Markov Random Fields and graphcut optimization

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Introduction

- **Historiy**
  - Statistical physics (crystal structure)
  - Geman and Geman paper (84)
  - New interest with graph-cuts (2000)

- **Main principle of Markov Random Fields**

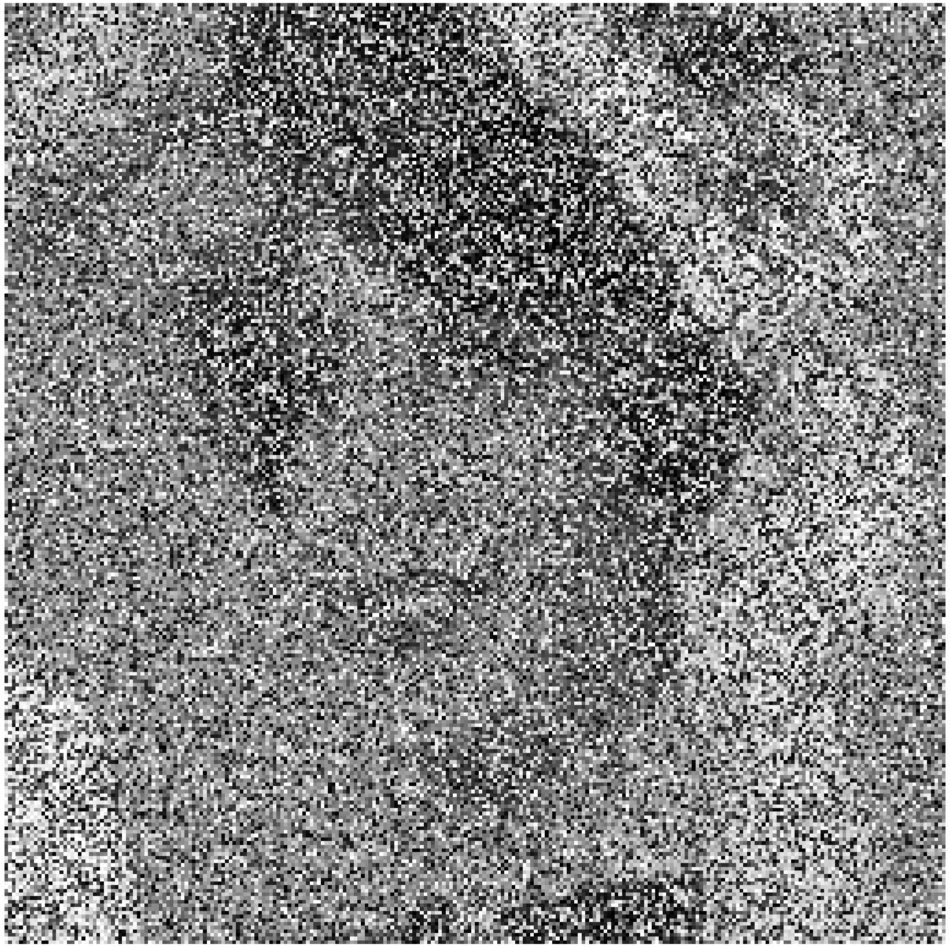
introducing contextual relationship in image processing

a local neighborhood is enough

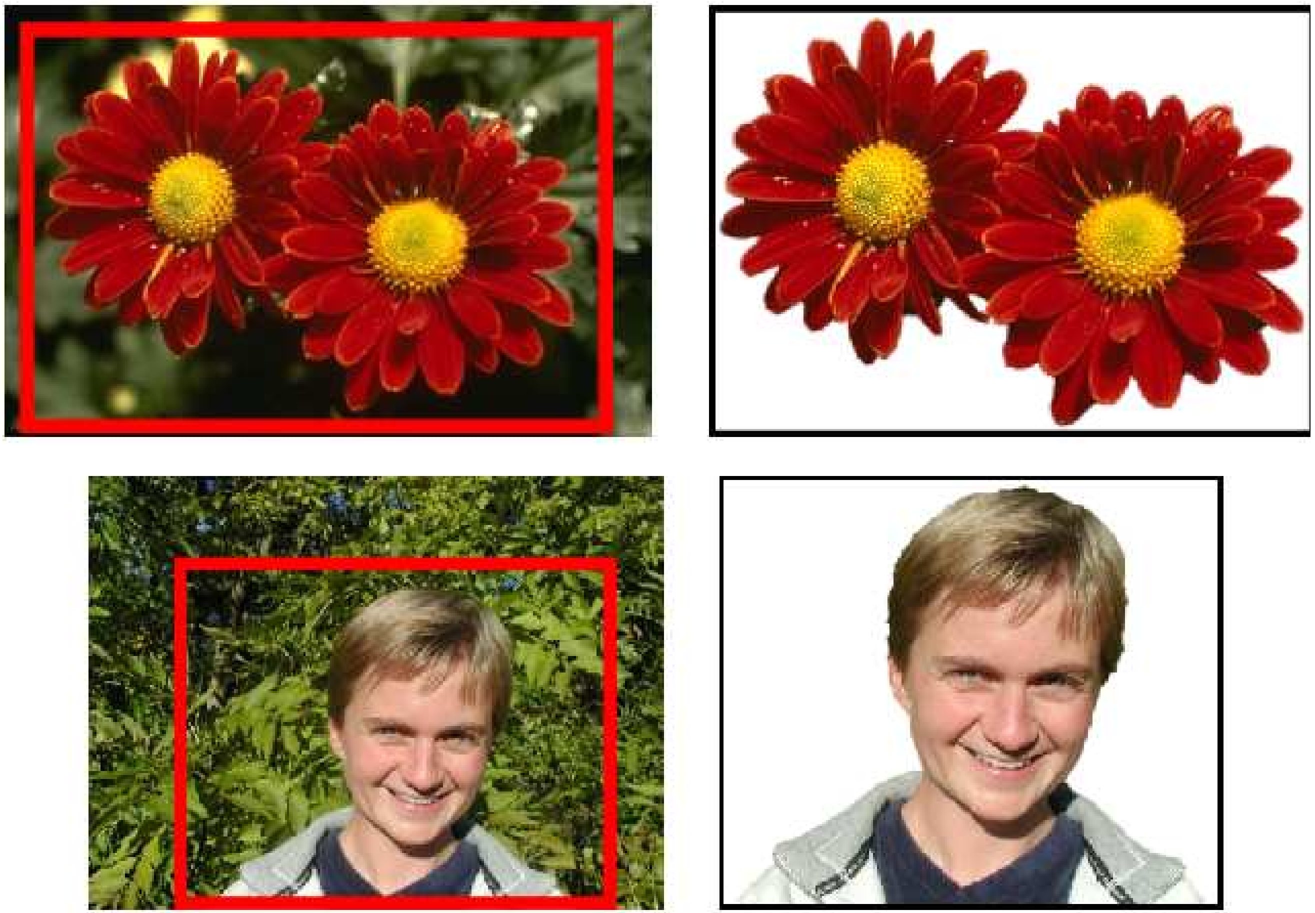
Prior model for natural images : spatial context

Illustrations - denoising (Darbon, Sigelle)

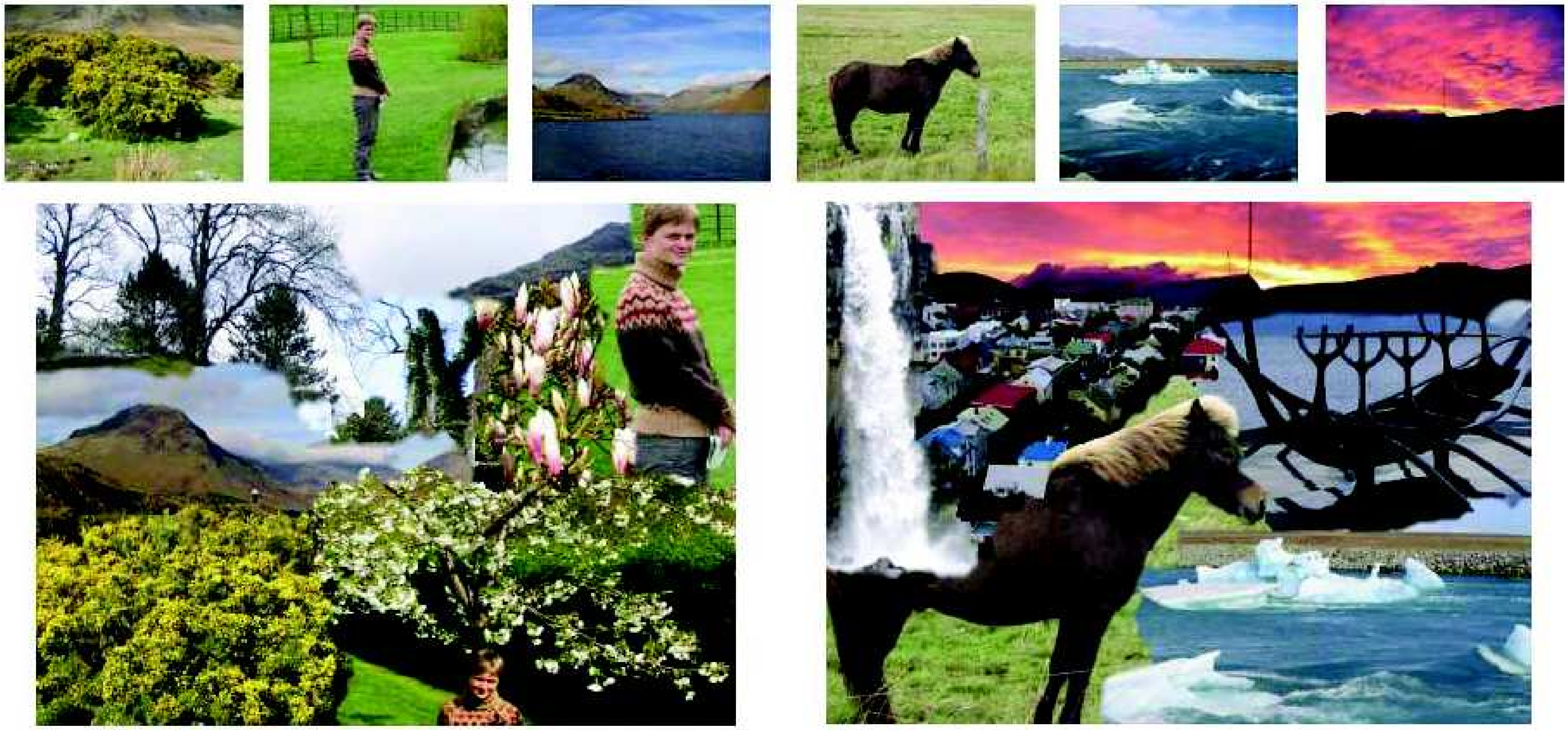
impulsive noise + TV



Illustrations - object selection (Rother, Kolmogorov et Blake)



Illustrations - digital tapestry (Rother et al.)



Illustrations - inpainting (Allène, Paragios)

Markov random fields and graph-cut based optimization

- **Bayesian analysis and markovian models**
- **Optimization**

Notations

◦ **Probabilistic model of images**

$S = \{s\} \subset \mathbf{Z}^d$ (finite) set of sites

$x_s \in E$ gray-level space

$(E = \{0..255\}$ $\{0..q-1\}$ (labels) $\mathbf{R})$

X_s random variable associated to s

$X = \{X_s\}_{s \in S}$ random field

$x = \{x_s\}_{s \in S} = \{x_s\} \cup x^s$ configuration (image)

$\Omega = E^{|S|}$ space of configurations

◦ **probabilities :**

$P(X_s = x_s)$ local probability

$P(X = x) = P(X_1 = x_1, X_2 = x_2 \ldots X_s = x_s \ldots)$ global (joint) law

$P(X_s = x_s | X_t = x_t, t \neq s)$(local) conditional probability

Bayesian analysis for image processing

Acquisition process of the observation y

$x \rightarrow$	Degradation Detection Measurement	$\rightarrow y$
original scene		observation

◦ **MAP (Maximum A Posteriori) crierion**

$$P(X|Y) \propto P(Y|X)P(X)$$

— $P(Y|X)$: likelihood term ("data attachment term")

— $P(X)$: prior term, choice of a model for the solution (regularization)

Energy formulation

◦ **Acquisition process**

$$P(Y = y|X = x) \quad = \quad \prod_{s \in S} P(Y_s = y_s|x) \quad = \quad \prod_{s \in S} P(Y_s = y_s|X_s = x_s)$$

◦ **Prior model : desired properties of the solution**

⇒ interaction between a site and its neighbors (regularity of the solution, ...)

⇒ X is a Markov random field

⇒ Hammersley-Clifford theorem

$P(X = x) = \dfrac{\exp - U(x)}{Z}$	Gibbs distribution
$U(x) = \sum_{c \in \mathcal{C}} U_c(x)$	global enregy
$U_c(x) = U_c(x_s, \ s \in c)$	clique potentials

A posteriori distribution

- **New Gibbs distribution**

$$P(X = x|Y = y) = \frac{\exp -\mathcal{U}(x|y)}{Z'}$$

$$\mathcal{U}(x|y) = \sum_{s \in S} - \ln(P(Y_s = y_s|X_s)) + U(x)$$

$$\mathcal{U}(x|y) = \sum_{s \in S} V_c(y_s|x_s) + \sum_{\{s,t\}} V_c(x_s, x_t)$$

$$\max_{x \in \Omega} \Pr(X = x|Y = y) \Leftrightarrow \min_{x \in \Omega} \mathcal{U}(x|y)$$

Examples of Markovian models

- **Segmentation - classification**

Ising model : binary field ($E = \{0, 1\}$)

Potts model : multi-labels field ($E = \{0, ..., K\}$)

$$V_c(x_s, x_t) = \beta\delta(x_s \neq x_t)$$

- **Restoration** $\quad V_c(x_s, x_t) = \phi(x_s - x_t)$
  - gaussian model (quadratic) $\quad \phi(u) = u^2$
  - Geman and Mac Clure 85 $\quad \phi(u) = \dfrac{u^2}{1+u^2}$
  - Hebert and Leahy 89 $\quad \phi(u) = \log(1 + u^2)$
  - Charbonnier 94 $\quad \phi(u) = 2\sqrt{1+u^2} - 2$
  - TV (Total Variation) model $\quad \phi(u) = |u|$

Markov random fields and graph-cut based optimization

- ◦ **Bayesian analysis and markovian models**
- ◦ **Optimization**

Optimization methods

◦ **Difficulties**

Configuration space Ω is huge : $Card(\Lambda)^{(np\times nl)}$!

◦ **Methods**

— Simulated annealing (Geman et Geman 84) : iterative stochastic algorithm, global minimum, slow algorithm (many Gibbas samplers)
— ICM (Iterated Conditional Modes) : deterministic algorithm, local minimum, very fast, needs a good initialization
— Research of minimum cut in a graph : fast and global minimum ! but for a restricted class of energies ...

Graph-cut based optimization methods

- **Introduction - reminders on graphs**
- **Binary case**
- **Approximate algorithms**
- **Exact algorithms**

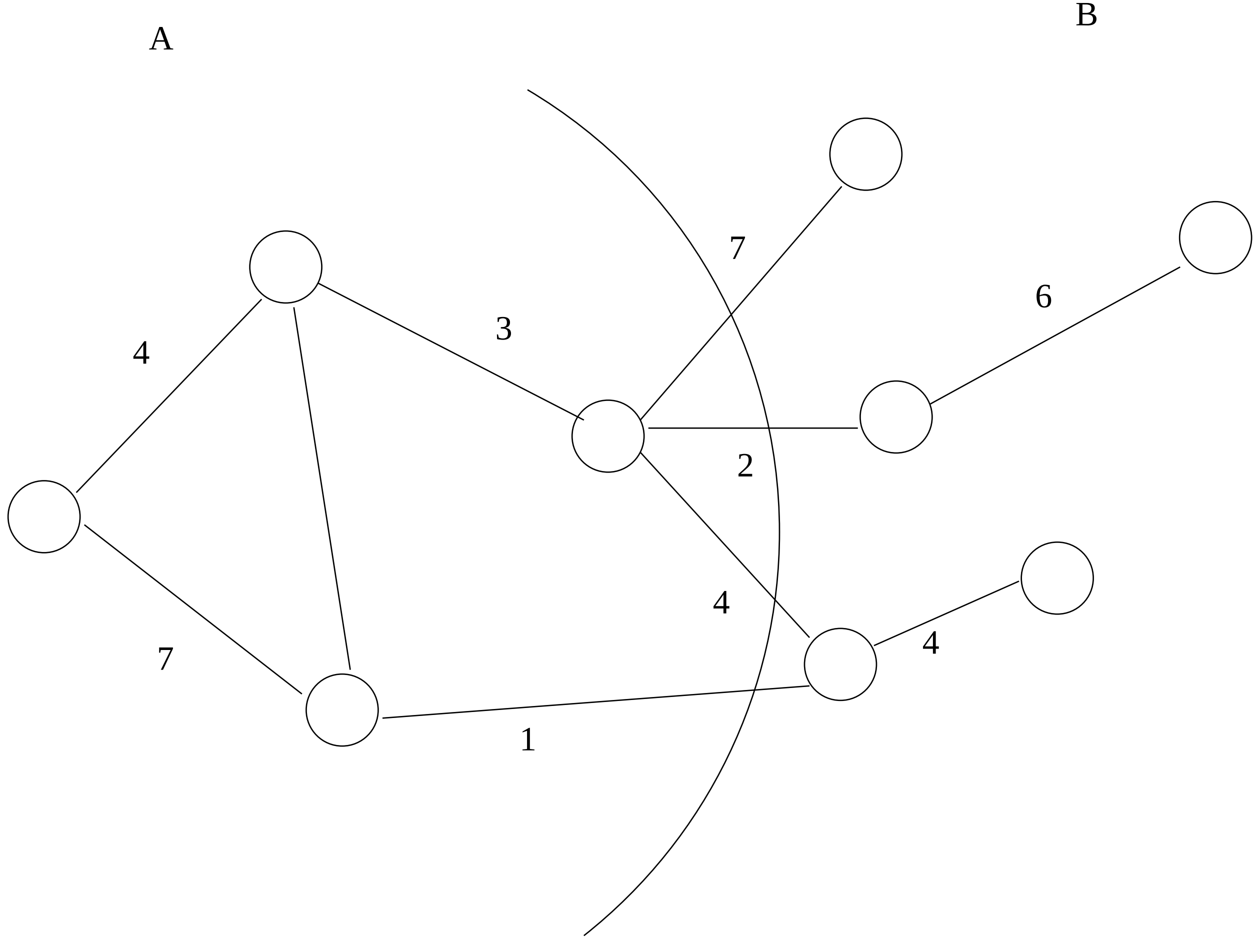
Graph theory and cuts

◦ Cut of a graph

— graph $G = (X, E)$

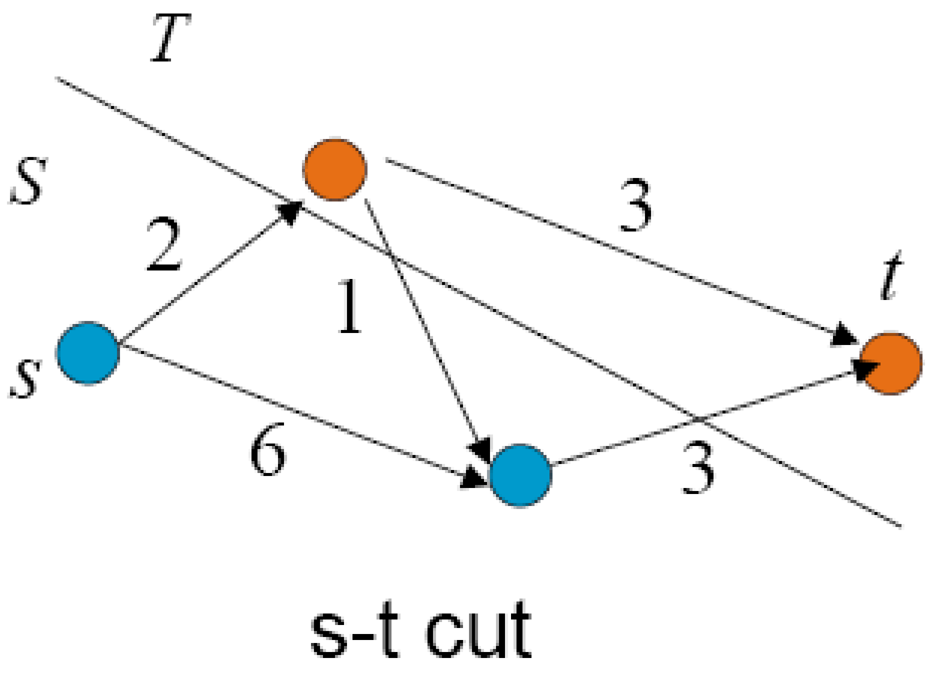
— partitioning in two parts A and B ($A \cup B = X$, $A \cap B = \emptyset$)

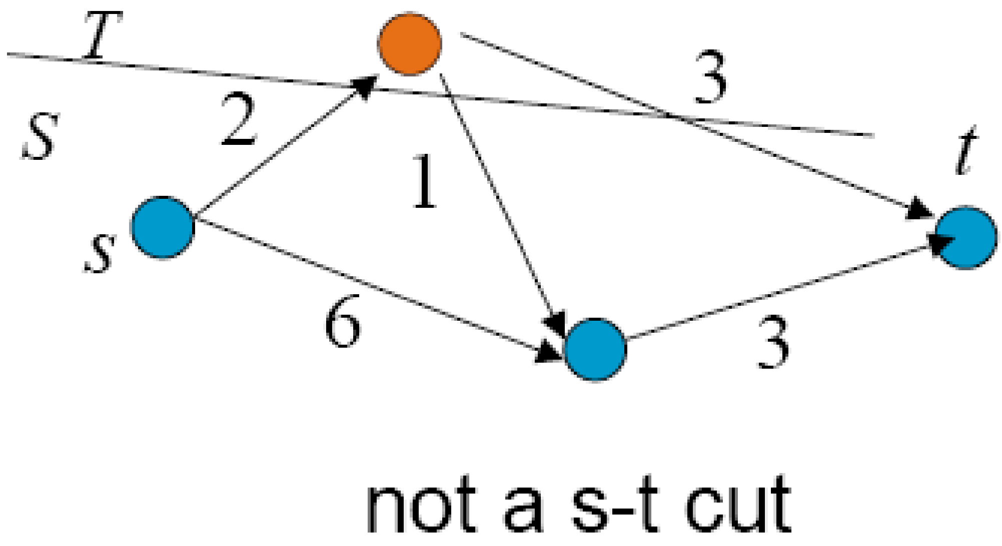
— $cut(A, B) = \sum_{x \in A, y \in B} w(x, y)$



Graph theory and cuts

- **Cut of a graph with 2 terminal nodes**
  - adding of 2 particular nodes : source s, sink t
  - partitioning in 2 parts S and T, one containing the source and the other the sink : st-cut
  - $cut(S,T) = \sum_{x \in S, y \in T} w(x,y)$

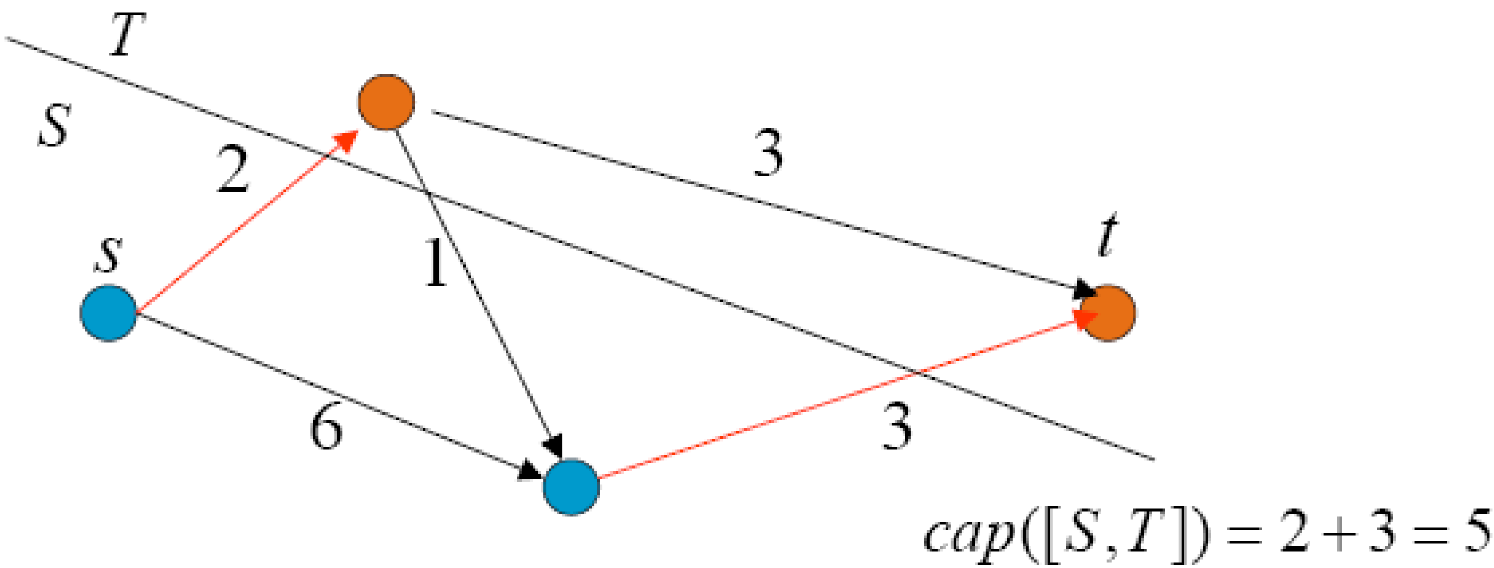




Graph theory and cuts

◦ **Min-cut of a graph**

Min-cut : Among all the s-t cuts, the one with the minimum cost



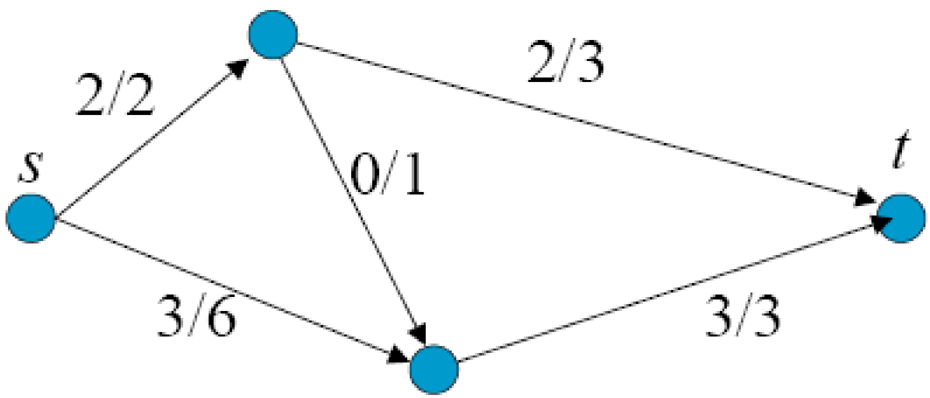
Graph theory and cuts

◦ **Flow**

— flow(p,q) $\leq$ w(p,q)

— in a node : in-going flow = out-going flow

— search for the max flow between s and t for given positive capacities of the edges

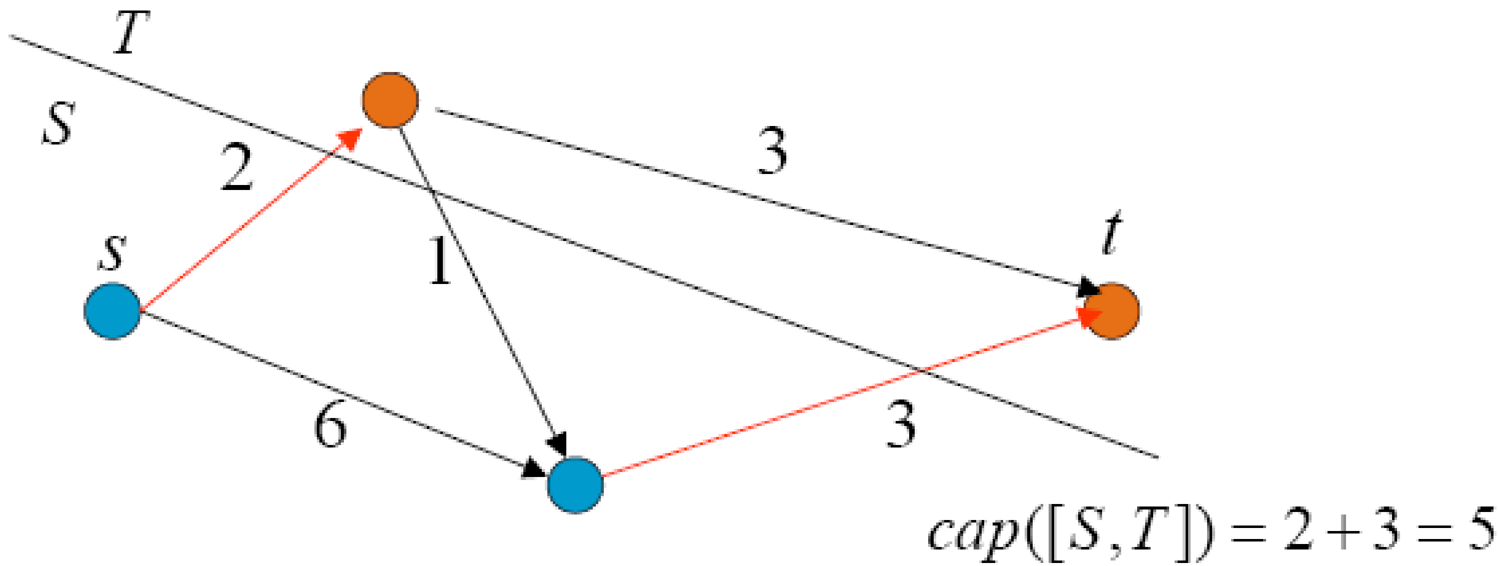


An example of flow

Graph theory and cuts ◦ MinCut = MaxFlow

— maximum flow = cut of minimum capacity

— value of the flow = cost of the cut



Graph theory and cuts

◦ **Ford and Fulkerson algorithm (62)**

notion of residual graph and search for the shortest path

algorithm in $O(nmc_{max})$ (n number of nodes, m number of edges and c_{max} maximum capacity of the edges)

◦ **"Push - relabel" algorithm (Goldberg et Trajan)**

does not respect in-going flow = out-going flow

algorithm in $O(n^3)$ or $O(n^2\sqrt{m})$

Graph theory and cuts

◦ **Specific algorithm for image processing (Boykov et Kolmogorov)**

— building of 2 trees, each one starting from a terminal node

— meeting of the 2 trees : existence of an increasing chain

— updating of the residual graph and iteration

in practice : much more adapted to graphs with few edges of image processing !

`http://www.adastral.ucl.ac.uk/~vladkolm/software.html`

Graph-cut based optimization methods

- **Introduction - reminders on graphs**
- **Binary case**
- **Approximate algorithms**
- **Exact algorithms**

Binary case - Ising model (Greig et al. 89)

◦ **Ising model**

two labels 0 (black) and 1 (white)

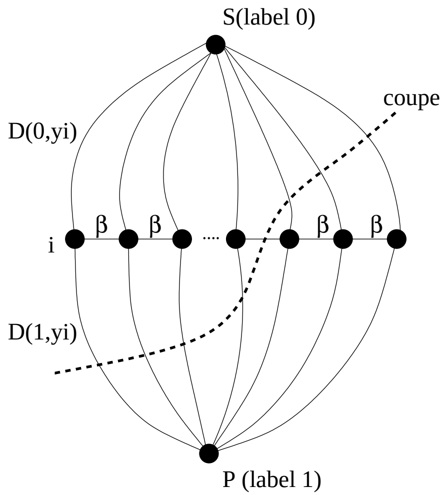
energy :

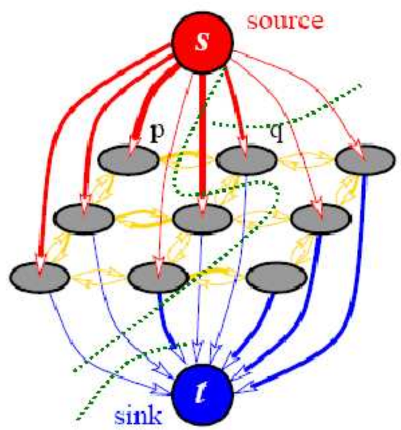
$$U(x|y) = \sum_s \mathcal{D}(x_s, y_s) + \sum_{(s,t)} \beta\delta(x_s \neq x_t)$$

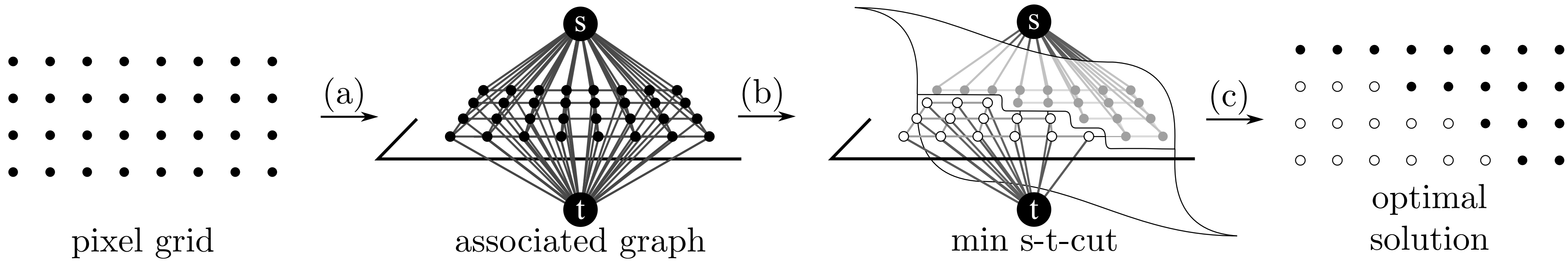
◦ **Graph building**

- nodes = all the pixels p of the image
- addition of two terminal nodes (source : label 0, sink : label 1)
- edges :
  1. link with the source with weight : $w(p, s) = \mathcal{D}(0, y_p)$
  2. link with the sink with weight : $w(p, t) = \mathcal{D}(1, y_p)$
  3. if two pixels p and q are neighbors in 4 connexity : edge of weight $w(p, q) = \beta$

Binary case 1D and 2D (Greig et al. 89)







Binary case - Ising model (Greig et al. 89)

◦ **Cost of a cut**

S set of pixels linked to the source

T set of pixels linked to the sink

cost of the cut :

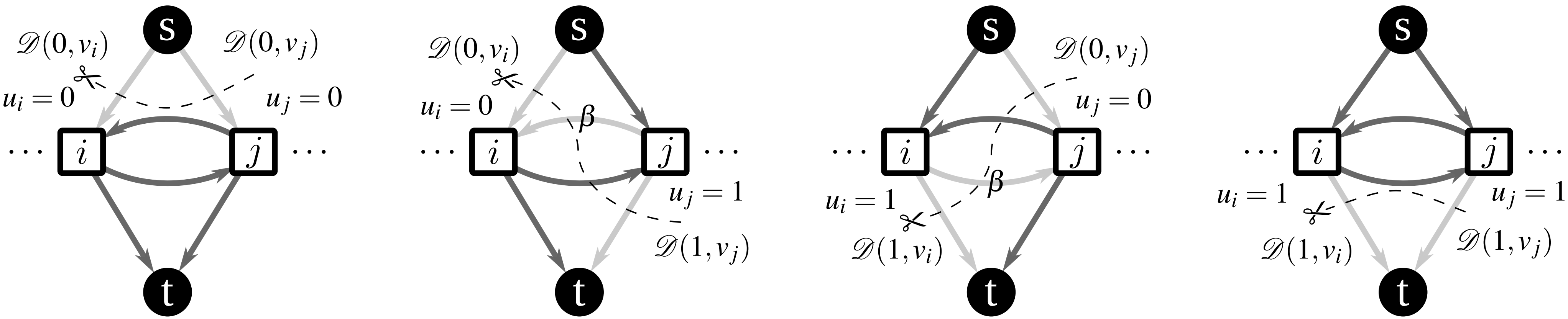
$$C(S,T) = \sum_{p\in S} \mathcal{D}(1,y_p) + \sum_{p\in T} \mathcal{D}(0,y_p) + \sum_{(s\in S, t\in T)} \beta$$

$\Rightarrow C(S,T) = U(x|y)$ for a labeling x defined by

— if $p \in S : x_p = 1$

— if $p \in T : x_p = 0$

Binary case 1D and 2D (Greig et al. 89)



$\cdots + \mathscr{D}(0,v_i) + \mathscr{D}(0,v_j) + 0 + \cdots$

$\cdots + \mathscr{D}(0,v_i) + \mathscr{D}(1,v_j) + \beta + \cdots$

$\cdots + \mathscr{D}(1,v_i) + \mathscr{D}(0,v_j) + \beta + \cdots$

$\cdots + \mathscr{D}(1,v_i) + \mathscr{D}(1,v_j) + 0 + \cdots$

Binary case : generalization (Kolmogorov et Zabih, 2004)

- **Energy formulation**

$$U(x|y) = \sum_{s} \mathcal{D}(x_s, y_s) + \sum_{(s,t)} V_c(x_s, x_t)$$

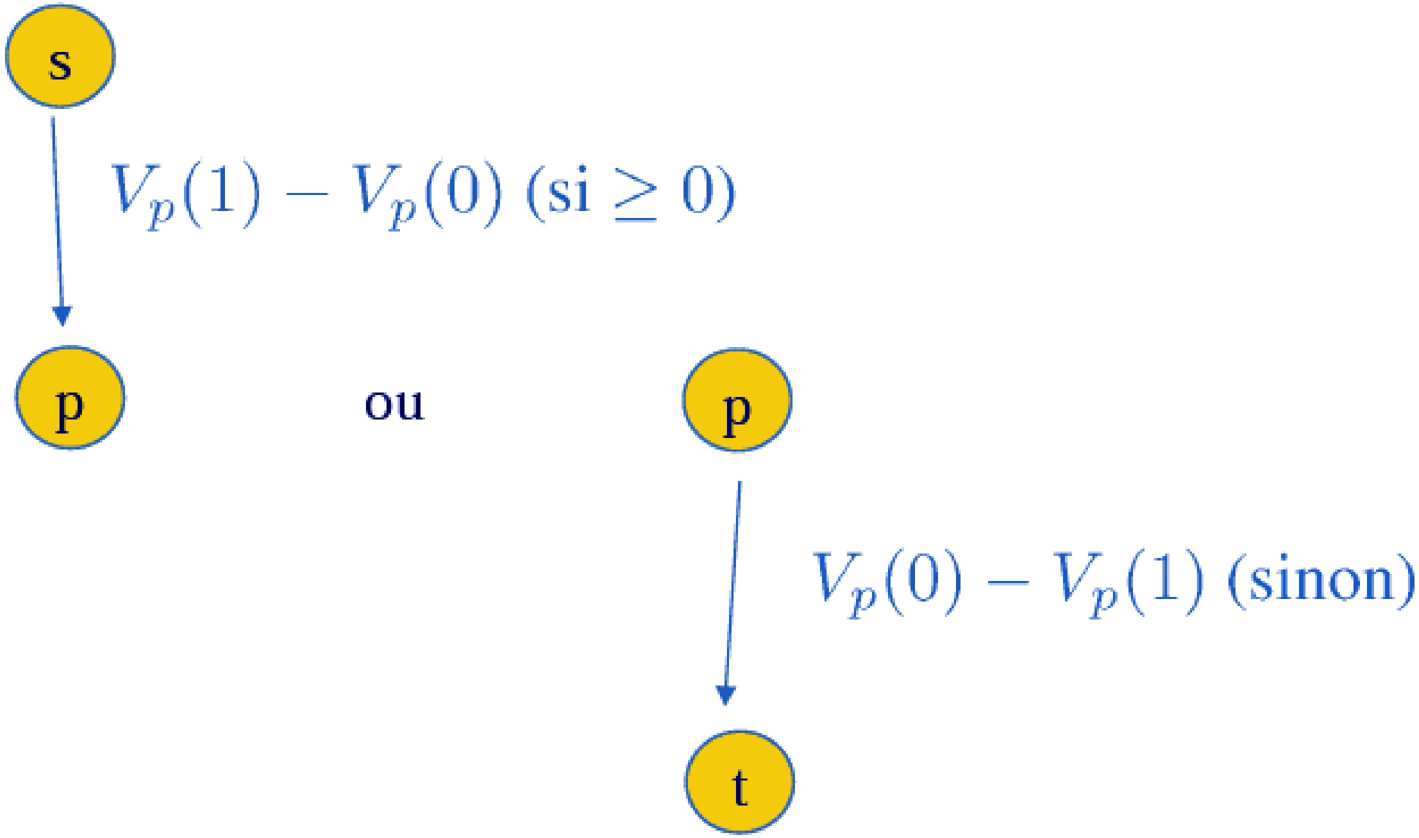
- **Condition for a graph-representable energy**

$$V_c(0,0) + V_c(1,1) \leq V_c(0,1) + V_c(1,0)$$

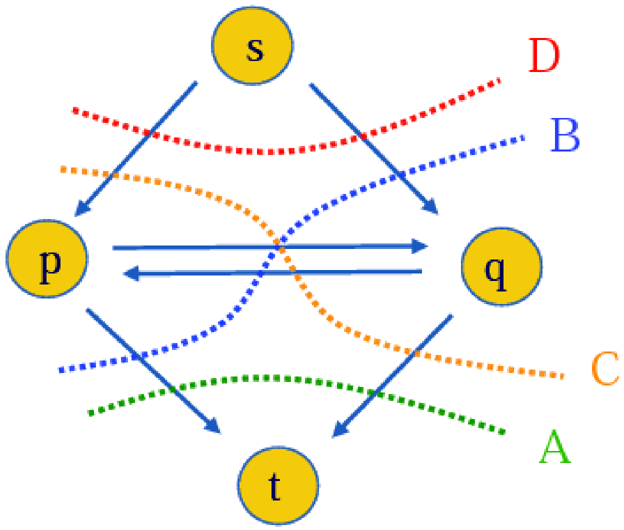
V_c “sous-modular” functions

Building of the graph (Kolmogorov et Zabih)

(if $p \in S$ $x_p = 0$)



Building of the graph (Kolmogorov et Zabih)

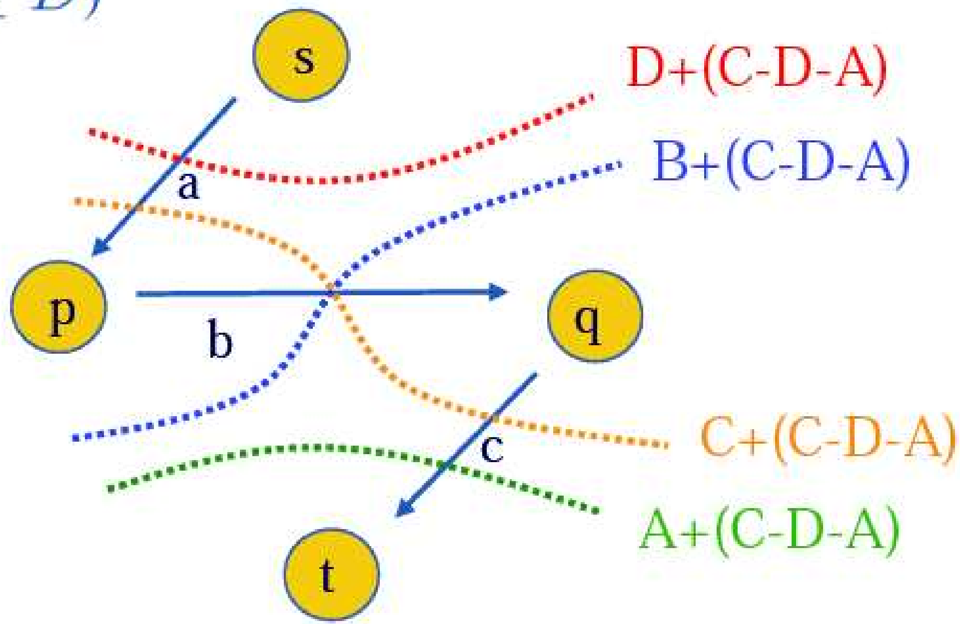


$A = V(0,0), B = V(0,1), C = V(1,0), D = V(1,1)$

$\Rightarrow$ Arcs ?

Building of the graph (Kolmogorov et Zabih)

(Cas $C \geq A$ et $C \geq D$)



$$a = C - A, b = B + C - D - A, c = C - D$$

On a bien $b \geq 0$ car

$$V(0,0) + V(1,1) \leq V(0,1) + V(1,0)$$

soit

$$A + D \leq B + C$$

Interactive segmentation : “hard” constraints

◦ Principle

the user manually defines the object and the background

$\Rightarrow$ minimization of the energy of a binary classification with “hard” constraints (= pixels that can not change their class)

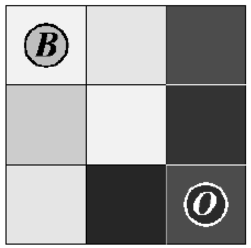
◦ Method

Search for the minimum cut with high weights for some edges guaranteeing that these edges will not belong to the cut

◦ Advantages

— allow to introduce “hard” constraints not easy to introduce when optimizing with the simulated annealing algorithm
— the areas belonging to the object and background can be used to do some supervised learning of the likelihood term (“data attachment term”)
— very fast algorithm if new markers are introduced

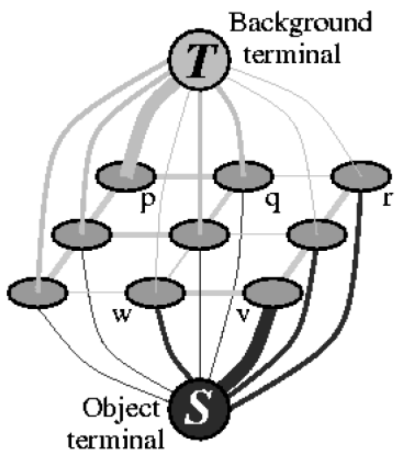
Building of the graph (Boykov et Jolly)



(a) Image with seeds.

(d) Segmentation results.

⇓ ⇑



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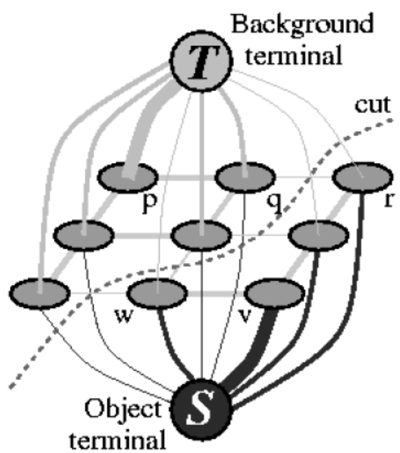
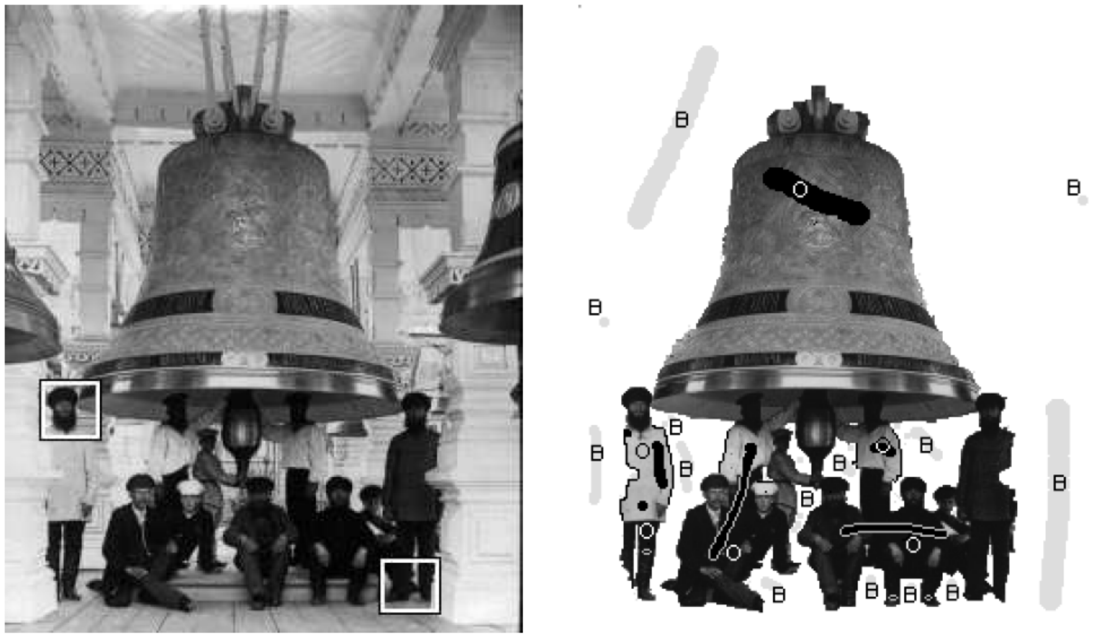


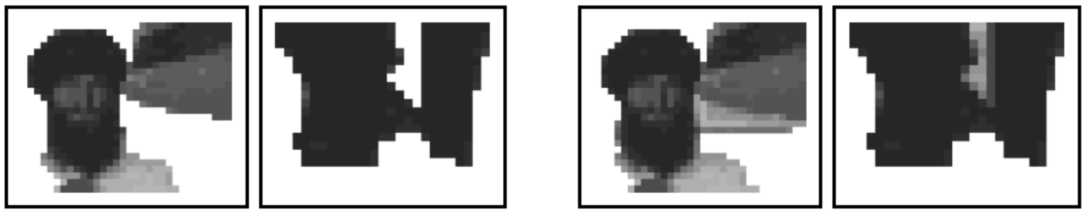
Weights of the graph (Boykov et Jolly)

edge	weight (cost)	for
$\{p,q\}$	$B_{\{p,q\}}$	$\{p,q\} \in \mathcal{N}$
$\{p,S\}$	$\lambda \cdot R_p(\text{"bkg"})$	$p \in \mathcal{P},\ p \notin \mathcal{O} \cup \mathcal{B}$
	K	$p \in \mathcal{O}$
	0	$p \in \mathcal{B}$
$\{p,T\}$	$\lambda \cdot R_p(\text{"obj"})$	$p \in \mathcal{P},\ p \notin \mathcal{O} \cup \mathcal{B}$
	0	$p \in \mathcal{O}$
	K	$p \in \mathcal{B}$

Illustrations (Boykov et Jolly)



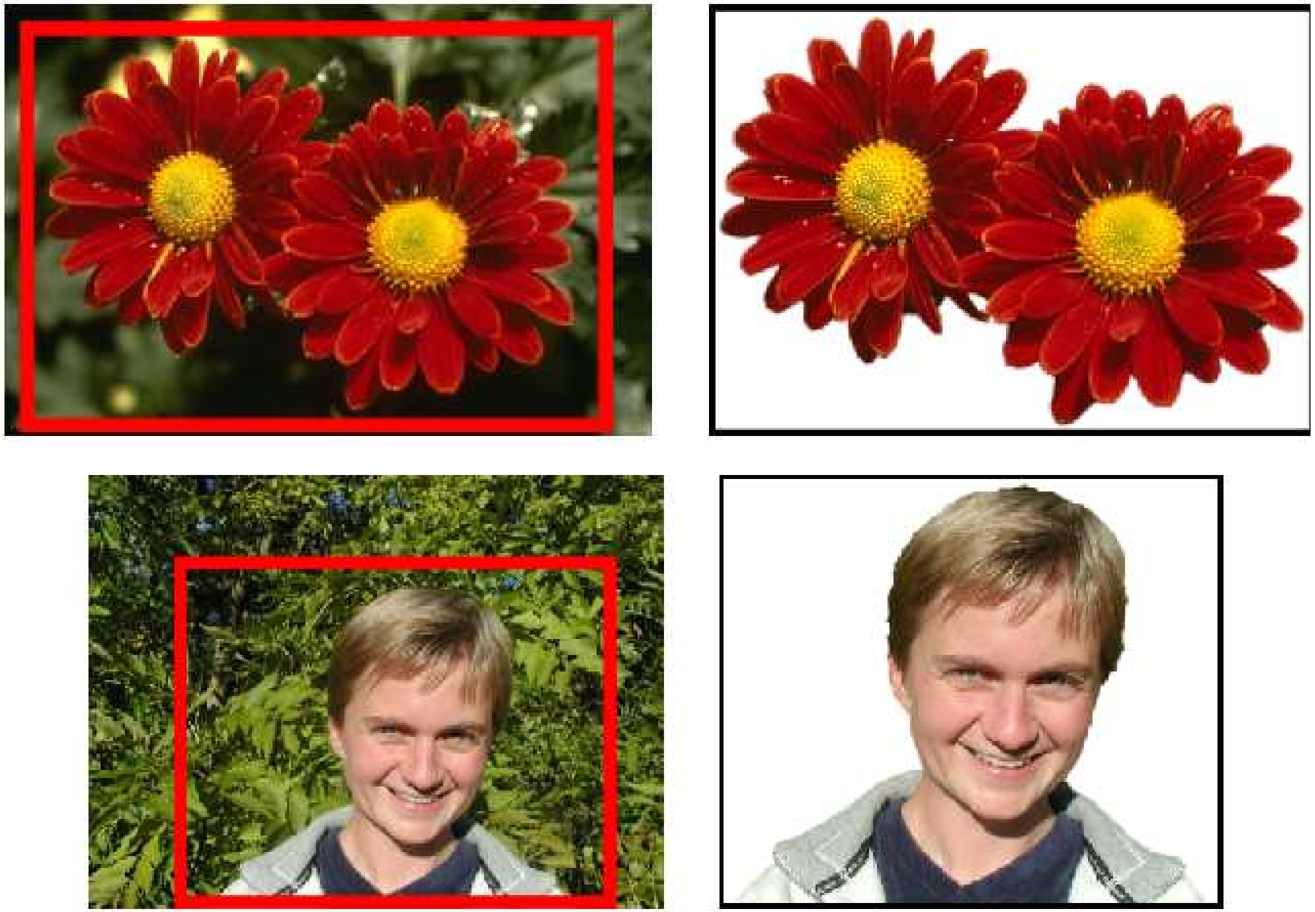
(a) Original B&W photo (b) Segmentation results



Interactive methods with graph-cuts

- **Grab-cut (Rother et al.)**
  - take into account the colors
  - two classes (one for the object and the other one for the background) but with gaussian mixture models to define their distributions (many gaussian components for the background and the object)
  - regularization term weighted by the gradient between neighboring pixels
  - learning of the distribution parameters in a semi-supervised way : initialization by the user (definition of the backgroynd class) and then iteratively after each cut refinement

Illustrations -GrabCut- (Rother, Kolmogorov et Blake)



Graph-cut based optimization methods

- **Introduction - reminders on graphs**
- **Binary case**
- <u>**Approximate algorithms**</u>
- **Exact algorithms**

Extension to the multi-labels case (Boykov et al)

$$U(x|y) = \sum_p \mathcal{D}(x_p, y_p) + \sum_{(p,q)} V_c(x_p, x_q)$$

$x_p \in E$ finite 1D set

- **Idea :** go back to the ... binary case !

- **Constraints on the regularization functions**

V_c is a metric or a semi-metric

Semi-métric $\forall \alpha, \beta \in E^2$:

— $V_c(\alpha, \beta) = V_c(\beta, \alpha) \geq 0$

— $V_c(\alpha, \beta) = 0 \Leftrightarrow \alpha = \beta$

Metric if $V_c(\alpha, \beta) \leq V_c(\alpha, \gamma) + V_c(\gamma, \beta)$

Examples : troncated quadratic (semi-), Potts model,...

- **Limits**

approximate solution (local minimum) !

Extension to the multi-labels case : $\alpha - \beta$ swap

◦ **$\alpha - \beta$ swap definition**

• labeling = partitioning of the image $\mathbf{P} = \{P_l | l \in E\}$ with $P_l = \{p \in I | x_p = l\}$

• $\alpha - \beta$ swap : movement from the partitioning $\mathbf{P}$ to the partitioning $\mathbf{P'}$ such that $P_l = P'_l \; \forall l \neq \alpha, \beta$ (some labeled pixels α will be labeled β and vice-versa)

Extension to the multi-labels case : $\alpha - \beta$ swap

◦ **Optimization of the $\alpha - \beta$ swap by min cut**

— building of the graph using only the pixels labeled α or β as nodes ($S_{\alpha\beta}$)

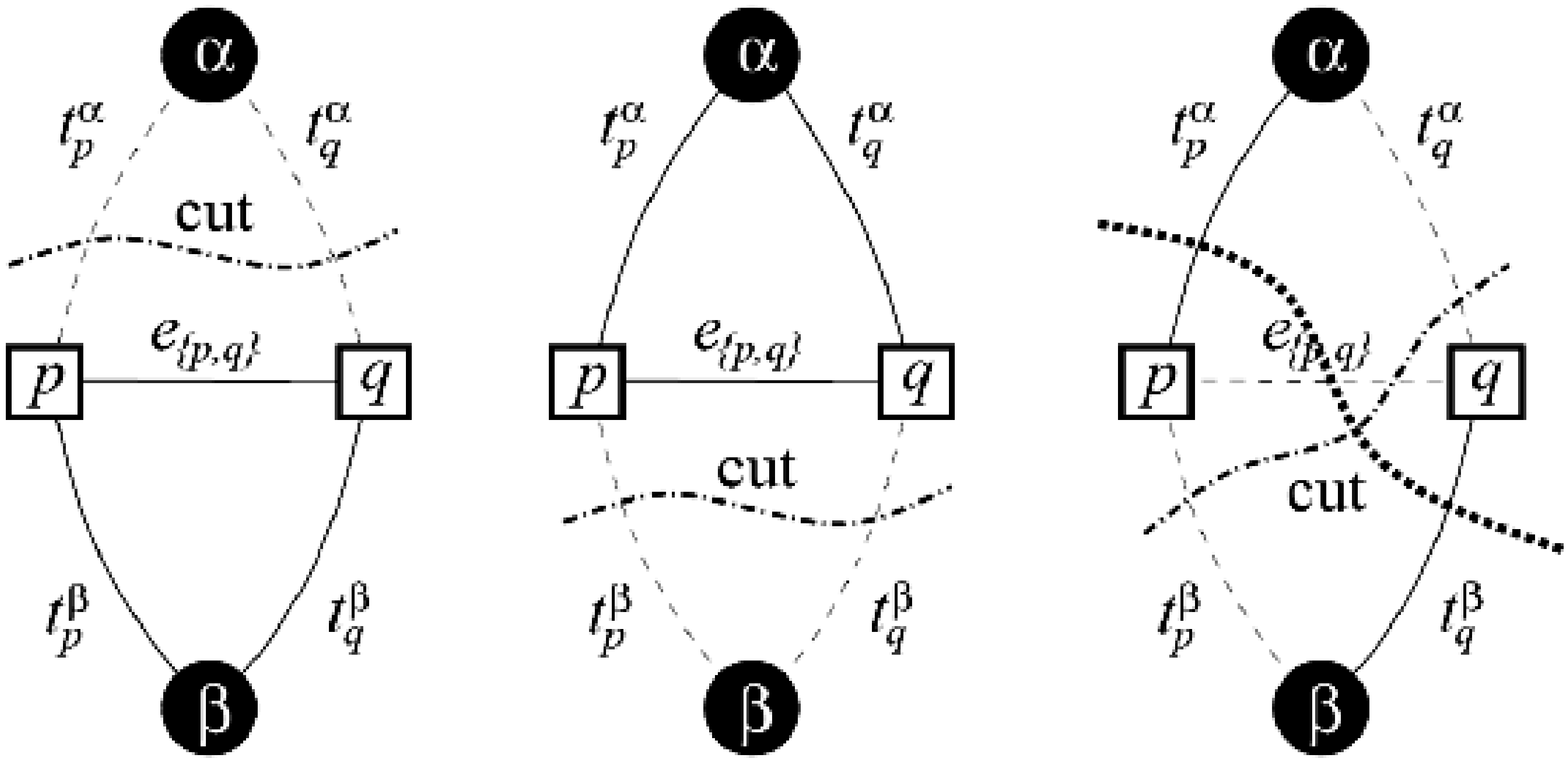
— addition of two terminal nodes one for α, the other one for β

— edges :

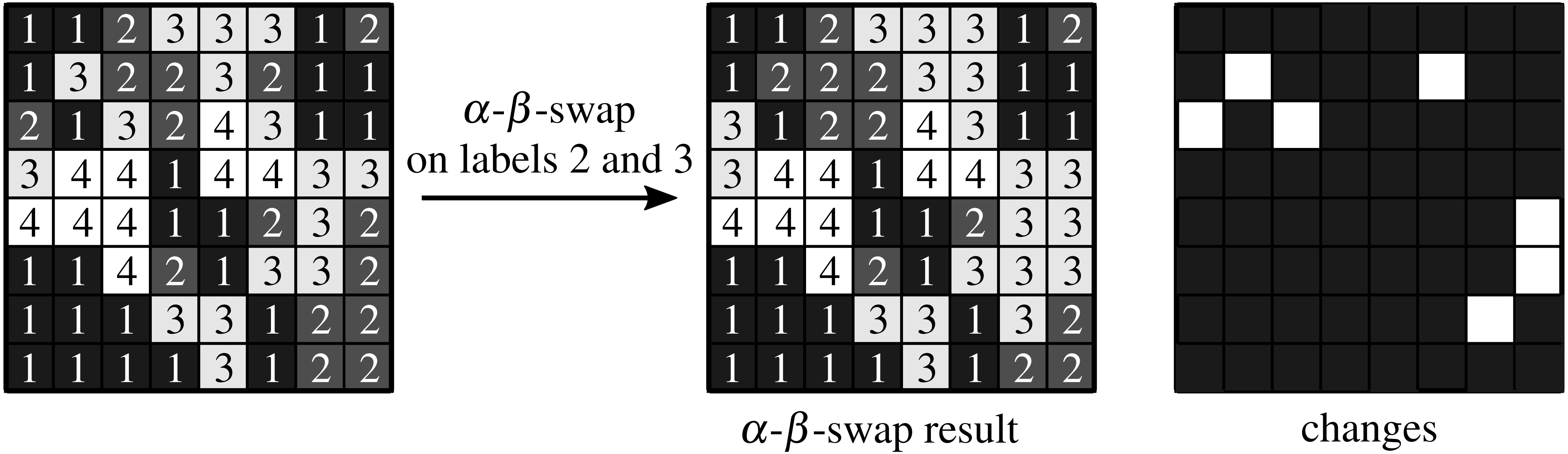
1. link with the α node with the weight :
   $w(p,\alpha) = \mathcal{D}(\alpha, y_p) + \sum_{q|q\in N_p,\;\; q\notin S_{\alpha\beta}} V_c(\alpha, x_q)$
2. link with the β node with the weight :
   $w(p,\beta) = \mathcal{D}(\beta, y_p) + \sum_{q|q\in N_p,\;\; q\notin S_{\alpha\beta}} V_c(\beta, x_q)$
3. if two pixels p et q are neighbors in 4-neighborhood and in $S_{\alpha\beta}$:
   edge with weight poids $w(p,q) = V_c(\alpha, \beta)$

— the final label of a pixel corresponds to the one of the cut edge

Extension of the multi-labels case : $\alpha - \beta$-swap



Extension of the multi-labels case : $\alpha - \beta$-swap



Extension of the multi-label case : α-expansion

◦ **Definition of α expansion**

• α-extension : mouvement d'une partition $\mathbf{P}$ à une partition $\mathbf{P}'$ telle que les pixels étiquetés à α le restent et d'autres peuvent prendre l'étiquette α

Extension of the multi-label case : α-expansion

- **Optimization of the α-expansion by min cut** (V_c must be a metric)
  - — building of the graph with all the pixels
  - — addition of two terminal nodes, one for α, the other one for $\overline{\alpha}$ (the current label)
  - — the final label is the one of the cut edge
  - — it is necessary to add an auxiliary node and edges to take into account all the situations !

Extension to the multi-labels case : α-expansion

◦ **Graph-cut representable**

α label 0

$\overline{\alpha}$ label 1 (pixel p keeps the current label $x_p = \overline{\alpha}(p)$, pixel q keeps the curent label $x_q = \overline{\alpha}(q)$)

Sub-modularity condition :

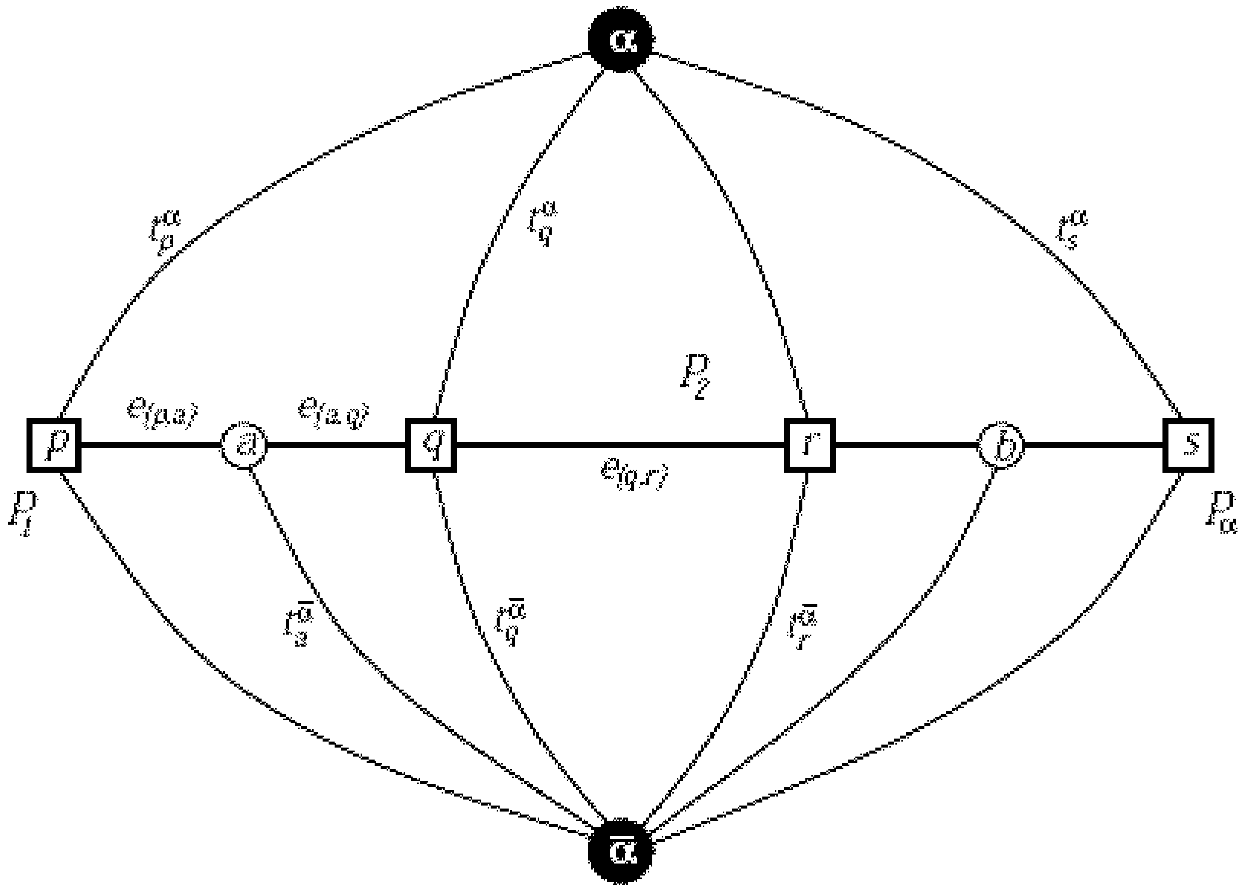
$V_c(0,0) + V_c(1,1) \leq V_c(0,1) + V_c(1,0)$

$\Rightarrow V_c(\alpha,\alpha) + V_c(\overline{\alpha}(p),\overline{\alpha}(q)) \leq V_c(\alpha,\overline{\alpha}(p)) + V_c(\overline{\alpha}(q),\alpha)$

$\Rightarrow 0 + V_c(x_p,x_q) \leq V_c(\alpha,x_p) + V_c(x_q,\alpha)$

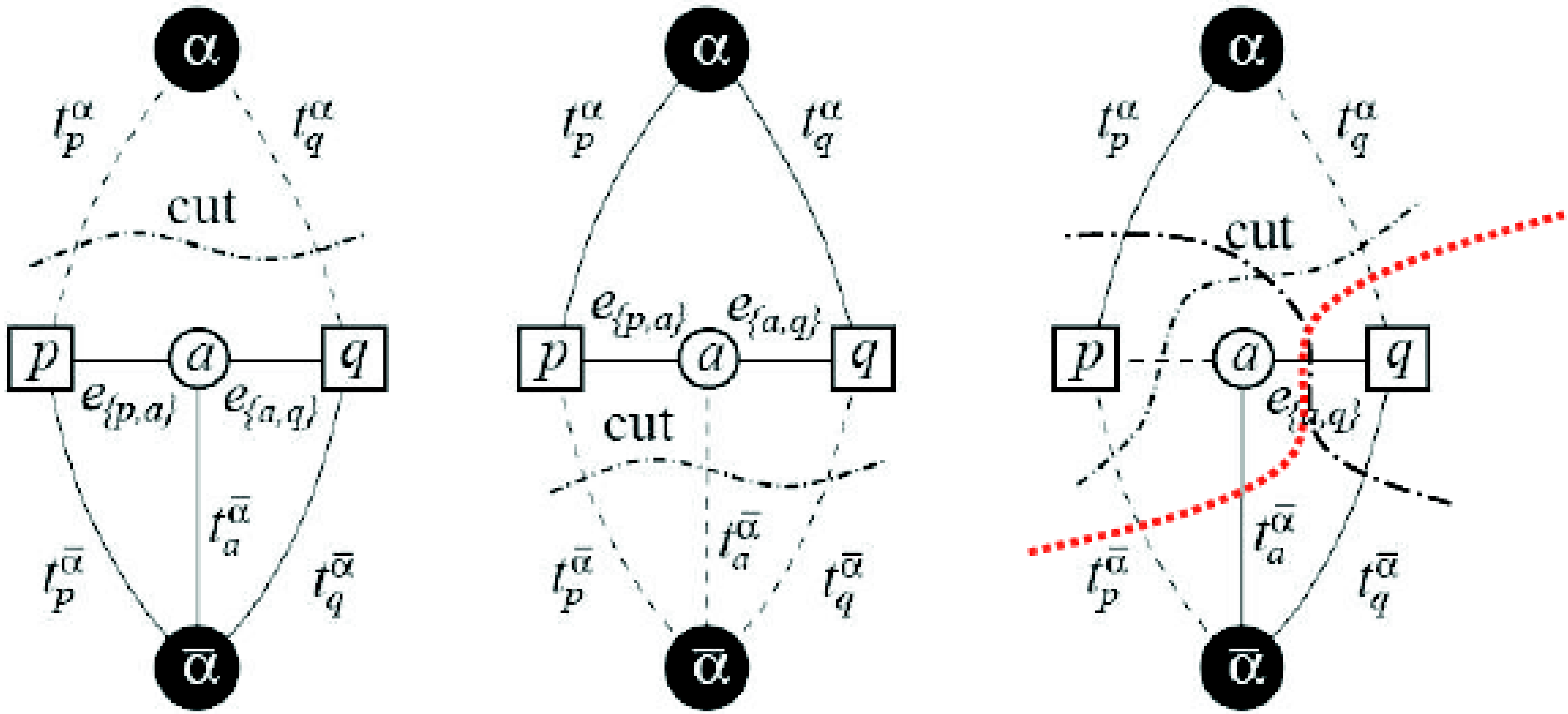
verified when V_c is a metric !

α-expansion move



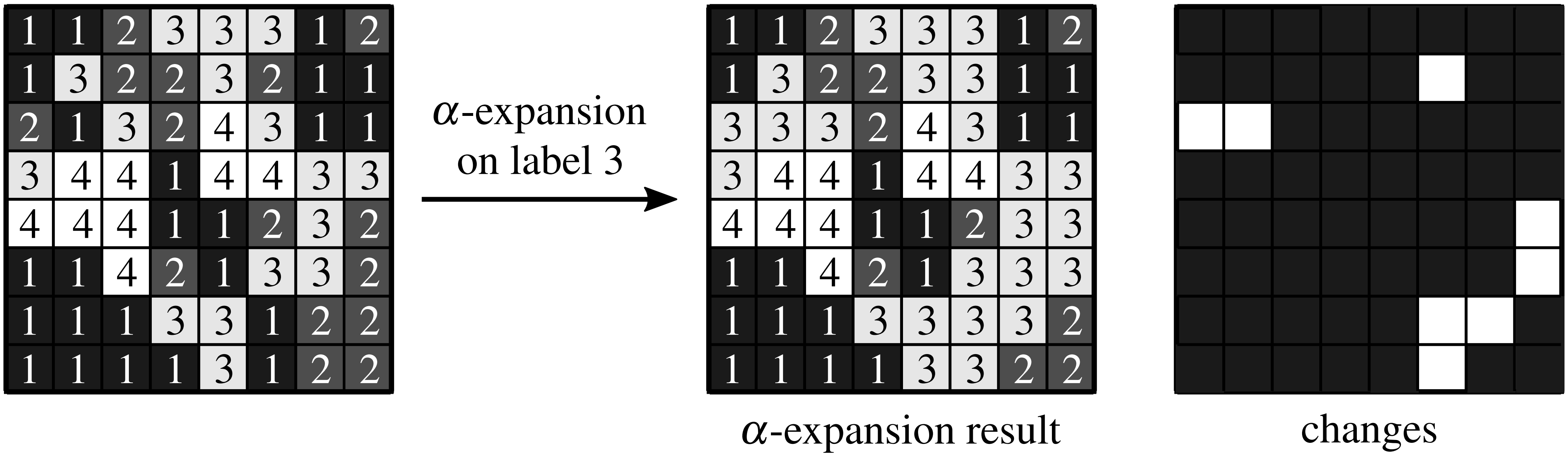
α-expansion move

$e(p,a) = V(l_p, \alpha)$, $e(a,q) = V(\alpha, l_q)$, $t_a^{\overline{\alpha}} = V(l_p, l_q)$

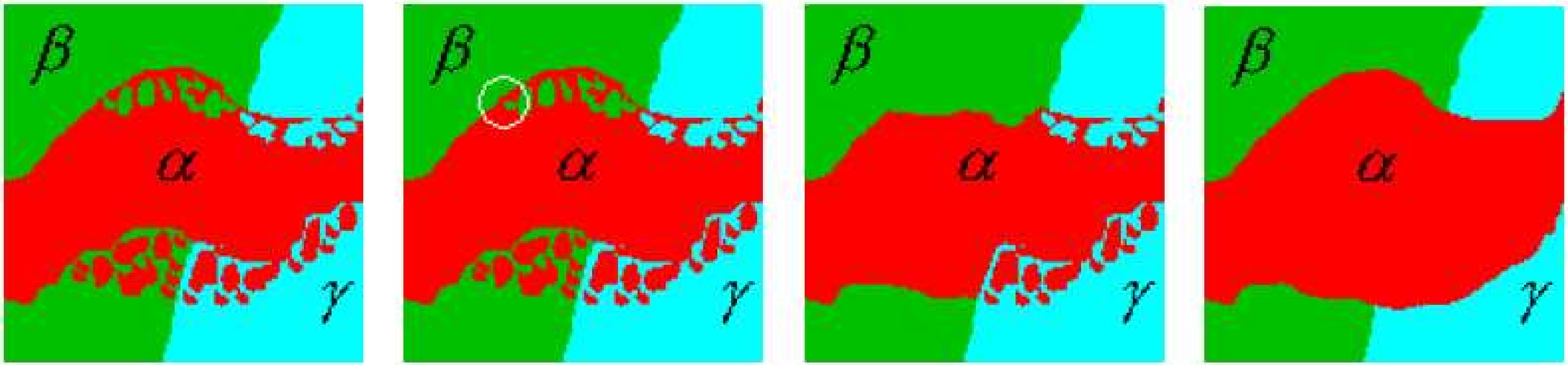


red cut is impossible because V is a metric $(V(l_p, \alpha) \leq V(l_p, l_q) + V(l_q, \alpha))$

α-expansion move



Illustrations (Boykov et al.)



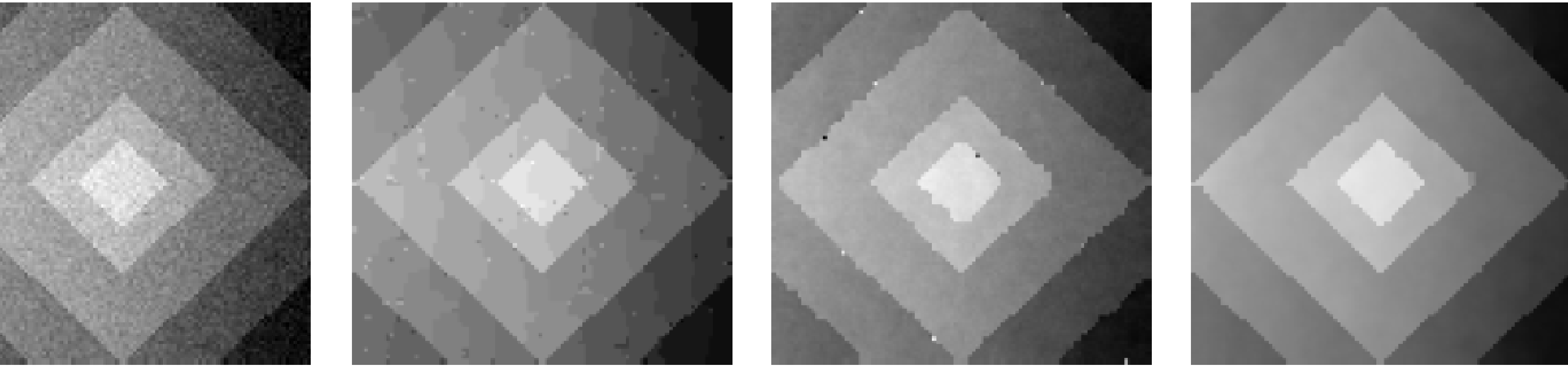
(a) initial labeling (b) standard move (c) α-β-swap (d) α-expansion

Results

- **Algorithms**
  - $\alpha - \beta$ swap : semi-metric potential
  - α-extension : metric potential
- **Performances**
  - converges towards a local minimum (some iterations)
  - much faster than simulated annealing
  - allows much bigger moves in the energetic landscape
  - theoretical results on the distance to the global minimum

Illustrations (Boykov et al.)

(E_2 Potts, E_1 troncated quadratic)



mond image (input) | Our method (E_2) | Annealing (E_1) | Our method (E_1)

Graph-cut based optimization methods

- **Introduction - reminders on graphs**
- **Binary case**
- **Approximated algorithms**
- **Exact algorithms**

Case of image restoration

◦ **Energy formulation**

$$U(x|y) = \sum_p \mathcal{D}(u_p, v_p) + \sum_{(p,q)} g(u_p - u_q)$$

data attachment term (likelihood) + regularization (prior)

◦ **Choice of the regularization potential**

— quadratic $(u_p - u_q)^2$
— troncated quadratic $min((u_p - u_q)^2, k)$
— Phi-fonction (conditions on the derivatives)
— Total variation (continuous domain $\int_\Omega |\nabla u|$)

restoration case

$$U(u|v) = \sum_p \mathcal{D}(u_p, v_p) + \beta \sum_{(p,q)} g(u_p - u_q)$$

data attachment term (likelihood) + regularization (prior)

◦ **Total Variation minimization** $\int_\Omega |\nabla u|$

Anisotropic model :

$$||\nabla u|| = \sqrt{(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2}$$

$$||\nabla u|| \approx |\frac{\partial u}{\partial x}| + |\frac{\partial u}{\partial y}|$$

$$||\nabla u|| \approx |u(i+1, j) - u(i, j)| + |u(i, j+1) - u(i, j)|$$

corresponds to $g(u_p - u_q) = |u_p - u_q|$

corresponds to a L_1 norm on the gradient

Restoration case - exact solution (Ishikawa)

V set of pixels, L set of labels

◦ **Hypotheses on g**

g is a convex function (on integers)

◦ **Method**

- Graph building
  - — nodes : $X = V \times L \cup \{s, t\}$ (n_{pi} node of pixel p for label i) ;
  - — edges : from s to all the nodes pixels-first label, then from all the nodes pixels-label i to nodes pixels-label i+1, etc.
  - — weights of edges "in column" : $c(s, n_{p(L-1)}) = \mathcal{D}(L, y_p)$, $c(n_{p(i+1)}, n_{pi}) = \mathcal{D}(i, y_p)$, $c(n_{p1}, t) = \mathcal{D}(1, y_p)$ ($c(n_{pi}, n_{p(i+1)}) = +\infty$ to hinder loops in the cut)

Restoration case - exact solution (Ishikawa)

regularization term : penalty weights

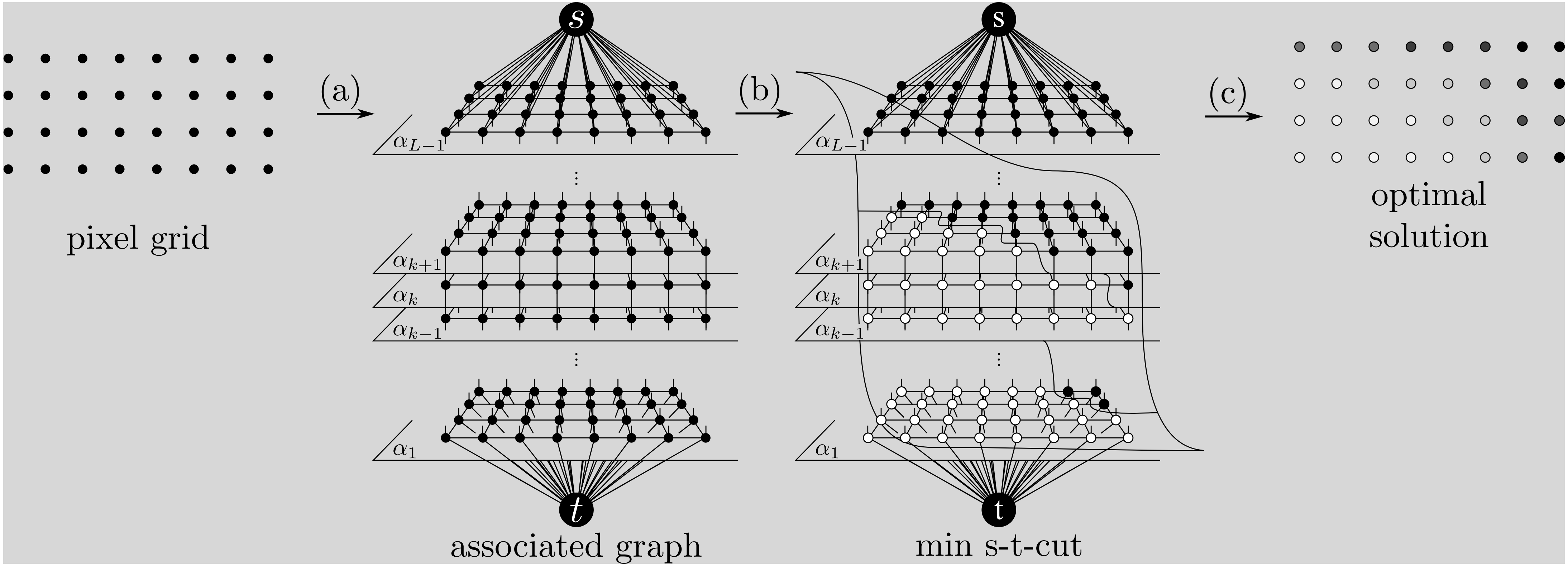
• Simplified case for TV regularization :

only “horizontal” edges with weight $\beta \Rightarrow g(x_p - x_q) = \beta |x_p - x_q|$

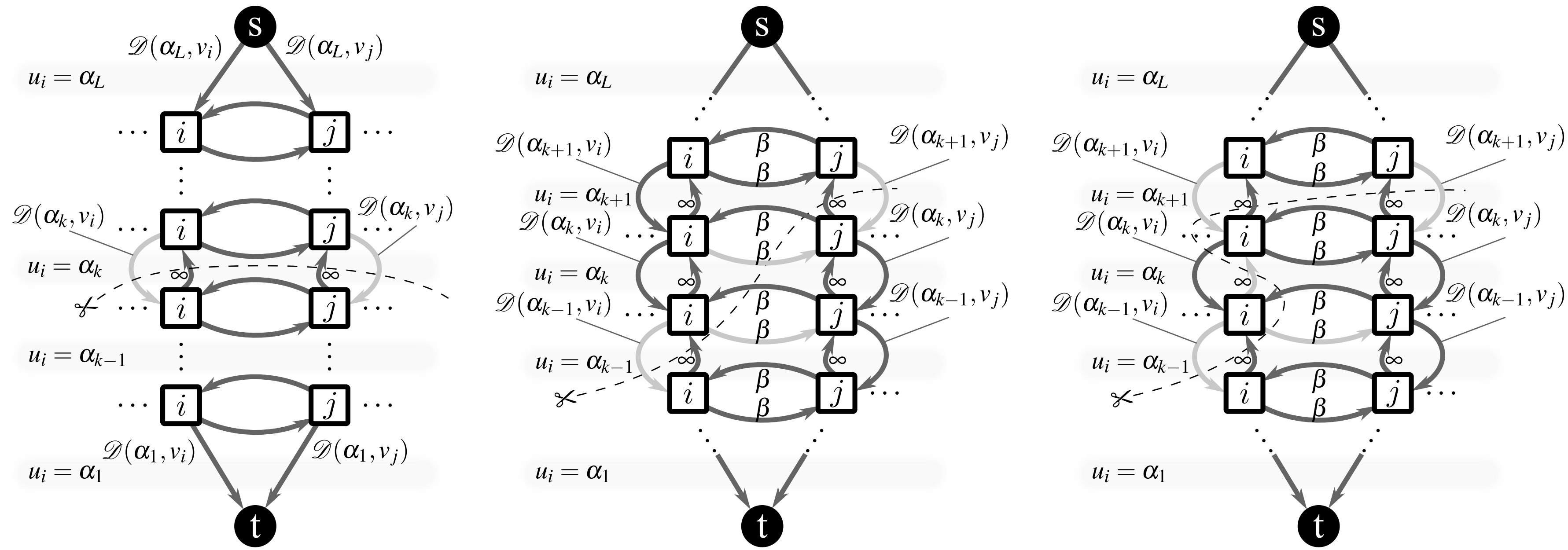
• General case :

set of edges linking pixel-nodes cut by the cut

Restoration case - exact solution (Ishikawa)



Restoration case - exact solution (Ishikawa)



$\cdots + \mathscr{D}(\alpha_k, v_i) + \mathscr{D}(\alpha_k, v_j) + 0 + \cdots$

(d)

$\cdots + \mathscr{D}(\alpha_{k-1}, v_i) + \beta + \beta + \mathscr{D}(\alpha_{k+1}, v_j) + \cdots$

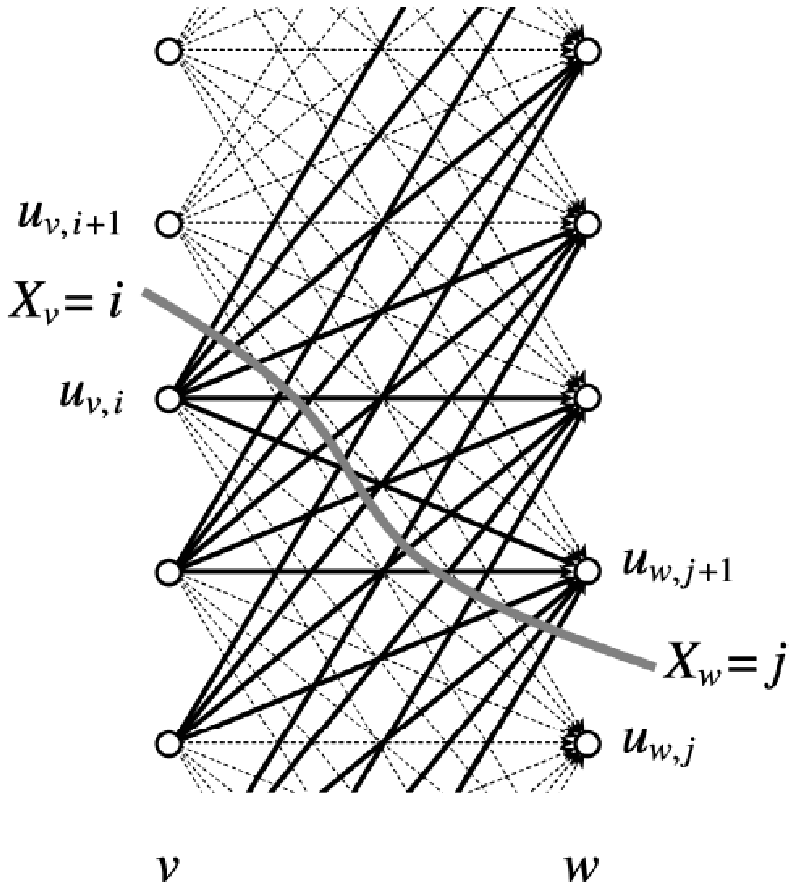
(e)

$\cdots + \mathscr{D}(\alpha_{k-1}, v_i) + \mathscr{D}(\alpha_{k+1}, v_i) + \beta + \infty + \mathscr{D}(\alpha_{k+1}, v_j) + \cdots$

(f)

Restoration case - exact solution (Ishikawa)

◦ **Cas général**



Restoration case - exact solution (Ishikawa)

◦ **Penalty term of the cut** :

$$g(i,j) = \sum_{a=1}^{i} \sum_{b=j+1}^{k} c(n_{va}, n_{wb}) + \sum_{a=i+1}^{k} \sum_{b=1}^{j} c(n_{wb}, n_{va})$$

◦ **Proposition :** if $g(i,j)$ defined as the sum of the capacities of neighboring pixels depends only of $i-j$, $g(i,j) = \tilde{g}(i-j)$ then $\tilde{g}$ is necessarily convex.

Reciprocally if g is convex then the edge capacities (for the regularization term) can be defined by :

$$c(n_{vi}, n_{wj}) = \frac{\tilde{g}(i-j+1) - 2\tilde{g}(i-j) + \tilde{g}(i-j-1)}{2}$$

the capcity becomes 0 for differences high enough between gray levels

NB : no constraint on the data attachment term

Restoration case - exact solution (Darbon, Sigelle 2006)

$$U(x|y) = \sum_p \mathcal{D}(x_p, y_p) + \sum_{(p,q)} w_{pq}|x_p - x_q|$$

◦ **Principle**

Decomposition of x on the level sets (thresholded versions of x)

⇒ reformulation using binary Markov random fields

⇒ reconstruction formula under certain hypotheses

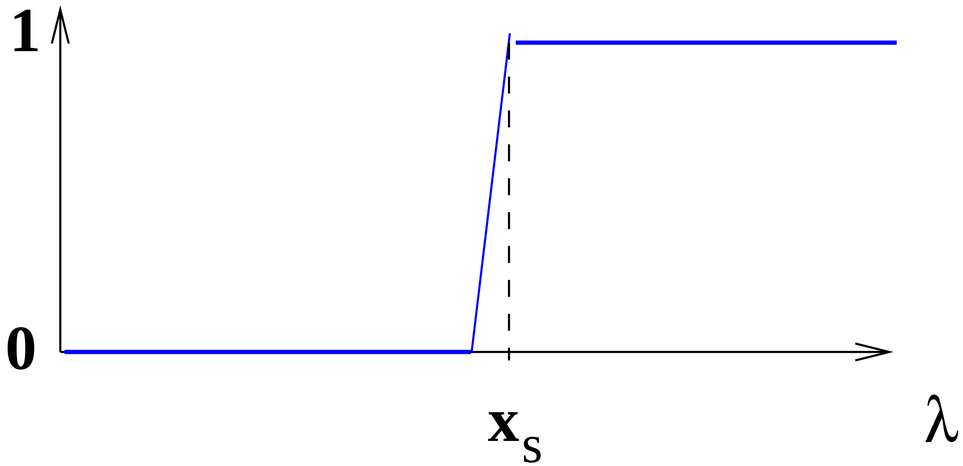
Level set decomposition

◦ **Definitions**

$x_s^\lambda = \mathbb{1}_{x_s \leq \lambda}$

$x_s = \min\{\lambda / x_s^\lambda = 1\}$

$x^\lambda = \{x_s^\lambda\}$ set of level λ



Level set decomposition

◦ **Reformulation of the energy**

- Regularization term :

$$TV(x) = \sum_{\lambda=0}^{L-2} \sum_{(s,t)} w_{st} \; |x_s^\lambda - x_t^\lambda|$$

$$TV(x) = \sum_{\lambda=0}^{L-2} \sum_{(s,t)} w_{st} \left[(1 - 2x_t^\lambda) \; x_s^\lambda \; + \; x_t^\lambda \right]$$

- Data attachment term :

$$f(y_s|x_s) = g_s(x_s) = \sum_{\lambda=0}^{L-2} (g_s(\lambda+1) - g_s(\lambda)) \; \underbrace{\mathbb{1}_{\lambda < x_s}}_{(1-x_s^\lambda)} + g_s(0)$$

Level set decomposition

∘ **Reformulation of the energy using the level sets**

$$U(x|y) = \sum_{\lambda=0}^{L-2} E^{\lambda}(x^{\lambda})$$

$$E^{\lambda}(x^{\lambda}) = \sum_{(s,t)} w_{st} \left[(1-2x_t^{\lambda})x_s^{\lambda} + x_t^{\lambda}\right] + \sum_{s} \left(g_s(\lambda+1) - g_s(\lambda)\right)(1-x_s^{\lambda}) + g_s(0)$$

Optimization using the level sets

$E^\lambda(x^\lambda)$: binary field with ising model (ferro-magnetism)

Let $\hat{x}^\lambda$ be the global minimizer of $E^\lambda(x^\lambda)$ with fixed λ

To have $\{\hat{x}^\lambda\}_{0\leq\lambda\leq L-1}$ the global minimum of $U(x|y)$ we need that :

$$\hat{x}^\lambda \leq \hat{x}^\mu \qquad \forall \lambda < \mu$$

The optimal solution is then given by :

$$\forall s \qquad \hat{x}_s = \min\{\lambda / \hat{x}_s^\lambda = 1\}$$

Conditions on the energies and associated graphs

- **Convexity condition for the local conditional energies**
  — reconstruction property guaranteed by the separate optimizations of the level sets
  — very fast algorithm by dichotomy on the grey-level set
- **Levelable regularization with non convex likelihood**
  — reconstruction property guaranteed by the addition of a coupling term between the grey-levels $\sum_s \alpha H(x_s^\lambda - x_s^{\lambda+1})$
  — graph different from Ishikawa but of similar size

Examples of results (Darbon, Sigelle)

gaussian noise (L2+TV)

Examples of results (Darbon, Sigelle)

gaussian noise (L2+TV)

Examples of results (Darbon, Sigelle)

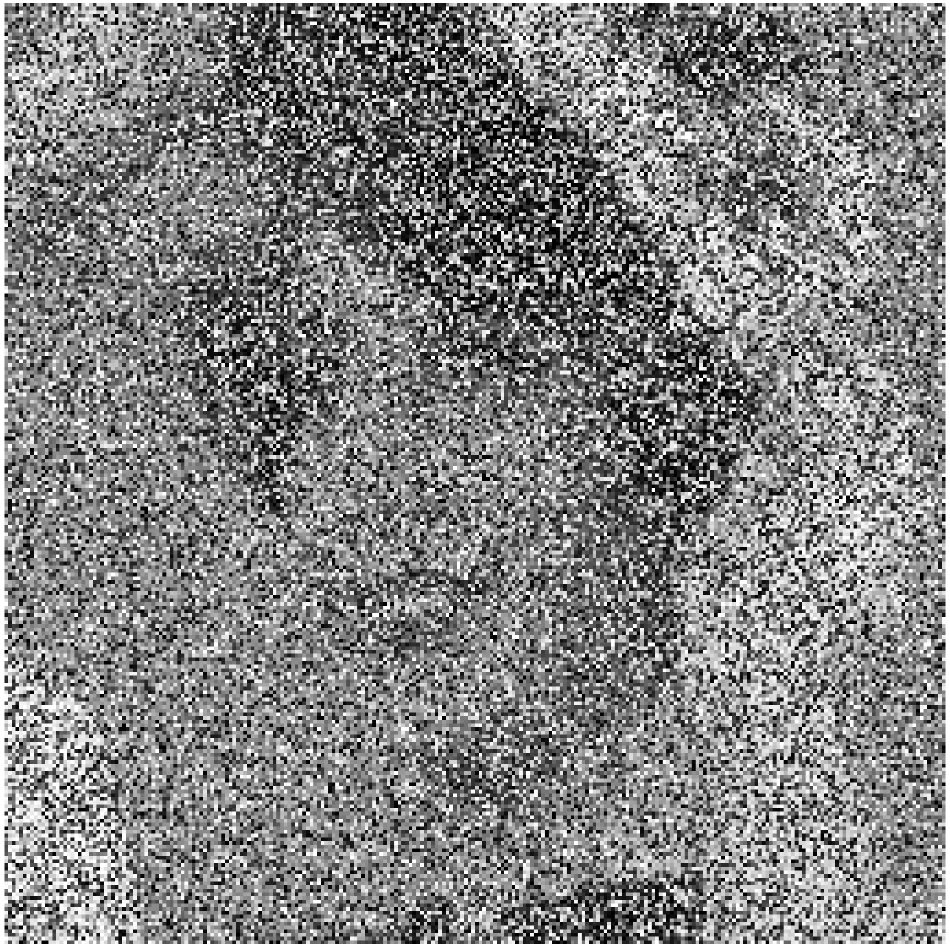
impulsive noise + TV

Examples of results (Darbon, Sigelle)

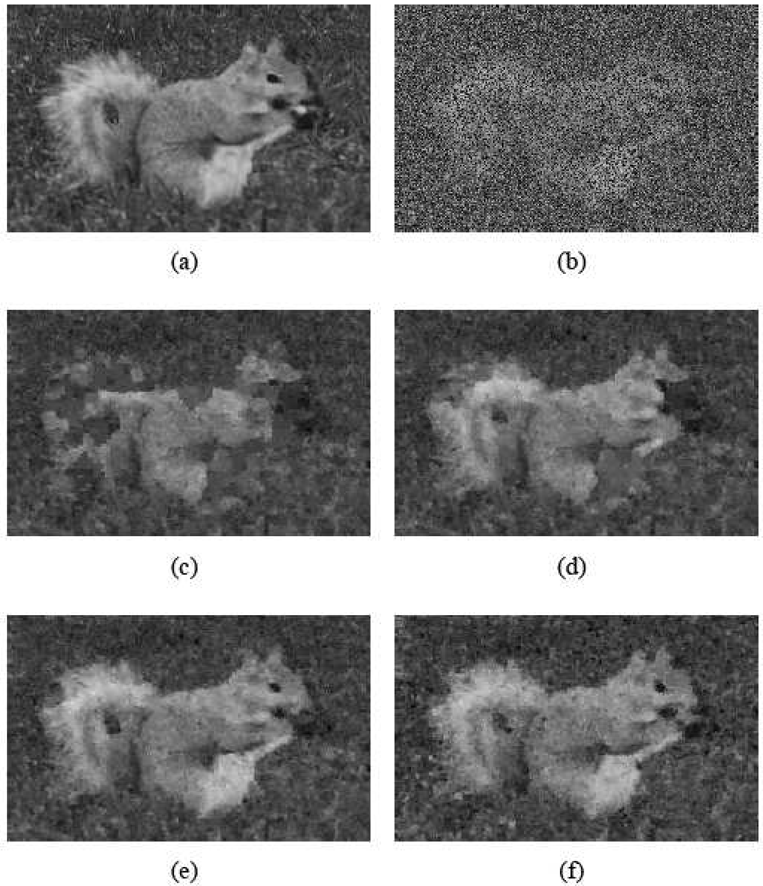
impulsive noise + TV

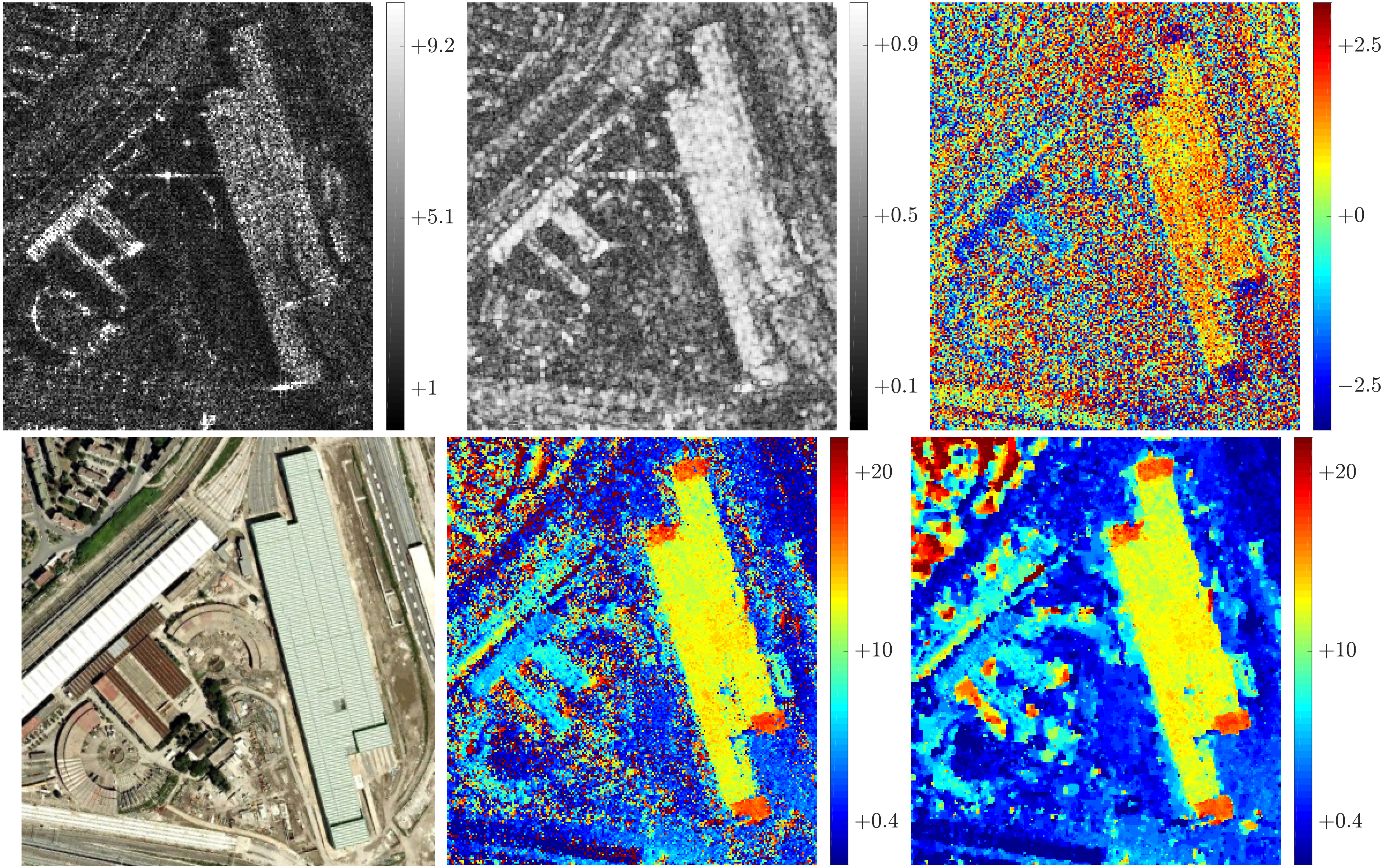
Examples de résultats (Darbon, Sigelle)

impulsive noise + TV




Examples - multi-labeling optimization




+9.2
+5.1
+1
+0.9
+0.5
+0.1
+2.5
+0
−2.5
+20
+10
+0.4
+20
+10
+0.4

Summary

Authors	Space	Régul.	Graph	Optimum
Greig et al.	binary	Ising	pixels + s,t	global
Kolmog. Zabih	binary	sub-modular	pixels + s,t	global
Freedman	binary	order 3		
Boykov et al.	multi	semi metric	sub-set +s,t	local*
		metric	pixels +s,t	local*
Ishikawa	multi	convex of $\|x_s - x_t\|$	S*n+s,t	global
Darbon et al.	multi	convex loc. en.	dichotomy	global
		levelable	S*n+s,t	global

Bibliography and figures

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