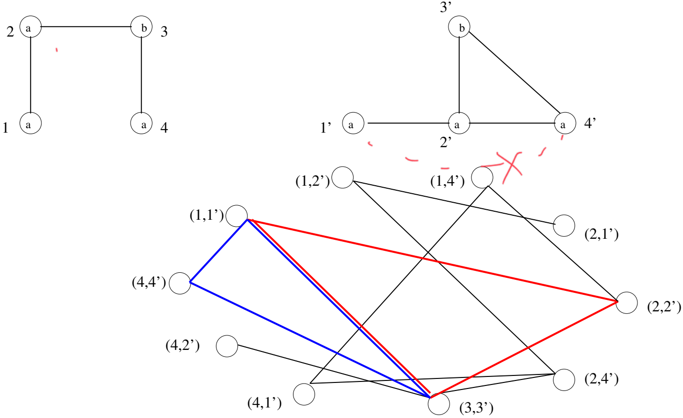

Sub-graph isomorphism: Ullman algorithm

- Principle : extension of the association set (v_i, w_{x_i}) until the G graph has been fully explored. In case of failure, go back in the association graph ("backtrack").
  Acceleration: "forward checking" before adding an association.
- Algorithm:
  - matrix of node associations
  - matrix of future possible associations for a given set of associations matrice
  - list of updated associations by "Backtrack" et "ForwardChecking"
- Complexity : worst case $O(m^n n^2)$ (n ordre de X, m de X', $n < m$)

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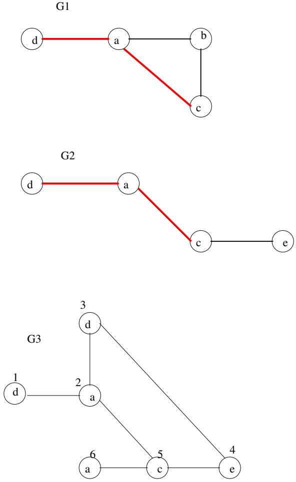
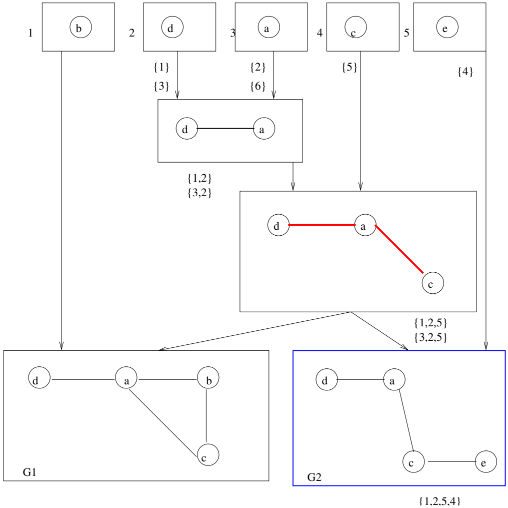
Error tolerant graph-matching

- Real world: noisy graphs, incomplete graphs, distorsions
- Distance between graphs (editing, cost function,...)
- Sub-graph isomorphism with error tolerance: search of the sub-graph G' with the minimum distance to G
- Optimal algorithms: A*
- Approximate matching: genetic algorithms, simulated annealing, neural networks, probablistic relaxation,...
  - iterative minimistion of an objective function
  - better adapted for big graphs
  - problem of convergence and local minima

Decomposition in common sub-graphs

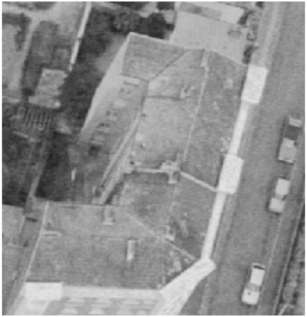
Messmer, Bunke

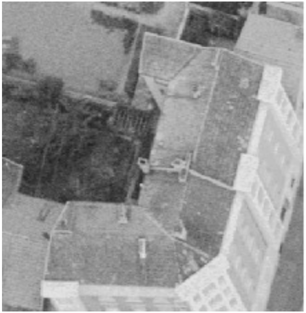


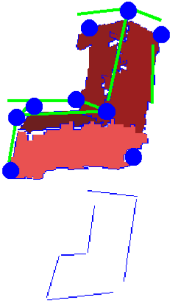


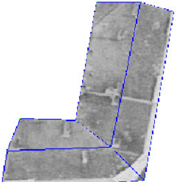
Example

3D reconstruction by graph matching between a graph (data) and a library of model graphs (IGN)



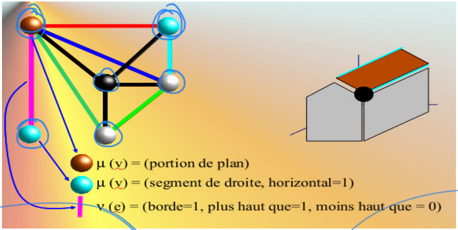


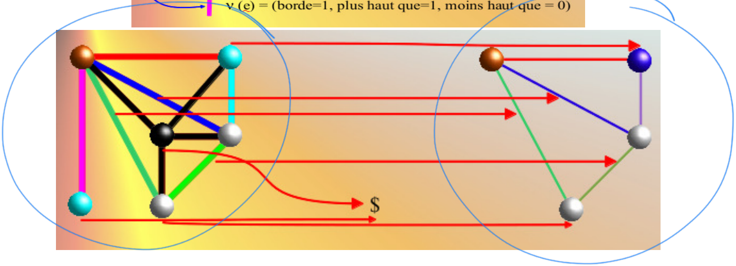




Example - building reconstruction

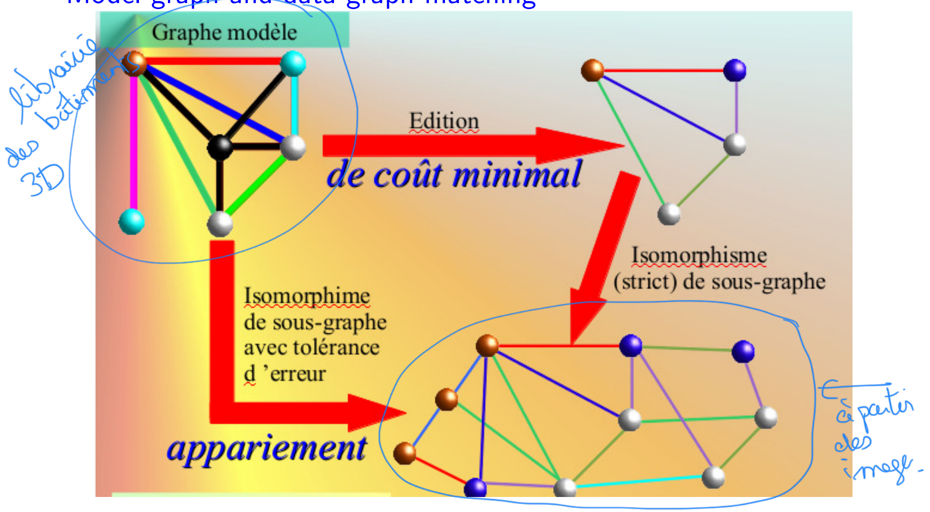
Model graph





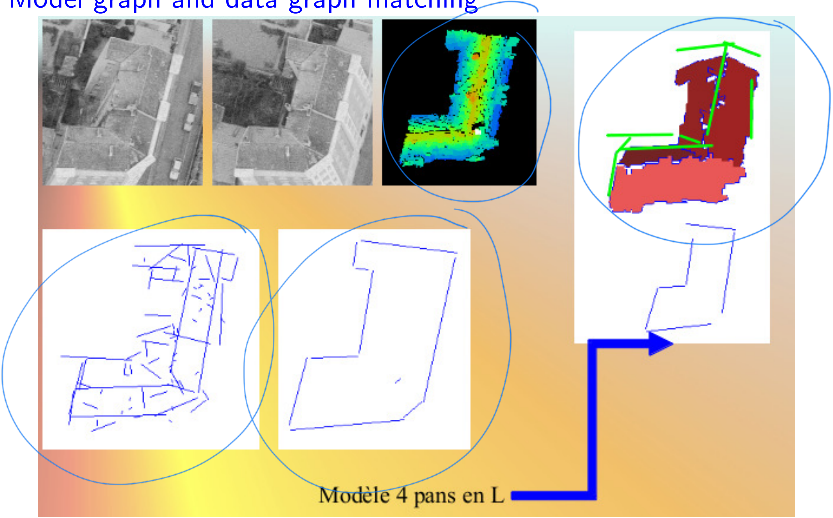
Example - building reconstruction

Model graph and data graph matching



Example - building reconstruction

Model graph and data graph matching



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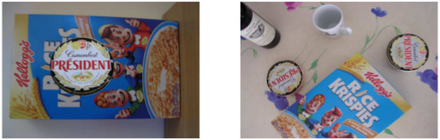
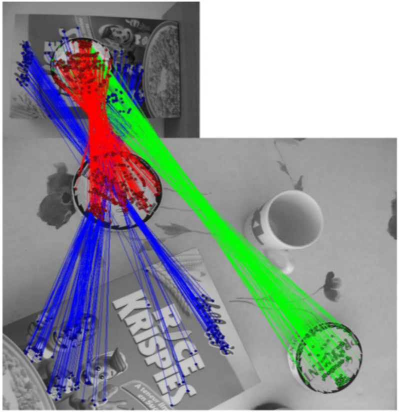
Matching with geometric transformation

- Graph = representation of the spatial information
- Matching = computation of the geometric transformation
  - polynomial deformation
  - elastic transformation (morphing)
- Matching approaches :
  - translation: maximum of correlation
  - Hough transform (in the parameter space)
  - RANSAC method: select randomly a set of matching points, compute the transformation, compute the score (depends on the number of matched pairs for the transformation)
  - AC-RANSAC: RANSAC + a contrario framework reducing the number of parameters (NFA to be set)

Example - MAC-RANSAC (PhD Julien Rabin)



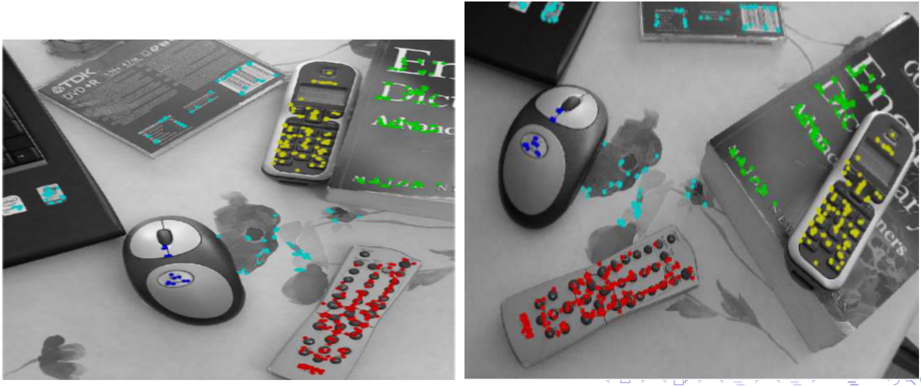
(a) Paire d'images analysée.



(b) Reconnaissance de chacun des objets superposés.

Example - MAC-RANSAC (PhD Julien Rabin)

(a) Paire d'images utilisée



Inexact matching

Optimization of a cost function

- Dissimilarity cost beween nodes

$$c_N(a_D, a_M) = \sum \alpha_i d(a_i^N(a_D), a_i^N(a_M)) \quad \sum \alpha_i = 1$$

- Dissimilarity cost between edges

$$C_E((a_D^1, a_D^2), (a_M^1, a_M^2)) = \sum \beta_j d(a_j^A(a_D^1, a_D^2), a_j^A(a_M^1, a_M^2)) \quad \sum \beta_j$$

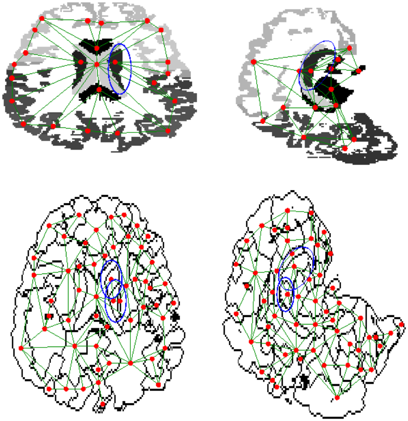
- Matching cost function h :

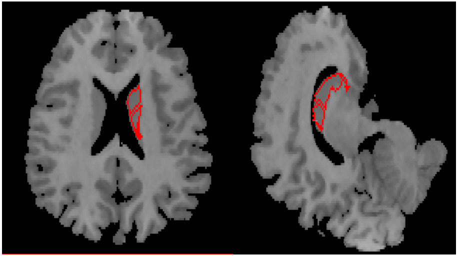
$$f(h) = \frac{\alpha}{|N_D|} \sum_{a_D \in N_D} c_N(a_D, h(a_D)) + \frac{1-\alpha}{|E_D|} \sum_{(a_D^1, a_D^2) \in E_D} c_E((a_D^1, a_D^2), (h(a_D^1),$$

Optimization methods:

- Tree search
- Expectation Maximization
- Genetic algorithms
- ...

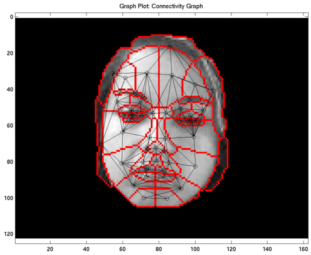
Example: brain structures (A. Perchant)

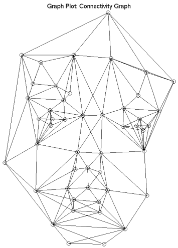


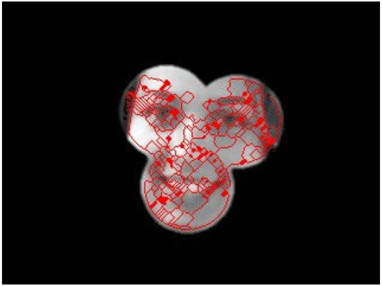


Example : face structures (R. Cesar et al.)

*1







Spectral method for graph matching (1)

Optimization of a cost function

- weighted adjacency matrix M
- nodes = potential assignments $a = (i, i')$ (can be selected by descriptor matching)
- edges = $M(a, b)$ agreement between the pairwise matchings a and b (geometric constraints)
- correspondance problem = finding a cluster C of assigments maximizing the inter-cluster score $S = \sum_{a,b \in C} M(a, b)$ with additional constraints
- cluster C = vector x (with $x(a) = 1$ if $a \in C$ and 0 else)

$$S = \sum_{a,b \in C} M(a, b) = x^T M x$$

$$x^* = argmax(x^T M x)$$

+ constraints (one to one mapping)

Spectral method for graph matching (2)

Search of the optimal cluster

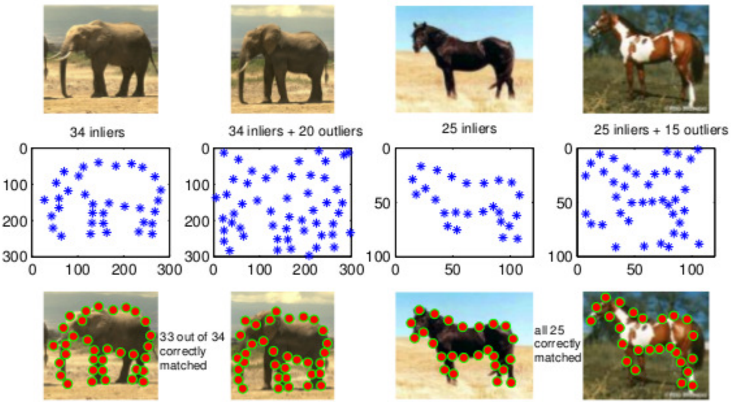
- number of assigments
- inter-connection between the assignments
- weights of the assignment

Spectral method: relaxation of the constraints on x

$$x^* = \textit{principal eigenvector}(x^T M x)$$

$+$ introduction of the one-to-one correspondance constraints
(iterative selection of $a^* = argmax_{a \in L}(x^*(a))$
and suppression in x^* of the incompatible assignments)

Example: point matching (Leordeanu, Hebert)



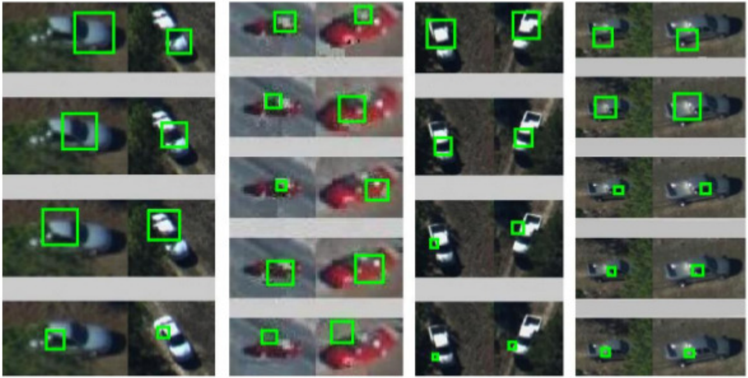
$d_{ab} = \frac{d_{ij}+q}{d_{i'j'}+q}$

$\alpha_{ab} =$ *angle* between the matchings

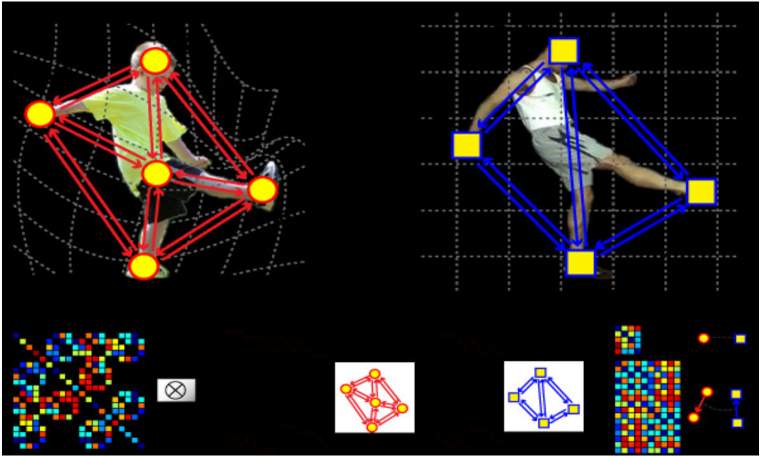
(with centring and normalization)

$M(a, b) = (1-\gamma)c_\alpha + \gamma c_d$

Example:feature matching (Leordeanu, Hebert)

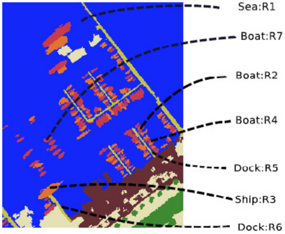


Example:factorized graph matching (Zhou, de la Torre)

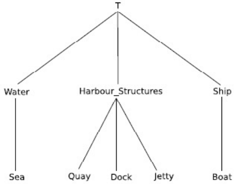


Spatial reasoning in images

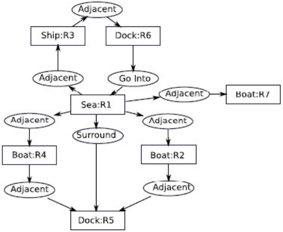
(a) Example image.



(b) Labeled image: The blue regions represent the sea, the red and orange represent ships or boats and the yellow regions represent the docks.



(c) Concept hierarchy T_C in the context of harbors.

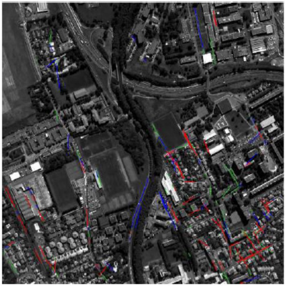


(d) Conceptual graph representing the spatial organization of some elements of Figure 5.8(b).

Spatial reasoning in images


(a)


(b)


(c)

References

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