A New Paradigm for Asynchronous Communication: Detection, Isolation, and Coding

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Outline

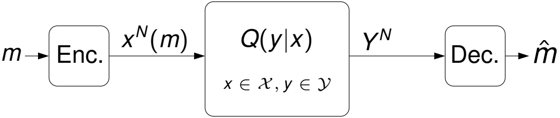
Information theory of synchronous channels

Studied for more than 60 years!

- A transmitter sends a message across an unreliable channel.
- The receiver tries to identify the sent message.

Shannon 1948: reliable identification is possible even if the channel is unreliable.

Classical probabilistic model



- message set $\{1, 2, \ldots, M\}$
- codebook $\{x^N(m)\}_{m=1,2,\ldots,M}$
- decoder (isolation rule): $Y^N \to \hat{m} \in \{1, 2, \ldots, M\}$

Performance criteria

- Rate: $R = \dfrac{\log M}{N}$ “information sent per channel use”
- error probability: $\max_{m=1,2,\ldots,M} \mathbb{P}(\hat{m} \neq m|m)$

R is “achievable” if, for any $\varepsilon > 0$, there is a rate R codebook and a decoder such that the error probability $\leq \varepsilon$.

Theorem (Shannon 1948)

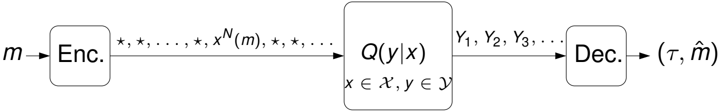
The maximum achievable rate is the “channel capacity”

$$C(Q) = \max_P I(PQ)$$
$$\triangleq \max_P \sum_x \sum_y P(x)Q(y|x) \log \frac{P(x)Q(y|x)}{P(x)P_Y(y)}$$

Asynchronous communication

- What if the codeword is sent at an unknown time?
- How does Shannon's result generalize?

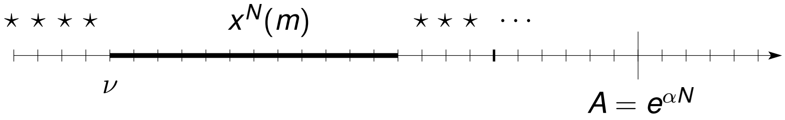
A simple model



- Transmitter: starts sending a codeword $x^N(m)$ at a random time $\nu \in [1, 2, \ldots, A]$. $A =$ "asynchronism level".
- Receiver:
  - within $[\nu, \nu+1, \nu+N-1]$ observes a noisy version of $x^N(m)$
  - otherwise observes "noise"
  - decodes on the basis of a *sequential decoder*

Remark: "$\star \in \mathcal{X}$" is a parameter of the channel, it is given to the encoder/decoder designer; only a *single* message is sent.

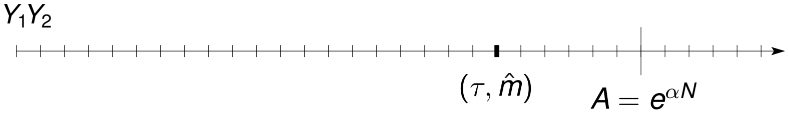
Transmitter:



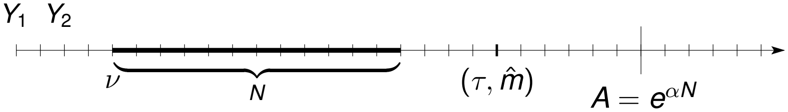
Channel:

$$Q(y|x),\ (x,y) \in \mathcal{X} \times \mathcal{Y}$$

Sequential decoder:



Natural rate definition: $R = \dfrac{\log M}{\mathbb{E}(\tau - \nu)^+}$ (instead of $\log M/N$)
with $\mathbb{E}(\tau - \nu)^+ = \max_m \mathbb{E}_m(\tau - \nu)^+$



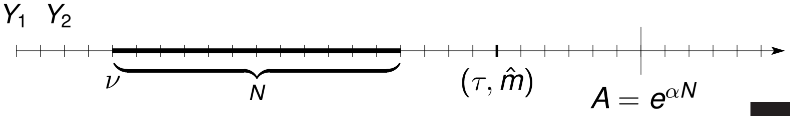
Performance criterion

(R, α) is achievable if, for any $\varepsilon > 0$, there exist length N codewords that

- operate under asynchronism level $A = e^{\alpha N}$
- achieve rate R
- yield error probability $\leq \varepsilon$.

Goal: capacity region

Find $R(\alpha) \triangleq \sup\{R : (R, \alpha) \text{ is achievable}\}$
Alternatively, find $\alpha(R) \triangleq \sup\{\alpha : (R, \alpha) \text{ is achievable}\}$



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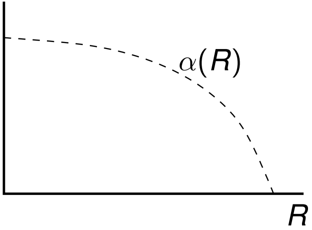
Hypothetical capacity region

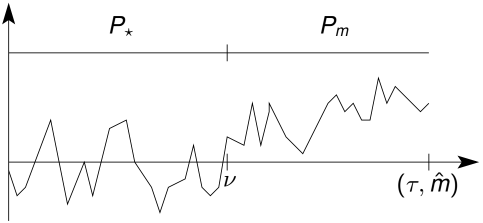


Asynchronous communication and the detection and isolation model (Nikiforov '95)

$$Y_1, Y_2, \ldots \text{ with } \begin{cases} Y_i \sim P_\star & i \leq \nu - 1 \\ Y_i \sim P_m & i \geq \nu \end{cases} \quad \text{with } m \in \{1, 2, \ldots, M\}$$

Goal:

find sequential test that minimizes $\mathbb{E}(\tau - \nu)^+$ while guaranteeing small error probability.

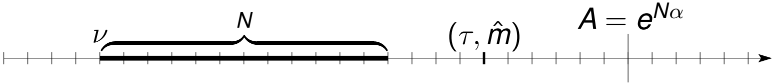


Asynchronous communication and the detection and isolation problem (cont.)

Differences:

- In the asynchronous communication problem, the change has limited duration: once the codeword is missed it is missed!
- The detection & isolation problem setting involves only decoding.
- The detection & isolation problem considers fixed number of hypothesis ($R = 0$).

Results



Theorem 1

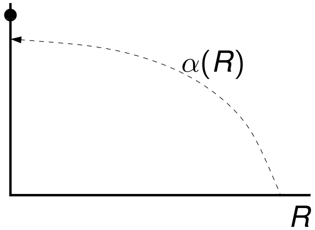
- If A grows sub-exponentially with N, there is no rate loss induced by lack of synchronization.
- The highest asynchronous exponent that can be achieved (regardless of the rate) is

$$\alpha(R=0) = \max_x D(Q(\cdot|x)||Q(\cdot|\star)) \,.$$

$\Rightarrow$ the interesting asynchronism regime is indeed $A = e^{N\alpha}$.

Theorem 2

In general, the capacity region is discontinuous at $R = 0$, i.e., $\lim_{R \downarrow 0} \alpha(R) \neq \alpha(R = 0)$



Theorem 3 (lower bound on the capacity region)

Given a channel Q, let $\alpha \geq 0$ and let P be a pmf such that for any pmf V at least one of the inequalities

$$D(V||(PQ)_Y) > \alpha$$

$$D(V||Q(\cdot|\star)) > \alpha$$

then the pair $(R = I(PQ), \alpha)$ is achievable. In other words, given P the pair

$$(R = I(PQ), \alpha = \min_V \max\{D(V||(PQ)_Y), D(V||Q(\cdot|\star))\}$$

is achievable.

Summary

- New asynchronous communication model at the edge between information theory (encoding) and sequential analysis (decoding).
- Upper and lower bounds on the capacity region.
- Open problems:
  - Tight characterization of the capacity region.
  - What if with some probability no message is sent?
  - Continuous time channels.

Ref.:

- A.T., V. Chandar, and G. Wornell, "Communication under strong asynchronism, " to appear in IEEE Transactions on Information Theory.
- A.T., V. Chandar, and G. Wornell, "Fundamental tradeoffs in one-shot asynchronous communication," preprint.