A Note on Bursty MAC

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In this note we establish a capacity result for the multiple access extension of the point-to-point model described in [1]. The corollary [1, p.1217] states that the point-to-point asynchronous capacity of a channel is given by

$$C(\alpha)=\max_{X:D(Y||Y_\star)\geq\alpha} I(X;Y).$$

This expression characterizes the largest rate, defined in the limit of large codeword length n, that can be accommodated when the level of asynchronism is $A_n=2^{\alpha n}$. Without loss of generality

$$\alpha\leq\alpha_o$$

where

$$\alpha_o=\max_x D(Y_x||Y_\star)$$

represents the synchronization threshold above which reliable communication is impossible even for a single user [2] .

Consider the simple MAC extension of the point-to-point setting proposed in [3, Remark 3, Section III]. Fix the asynchronism exponent

$$\alpha\in[0,\alpha_o].$$

Suppose there are

$$U=2^{\upsilon n(1-o(1))}\qquad(n\to\infty) \tag{1}$$

transmitters who communicate to a common receiver. Here υ denotes the *occupation* parameter of the channel and throught this note we assume that

$$0\leq\upsilon\leq\alpha.$$

The reason for this is that even though there are exponentially many users, in this regime the number of user-collisions can still be neglected. As a consequence, the multi-user setup reduces to the single user setup since interference can be entirely ignored.

The messages arrival times $\{\nu_1,\nu_2,\ldots,\nu_U\}$ at the transmitters are jointly independent and uniformly distributed over

$$\{1,\ldots,A_n\}$$

with

$$A_n=2^{\alpha n}$$

as before. Communication takes place as in the point-to-point case, each user uses the same codebook, and transmissions start at the times $\{\nu_1,\nu_2,\ldots,\nu_U\}$. Whenever a user tries to access the channel while it is occupied, the channel outputs random symbols, independent of the input (collision model).

The receiver decodes according to U stopping times/decoding functions

$$(\tau_1,\phi_1),(\tau_1+\tau_2,\phi_2)\ldots,(\tau_1+\tau_2+\ldots+\tau_U,\phi_U)$$

to output U message estimates

$$\hat m_i=\phi_i(Y_1^{\tau_1+\ldots+\tau_i})\quad i=1,\ldots,U.$$

A rate tuple $\{R_1,R_2,\ldots,R_U\}$ is achievable if the receiver can recover all except perhaps a vanishing fraction of the sent messages. In other words if

$$|\{m_1,m_2,\ldots,m_U\}\cap\{\hat m_1,\hat m_2,\ldots,\hat m_U\}|=(1-o(1))U$$

with probability $1-o(1)$.

Theorem 1 (Anonymous senders)**.** *Fix the asynchronism exponent* $0\leq\alpha\leq\alpha_o$ *and the occupancy exponent* $0\leq\upsilon\leq\alpha$. *Then the MAC capacity region is given by the hypercube*

$$R_i\leq C(\alpha,\upsilon)\overset{def}{=}\max_{X:D(Y||Y_\star)\geq\alpha-\upsilon} I(X;Y)\qquad i=1,2,\ldots,U$$

Proof: For the achievability the proof essentially reduces to the point-to-point setup. Let the number of users be

$$U=\frac{1}{n^2}2^{\upsilon n}.$$

The coding scheme is based on a random common codebook among users and sequential typicality as in [1, Proof of Theorem 1].The only difference is that now the receiver reiterates U times the sequential typicality decoding procedure. Analysis is essentially straightforward and reveals that the

$$D(Y||Y_\star)\geq\alpha-\upsilon$$

constraint refers to the probability of false-alarm with U users. Compared to the single user case, exponentially many users in fact decreases the per-user probability of false-alarm since the pure noise period is decreased from $2^{\alpha n}$ to $2^{\alpha n-\nu(n-o(1))}$. The converse argument follows [1, Proof of Theorem 1]. ■

In the previous setup, the receiver is not required to identify the transmitters, only the messages. If user identity is required then capacity is given by:

Theorem 2 (Senders with identity)**.** *Fix the asynchronism exponent* $0\leq\alpha\leq\alpha_o$ *and the occupancy exponent* $0\leq\upsilon\leq\alpha$. *Then the MAC capacity region is given by the hypercube*

$$R_i\leq\frac{1}{1+\upsilon}C(\alpha,\upsilon)\qquad i=1,2,\ldots,U.$$

To see this simply note if users commuicate at message rate nR bits, the actual transmitted information is $nR/n(1+\upsilon)$ [3, Remark 3, Section III].

References

[1] V. Chandar, A. Tchamkerten, and D. Tse, "Asynchronous capacity per unit cost," *Information Theory, IEEE Transactions on*, vol. 59, no. 3, pp. 1213–1226, 2013.

[2] A. Tchamkerten, V. Chandar, and G. Wornell, "Communication under strong asynchronism," *Information Theory, IEEE Transactions on*, vol. 55, no. 10, pp. 4508–4528, 2009.

[3] A. Tchamkerten, V. Chandar, and G. Caire, "Energy and sampling constrained asynchronous communication," *Information Theory, IEEE Transactions on*, vol. 60, no. 12, pp. 7686–7697, Dec 2014.