SAR Image Regularization with Graph-Cuts Based Fast Approximate Discrete Minimization.

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Salient features

- **SAR images**: coherent imagery in the radio wavelength range
  → speckle

- **High resolution interferometry**: amplitude and phase images
  small baseline → no phase unwrapping problem

- **Urban areas**: strong, coupled amplitude and phase discontinuities
  → joint amplitude-phase, TV-like (regularization) prior

- **Fast and memory costless graph cut algorithm**
  → efficient approximate optimization
SAR and InSAR image formation: MRF formulation

○ distribution of the amplitude: Rayleigh - Nakagami ($M$-look)

$$p(a_s \mid \hat{a}_s) = \frac{2M^M}{\Gamma(M)} \hat{a}_s^{2M-1} a_s^{2M-1} \exp \left( -\frac{Ma^2_s}{\hat{a}_s^2} \right) \quad U(a_s \mid \hat{a}_s) = M \cdot \left[ \frac{a^2_s}{\hat{a}_s^2} + 2 \log \hat{a}_s \right]$$

heavy-tailed speckle distribution $\rightarrow$ non-convex (quasi-convex) energy

○ distribution of the interferometric phase: Gaussian/uniform

$$p(\phi_s \mid \hat{\phi}_s) \quad U(\phi_s \mid \hat{\phi}_s)$$

$s \notin$ Shadows:

$$\frac{1}{\sqrt{2\pi}\hat{\sigma}_\phi} \exp -\frac{(\phi_s - \hat{\phi}_s)^2}{\hat{\sigma}_\phi^2} \quad \frac{(\phi_s - \hat{\phi}_s)^2}{\hat{\sigma}_\phi^2}$$

$s \in$ Shadows:

$$\frac{1}{2\pi} \quad 0$$
Independent and coupled TV regularization priors

- **Independent phase-amplitude TV**

  \[
  E(\hat{a}, \hat{\phi})_{(s,t)} = \beta_a |\hat{a}_s - \hat{a}_t| + \beta_\phi |\hat{\phi}_s - \hat{\phi}_t| 
  \]

- **Coupled phase-amplitude TV: first**

  \[
  E(\hat{a}, \hat{\phi})_{(s,t)} = \max(|\hat{a}_s - \hat{a}_t|, \gamma|\hat{\phi}_s - \hat{\phi}_t|)
  \]

- **Coupled phase-amplitude TV: second**

  \[
  E(\hat{a}, \hat{\phi})_{(s,t)}
  \begin{align*}
  s \notin \text{Shadows} \text{ and } t \notin \text{Shadows} & \quad \max(|\hat{a}_s - \hat{a}_t|, \gamma|\hat{\phi}_s - \hat{\phi}_t|) \\
  s \in \text{Shadows} \text{ and } t \notin \text{Shadows} \text{ and } \hat{\phi}_s \leq \hat{\phi}_t & \quad |\hat{a}_s - \hat{a}_t| + \gamma|\hat{\phi}_s - \hat{\phi}_t| \\
  s \in \text{Shadows} \text{ and } t \notin \text{Shadows} \text{ and } \hat{\phi}_s > \hat{\phi}_t & \quad |\hat{a}_s - \hat{a}_t| + 2 \gamma|\hat{\phi}_s - \hat{\phi}_t| \\
  s \in \text{Shadows} \text{ and } t \in \text{Shadows} & \quad |\hat{a}_s - \hat{a}_t| + \gamma \left(\hat{\phi}_s - \hat{\phi}_t\right)^2
  \end{align*}
  \]

  \[\Rightarrow \text{always convex!}\]
Global energy - convexity and submodularity

- **total energy (to be minimized)**

\[
E = \sum_{s} U(u_s | \hat{u}_s) + \beta \sum_{(s,t)} \psi(\hat{u}_s, \hat{u}_t) \quad u_s = (a_s, \phi_s) \quad \hat{u}_s = (\hat{a}_s, \hat{\phi}_s)
\]

- **submodular function of (two) binary variables**

\[
\chi(0,1) + \chi(1,0) \geq \chi(0,0) + \chi(1,1)
\]

- **applying to** \(\psi(\hat{u}_s + k_s \ d, \hat{u}_t + k_t \ d) \leftarrow \text{local displacement}\)

\[
\psi(\hat{u}_s, \hat{u}_t + d) + \psi(\hat{u}_s + d, \hat{u}_t) \geq \psi(\hat{u}_s, \hat{u}_t) + \psi(\hat{u}_s + d, \hat{u}_t + d)
\]

- **\(\psi\) depending on the difference \(\hat{u}_s - \hat{u}_t\)**

\[
\psi(\hat{u}_s - \hat{u}_t - d) + \psi(\hat{u}_s - \hat{u}_t + d) \geq 2\psi(\hat{u}_s - \hat{u}_t) \quad \forall d
\]

\(\rightarrow\) \(\psi\) convex
Proposed algorithm: local minimization

\[ \hat{u}^{(n+1)} = \arg \min_{\{k_s\}_{s \in S}} \sum_s U(u_s | \hat{u}_s^{(n)} + k_s \mathbf{d}) + \beta \sum_{(s,t)} \psi(\hat{u}_s^{(n)} + k_s \mathbf{d}, \hat{u}_t^{(n)} + k_t \mathbf{d}) \]
Graph cut energy minimization of binary images
Proposed algorithm: approximate global minimization

- **define** $d_i \in \mathcal{I}(d_i) \overset{\text{def}}{=} \{0, -d_i, +d_i\}^N / \{0, \ldots, 0\}$

1: for all $s \in S$ do
2: \[ \hat{u}_s^{(0)} \leftarrow \{L/2, \ldots, L/2\} \]
3: end for
4: $n \leftarrow 0$
5: for $i = 1$ to precision do
6: \[ d_i \leftarrow L/2^i \]
7: for all $d_i \in \mathcal{I}(d_i)$ do
8: \[ \hat{u}^{(n+1)} \leftarrow \arg\min_{\hat{u}^{(n+1)} \in \mathcal{I}(\hat{u}^{(n)})} E(\hat{u}^{(n+1)} \mid u) \]
9: \[ n \leftarrow n + 1 \]
10: end for
11: end for

- $\rightarrow$ exact for the convex case
Numerical simulations

- Noisy image
- Ground truth (4 regions a, b, c, and d with increasing constant level)
- Regularized image (ICM at convergence)
- Regularized image (alpha-expansion at convergence)
- Regularized image (proposed algorithm at convergence)

Energy vs. elapsed time (s)
### Comparison with other algorithms

\( N = \#\text{pixels} \quad L = \#\text{grey levels} \quad 8\text{-connectivity} \)

<table>
<thead>
<tr>
<th></th>
<th>( \alpha)-expansion</th>
<th>exact minimization</th>
<th>our</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum</td>
<td>local</td>
<td>global</td>
<td>local</td>
</tr>
<tr>
<td>graph size</td>
<td>( N ) nodes</td>
<td>( N \times L ) nodes</td>
<td>( N ) nodes</td>
</tr>
<tr>
<td></td>
<td>( 4 \times N ) arcs</td>
<td>( 5 \times N \times L ) arcs</td>
<td>( 4 \times N ) arcs</td>
</tr>
<tr>
<td>#cuts</td>
<td>( \propto L )</td>
<td>1</td>
<td>( \log_2(L/2) )</td>
</tr>
<tr>
<td>( D ) channels</td>
<td>( \propto L^D ) cuts</td>
<td>hardly possible</td>
<td>( (3^D - 1) \log_2(L/2) ) cuts</td>
</tr>
<tr>
<td>( D = 1 )</td>
<td>256</td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>( D = 2 )</td>
<td>65536</td>
<td></td>
<td>256</td>
</tr>
<tr>
<td>CPU time</td>
<td>22s (2 steps)</td>
<td>30s (ICM)</td>
<td>( \leq 3 )s</td>
</tr>
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Urban area InSAR image (2-look, $1200 \times 1200$)

amplitude

interferometric phase
Urban area InSAR image (followed)

filtered amplitude

filtered phase
True interferometric complex coherence

\[ \rho_s = \rho_s \exp i \phi_s = \frac{\sum_{i=1}^{W} z_{1i} z_{2i}^*}{\left( \sum_{i=1}^{W} |z_{1i}|^2 \cdot \sum_{i=1}^{W} |z_{2i}|^2 \right)^{1/2}} = \frac{\frac{1}{M} \sum_{i=1}^{M} z_{1i} z_{2i}^*}{\left( \frac{1}{M} \sum_{i=1}^{M} |z_{1i}|^2 \cdot \frac{1}{M} \sum_{i=1}^{M} |z_{2i}|^2 \right)^{1/2}} \]

\[ U(I_s^{(1)}, I_s^{(12)}, I_s^{(2)}, \varphi_s | a_s, \phi_s, \rho_s) = 4 \log a_s + \frac{I_s^{(1)} + I_s^{(2)} - 2 \cdot I_s^{(12)} \cdot \rho_s \cdot \cos(\phi_s - \varphi_s)}{a_s^2(1 - \rho_s^2)} \]

\[ U(a, \varphi) = \sum_{(s,t)} \max(\beta_a |a_s - a_t|, \beta_\phi |\phi_s - \phi_t|) \quad (no \ Shadows) \]
True complex coherence: Toulouse (1000 × 1000)
Conclusion

- **MRF model → joint phase-amplitude SAR image denoising**
  total variation minimization performs well in urban areas
  heavy-tailed amplitude distribution → non-convex minimization problem

- **optimization through graph-cut techniques**
  exact solution for small images
  approximate solution (”large moves”) for large images
  → efficient denoising algorithm in $D$ dimensions ($D = 1, 2$)

- **applications**
  - automatic classification
  - edge detection
  - joint regularization of optical and SAR images
Baseline interferometry

\[ \delta(\Delta \phi) \approx \frac{1}{4\pi} \frac{B}{h} \frac{\delta h}{\lambda} \]

Ex: \( h = 3 \text{ km} \) \( B = 1 \text{ m} \) \( \lambda = 3 \text{ cm} \) (9.5 GHz) \( \delta h = 10 \text{ m} \)