

Main characteristics of SAR images

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Backscattering mechanims

Relief effects and geometry influence

Speckle phenomenon

- Origine
- Modeling
- Multi-looking
- Multiplicative noise model

Texture and log-statistics





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Backscattering mechanisms: surface and volume



 \rightarrow Strong influence on the backscattered radiometries



Targets and object appearance

Bright targets :

 Trihedral / dihedral structures (manmade objects, urban areas)

Surface area:

- Depends on the roughness
- Depends on the geometric configuration
- Dielectric properties (water content, humidity)
- Many objects in the resolution cell:
 - Speckle





Volume scattering mechanisms



2) Trunk scattering

4) Attenuated soil scattering

6) Trunk-branch interaction

7) Soil-branch interaction

Examples of main backscattering mechanisms on the forest

Volume backscattering mechanisms generally rely on interaction mechanisms which are highly complex and still not well-known. Main trends:

→ Backscattering coefficient **>** when vegetation volume (biomass) **>**

→ Wavelength penetration **>** when frequency **>**, i.e. when wavelenght **>**

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Backscattering of a cell

$$U_{\omega}(P,t) \approx \frac{1}{R(P)} \iint_{\Sigma} e^{j4\pi \frac{x \sin \theta}{\lambda}} A(x,y) \, ds$$

- A(x,y) is characteristic of the imaged area
 A(x,y) can be complex :
 - Amplitude : backscattering coefficient
 - Phase : delays or delocalisation inside the pixel
- →Directivity of the backscattered signal : depends on A(x,y)
 - The diagram of the local ground antenna is not known



Backscattering of a cell

$$U_{\omega}(P,t) \approx \frac{1}{R(P)} \iint_{\Sigma} e^{j4\pi \frac{x \sin \theta}{\lambda}} A(x,y) \, ds$$

An object on the ground is defined by its RCS (Radar Cross Section) or SER (Section Efficace Radar) :

- Depends on the material (dielectric properties, roughness)
- Depends on the shape (geometry)
- SER
 - Ratio between emitted power and backscattered power
 - Depends of the antenna gain





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Influence of lateral viewing





Geometrical distorsions Variable incidence angle: variable resolution

 θ =6°, dx

θ=60°, dx/10



Airborn system: same δr , variable δx along the swath



Influence of relief on cell size



Effets de la variation de la case sol en fonction de la pente locale : le Cap Vert



Image ERS

TSI



Mosaïque sur Google



Cell size and local slope



ELECO

Ground range: Case $\alpha = \theta$



Range cell : ∆r

- Ground range : ∆x
- Influence of local slope



A and B in the same range cell Relation between X, H et θ

$$\tan \theta = \frac{H}{X}$$







- Weak slope : A first, then B
- Slope = incidence angle: A and B in the same cell

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- Strong slope : B first, then A : <u>« lay-over »</u>
 - Lay-over condition:

$$H > X \tan \theta$$



Relief – geometric distorsions







Example of a vertical post









Lateral viewing

Terrasar-X, θ~34°
 Relationship between
 h and BP

$$H = \Delta X \tan \theta$$
$$H = \frac{\Delta R}{\cos \theta}$$













Urban areas: optic / SAR









Pyramid example 30° and 60°









- Side : 232m
- Height : 146m
- Slope : 51°
- Incidence : 53°



Gizeh : incidence 40°





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Influence of the viewing direction



X-SAR image of Brooklyn, New-York, resolution 6.5m



Geometric effects

In urban areas:

- Shadow areas behind over-ground objects
- Overlay phenomenon (lay-over)
- Corner reflector wall / ground
- Strong backscattering of facades oriented towards the sensor



















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SAR data visualization

- Data: 1 complex number per pixel
- Amplitude (modulus of electro-magnetic field)

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- Dynamic:
 - Very widespread
 - Display of images on 8 bits gray-levels (0 to 255 values)
 - 8-bits coding should be adapted !

Dynamic truncation with

max = mean + 3sigma



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國務國際 Desert in Australia (Terrasar-X)





Sonar: underwater ground










國務 國 Desert of Mauritania (ERS)





「 図 の Sea – near Bergen (Terrasar-X Spotlight)











副多聞 Phenomenological analysis (Terrasar-X)











Size of resolution cell >> λ

• Elementary scatterers inside the resolution cell

Coherent sum of the waves:

- Each scatterer backscatters the e.m wave
- Phenomenon of interferences
- Vectorial addition in the complex plane













- No acces to the scatterers inside the resolution cells (even if it is deterministic!)
- Random variable modeling:

backscattering modeled by a r.v !

Why developing models for the backscattered field ?

- Prediction of the performances of image processings
- Choice of the thresholds
- Development of model based methods





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Speckle modeling

SAR data acquisition



Synthetic aperture radar (SAR) imagery

- Active sensor: emits a wave and measures its echoes
- SAR: At each pixel: complex amplitude of the echo $z = Ae^{j\varphi}$
- Amplitude: A = |z|
- Intensity: $I = A^2 = |z|^2$
- Phase: $\varphi = \arg z$



多数 の Speckle modeling

Origins of speckle in SAR / coherent imaging systems / interferences





see [Goodman, 1976]

Coherent summation of N punctual echos

$$z = \sum_{i}^{N} z_i, \quad z_i \in \mathbb{C}$$
 .

Goodman model (rough surfaces):

- 1 $z_1,...,z_N$ iid,
- 2 $\Re(z_i)$ and $\Im(z_i)$ iid,
- $|z_i|$ and $\arg z_i$ independent.

• By the law of large numbers wrt N

$$p(z|R) \triangleq p(\Re(z), \Im(z)|R)$$
$$= \frac{1}{\pi R} \exp\left(-\frac{|z|^2}{R}\right)$$

where R > 0 is the quantity of interest.

• R is linked to the radar cross-section (R = reflectivity).



Speckle modeling

Statistics of the circular complex Gaussian distribution



 $z = A e^{j \varphi}$ is distributed according to a complex circular Gaussian, thus

- $\varphi = \arg(z)$ uniformly distributed in $[-\pi, \pi]$,
- $I = |z|^2$ exponentially distributed:
- A = |z| Rayleigh distributed:
- *I* or *A* are sufficient statistics for *R*:

 $p(I \mid R) = \frac{1}{R} \exp\left(-\frac{I}{R}\right).$ $p(A \mid R) = \frac{2A}{R} \exp\left(-\frac{A^2}{R}\right).$

 $\mathbb{E}[I] = \mathbb{E}[A^2] = R.$

 \Rightarrow heavy right tail.

 \Rightarrow phase is non-informative.

 \Rightarrow many SAR applications focus only on |z|.





Coefficient of variation

$$p(I) = \frac{1}{R}e^{\left(-\frac{I}{R}\right)}$$

$$\mu_I = \sigma_I = R$$



$$\gamma_I = \frac{\sigma_I}{\mu_I} = 1$$



hererogeneity measure = coefficient of variation





Amplitude distribution:

$$p(A) = \frac{2A}{R} e^{\left(-\frac{A^2}{R}\right)}$$

Rayleigh pdf

$$R = 2\sigma^2 \propto \sigma^0$$
• $\mu_A = \sqrt{\frac{\pi R}{4}}$

•
$$\gamma_A = \frac{\sigma_A}{\mu_A} = \sqrt{\frac{4}{\pi}} - 1 \approx 0.523$$







Data	Pdf
Real part Imaginary part	Gaussian pdf 0 mean
	$R = 2\sigma^2 \propto \sigma^0$
Phase	Uniform pdf
Intensity	Negative exponential pdf
	$\mu_I = \sigma_I = R$
Amplitude	Rayleigh pdf
	$\mu_A = \sqrt{\frac{\pi R}{4}}$





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Multi-look processing

Averaging N i.i.d samples reduces the variance by N

$$X_1, X_2, \dots, X_N \text{ samples}$$

$$X^{ML} = \frac{1}{N} \sum_i X_i$$

$$Var X^{ML} = \frac{Var X_i}{N}$$

Which data ?

- Complex data ? $z_1, z_2, ..., z_N$
- Intensity data ? $I_1, I_2, ..., I_N$
- Amplitude data ? $A_1, A_2, ..., A_N$



Multi-vues

Multi-looking

• Increase signal to noise ratio using spatial/temporal average

$$I = I_{\mathsf{ML}} = \frac{1}{L} \sum_{t=1}^{L} |z_t|^2 = \frac{1}{L} \sum_{t=1}^{L} I_t$$

• As I_t are iid, I is gamma distributed

$$p(I \mid R, L) = \underset{t=1}{\overset{L}{\bigstar}} p(I_t \mid R) = \frac{L^L I^{L-1}}{\Gamma(L) R^L} \exp\left(-\frac{LI}{R}\right),$$

• $A = A_{ML} = \sqrt{I_{ML}}$ is Nakagami-Rayleigh distributed.



Intensity multi-looking

Averaging of L intensity samples:

- Convolution of neg. exp. pdf : Gamma pdf
- L: number of looks



Amplitude multi-looking

Square root of the average of L intensity samples

- Nakagami pdf
- L: number of looks

$$p(A|R) = \frac{2L^L}{\Gamma(L)} \frac{A^{2L-1}}{R^L} \exp(-L\frac{A^2}{R})$$

$$\gamma_A = \frac{0.523}{\sqrt{L}}$$







Which samples ?

- Historically :
 - Azimuth sub-band decomposition of the complex spectrum
 - Decrease of spatial resolution to improve radiometric resolution
- Spatial samples
 - Mean filter
 - Loss of spatial resolution
- Temporal samples
 - Not iid ?



Multi-looking : less speckle, less resolution



10x10 multi-looking : easier image interpretation



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Temporal multi-looking (13 images)







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Original TerraSAR-X image





Video of the temporal multi-looking





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Very efficient Very simple Only valid for stable areas Temporal average of 26 images TSX











Equivalent number of looks (ENL)

For a multi-look image:

- The number of looks is usually less than the theoretical number of views because of the correlation between samples
- ENL computation
 - Choice of a physically homogeneous area
 - Computation of the coefficient of variational
 - Inversion of the relationship (amplitude or intensity)
- Use:

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- To adjust statistical models
- To evaluate the efficiency of a filtering method





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Modeling of the speckle noise as multiplicative:

$$\begin{bmatrix} I = R.S & E(S) = 1\\ Var(S) = \frac{1}{L} \end{bmatrix}$$
$$p(S) = \frac{L^{L-1}}{\Gamma(L)} S^{L-1} \exp(-LS)$$

Texture modeling with scene pdf:

$$p(I=RS)=\int p(R)p(S=\frac{I}{R})\frac{1}{R}dR$$

$$p(I) = \int p(R)p(I|R))dR$$



Homomorphic approaches

Goodman model of fully developed speckle

SAR intensity is distributed according to a Gamma distribution:

$$p_{I}(I|R) = \frac{L^{L}I^{L-1}}{\Gamma(L)R^{L}} \exp\left(-L\frac{I}{R}\right) \text{ with } R \text{ the radar reflectivity.}$$
$$\rightarrow \mathbb{E}[I] = R$$
$$\rightarrow \operatorname{Var}[I] = R^{2}/L$$

The log of the intensity follows a Fisher-Tippett distribution:

$$p_{y}(y|x) = \frac{L^{L}}{\Gamma(L)} e^{L(y-x)} \exp\left(-Le^{y-x}\right)$$

$$\rightarrow \mathbb{E}[y] = x - \log L + \Psi(L)$$

$$\rightarrow \operatorname{Var}[y] = \Psi(1, L) \quad (\Psi : \operatorname{polygamma})$$





multiplicative speckle noise

additive stationary noise







approximate log-transformed speckle as additive white Gaussian noise \rightarrow not very good for small *L*: asymmetry towards lower values \rightarrow not centered (a debiaising step is needed)





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Texture modeling

- Use of a multiplicative model:
 Image (I) = texture (R) x speckle (S)
 Distribution modeling:
 - Speckle: fully developed speckle (Goodman model)
 - Texture: proposal of different distributions
- Probabilistic tools:
 - Mellin transform
 - Log-statistics


Mellin convolution and associated tools

Mellin convolution for positive r.v:

$$r(I) = \int_{0}^{\infty} p(R) \gamma\left(\frac{I}{R}\right) \frac{dR}{R}$$
$$= p \hat{*} \gamma$$

Modeling of many textures on SAR:

S distribution	R distribution	I distribution
Gamma	dirac	Gamma
Gamma	Gamma	Κ
Gamma	Gamma inverse	Fisher





Convolution and Fourier transform:



$$r = p * q = \int_{-\infty}^{\infty} p(u)q(x-u)du \Leftrightarrow TF(r) = TF(p).TF(q)$$

Adapted to additive noise

Mellin convolution and Mellin transform:



$$r = p \hat{*} q = \int_{0}^{\infty} p(u) q\left(\frac{x}{u}\right) \frac{du}{u} \Leftrightarrow TM(r) = TM(p).TM(q)$$

• Adapted to multiplicative noise



Statistics et log-statistics

Statistics : pdf defined on \Re

- Use of the Fourier transform
- Convolution: additive noise
- Characteristic functions
- Gaussian pdf: defined on \Re
- Log-statistics : pdf defined on ℜ⁺
 - Use of Mellin transform
 - Mellin convolution: multiplicative noise
 - Characteristic function of «second kind»
 - Gamma pdf : defined on \Re^+



Maracteristic functions, moments and cumulants



$$\Phi(f) = TF(p) = \int_{-\infty}^{+\infty} e^{2j\pi xf} p(x) dx$$

$$m_{k} = \left(-j\right)^{k} \frac{d^{k} \Phi(f)}{df^{k}} \bigg|_{f=0}$$

$$\kappa_{k} = \left(-j\right)^{k} \frac{d^{k} \log(\Phi(f))}{df^{k}} \bigg|_{f=0}$$

 $\widetilde{\Phi}(s) = TM(p) = \int_{0}^{\infty} x^{s-1}p(x)dx$ $\widetilde{m}_k = \frac{d^k \widetilde{\Phi}(s)}{ds^k}$ $\widetilde{\kappa}_{k} = \frac{d^{k} \log(\widetilde{\Phi}(s))}{ds^{k}}$

副多聞 Convolution and Mellin convolution

$$r = p * q = \int_{0}^{\infty} p(u)q(x-u)du$$
$$TF(r) = TF(p).TF(q)$$
$$\Phi[r] = \Phi[p]\Phi[q]$$

$$r = p \hat{*} q = \int_{0}^{\infty} p(u) q\left(\frac{x}{u}\right) \frac{du}{u}$$
$$TM(r) = TM(p) \cdot TM(q)$$
$$\widetilde{\Phi}[r] = \widetilde{\Phi}[p] \widetilde{\Phi}[q]$$



副務部 Estimation of moments and log-moments

$$\Phi(f) = TF(p) = \int_{-\infty}^{+\infty} e^{2j\pi xf} p(x) dx$$
$$m_{k} = (-j)^{k} \frac{d^{k} \Phi(f)}{df^{k}} \Big|_{f=0}$$

$$=\int_{-\infty}^{+\infty}x^{k}p(x)dx$$

$$\hat{m}_{k} = \frac{1}{N} \sum_{i=1}^{N} (x_{i})^{k}$$

$$\widetilde{\Phi}(s) = TM(p) = \int_{0}^{\infty} x^{s-1} p(x) dx$$
$$\widetilde{m}_{k} = \frac{d^{k} \widetilde{\Phi}(s)}{ds^{k}} \Big|_{s=1}$$
$$= \int_{0}^{+\infty} (\log(x))^{k} p(x) dx$$
$$\widetilde{\widehat{m}}_{k} = \frac{1}{N} \sum_{k=1}^{N} \log(x_{i})^{k}$$

 N_{i-1}

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Moments et log-moments



Example of Rayleigh-Nakagami pdf

$$G[L,\mu](x) = \frac{1}{\Gamma(L)} \frac{1}{\mu} \left(\frac{Lx}{\mu}\right)^{L-1} e^{-\frac{Lx}{\mu}}$$

$$\widetilde{\Phi}_{G}(s) = \mu^{s-1} \frac{\Gamma(L+s-1)}{L^{s-1}\Gamma(L)}$$

$$RN[L,\mu](x) = \frac{2}{\Gamma(L)} \frac{\sqrt{L}}{\mu} \left(\frac{\sqrt{L}x}{\mu}\right)^{2L-1} e^{-\left(\frac{\sqrt{L}x}{\mu}\right)^2}$$

$$\widetilde{\Phi}_{\rm RN}(s) = \mu^{s-1} \frac{\Gamma\left(L + \frac{s-1}{2}\right)}{L^{\frac{s-1}{2}}\Gamma(L)}$$

$$\widetilde{\kappa}_{1,G} = \log(\mu) + \Psi(L) - \log(L)$$
$$\widetilde{\kappa}_{2,G} = \Psi(1,L)$$

 $\widetilde{\kappa}_k = \Psi(k-1,L)$

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$$\widetilde{\kappa}_{1,\text{RN}} = \log(\mu) + \frac{1}{2} (\Psi(L) - \log(L))$$
$$\widetilde{\kappa}_{2,\text{RN}} = \frac{1}{4} \Psi(1,L)$$

$$\widetilde{\kappa}_{k,\mathrm{RN}} = \left(\frac{1}{2}\right)^k \Psi(k-1,L)$$

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Practical use of log-cumulants to analyze 習習習習 textures

Computation of log-cumulants:

$$\widehat{\kappa_{1}} = \frac{1}{N} \sum_{i=1}^{N} \ln(x_{i})$$

$$\widehat{\kappa_{2}} = \frac{1}{N} \sum_{i=1}^{N} (\ln(x_{i}))^{2} - \frac{1}{N^{2}} \left(\sum_{i=1}^{N} \ln(x_{i}) \right)^{2}$$

$$\widehat{\kappa_{3}} = \frac{1}{N} \sum_{i=1}^{N} (\ln(x_{i}))^{3}$$

$$- \frac{3}{N^{2}} \left(\sum_{i=1}^{N} \ln(x_{i}) \right) \left(\sum_{i=1}^{N} (\ln(x_{i}))^{2} \right)$$

$$+ \frac{2}{N^{3}} \left(\sum_{i=1}^{N} \ln(x_{i}) \right)^{3}$$

- For known pdf : inversion to recover pdf parameters
- For unknown pdf: positioning in the log-cum3 / logcum2 diagram



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Summary of the session (1)

Backscattering of the objects

 Parameters: roughness, geometric configuration, dielectric properties

Geometric effects in SAR imagery

• Especially for HR SAR and urban areas (shadows, layovers, corner reflectors)

Speckle phenomenon:

- Well modeled by Goodman model for rough surfaces
- Amplitude: Rayleigh Nakagami
- Intensity: Gamma
- Homogeneity measure: coefficient of variation



Summary of the session (2)

Multi-looking: averaging of L samples

- Incoherent (intensity)
- ENL deduced from coefficient of variation

Log-statistics

- Give more reliable estimates to compute the distribution parameters
- The log-cum1 / log-cum2 diagram allows an easy visualization of the distribution positioning

