Asynchronous Capacity per Unit Cost

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Outline

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Cost of synchronization

- Continuous communication (e.g., video, voice): negligible
- Bursty communication (e.g., sensor networks): ?

What is the minimum energy to send one bit asynchronously, equivalently, what is the asynchronous capacity per unit cost?

Energy-limited synchronous communication

Studied by Gallager and Verdù late 80's

- Memoryless channel: $\mathcal{X} \xrightarrow{Q(y|x)} \mathcal{Y}$
- Each $x \in \mathcal{X}$ has a cost $k(x) \geq 0$
- B bits of information available to the transmitter at time one
- Transmitter sends information from time one
- Receiver decodes at fixed time

Energy-limited synchronous communication (cont.)

Capacity per unit cost (Verdù)

$$\mathbf{R} = \frac{B}{k(\text{codeword})} \qquad \mathbf{C}_{\text{syn}} = \sup\{\mathbf{R} : \ \mathbf{R} \text{ is achievable}\}$$

- If there is a zero cost symbol:

$$\mathbf{C}_{\text{syn}} = \max_{x \in \mathcal{X}} \frac{D(Y_x || Y_0)}{k(x)} , \quad Y_x \sim Q(\cdot|x)$$

- General:

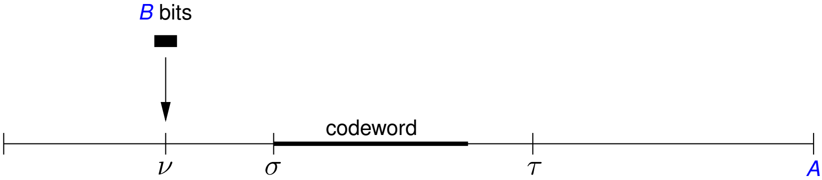
$$\mathbf{C}_{\text{syn}} = \max_{X} \frac{I(X;Y)}{\mathbb{E}(k(X))}$$

Outline

Energy-limited Asynchronous communication

- Information available at a random time at the transmitter
- Transmitter chooses when to start sending information
- Outside information transmission period, the transmitter stays idle and the receiver observes noise.
- Receiver decodes without knowing information arrival time

Model

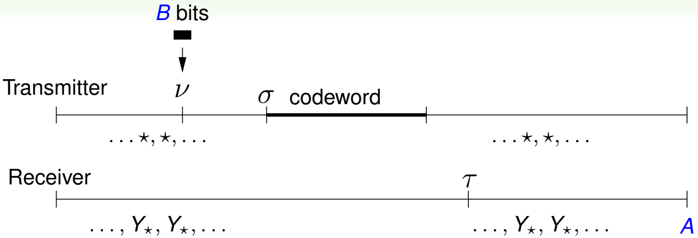


Transmitter

- B-bit message is revealed at random time $\nu \in [1, A]$
  $A =$ asynchronism level
- Starts sending codeword at time $\sigma = \sigma(\nu, \text{message})$

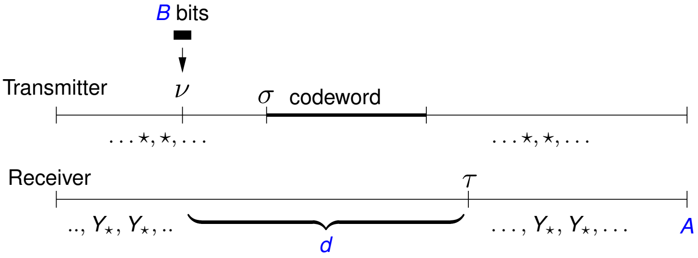
Receiver

- Decodes at a random time τ without knowing ν but knowing A



Channel

- $\mathcal{X} \cup \{\star\} \xrightarrow{Q(\cdot|\cdot)} \mathcal{Y}$
- $\mathcal{X}$: for codebook design, may or may not include $\star$
- $\star =$ idle input symbol, generates pure noise $Y_\star \sim Q(\cdot|\star)$
- Cost $k(x) \geq 0$ for all $x \in \mathcal{X}$



Delay constraint (for a meaningful problem):

- d: maximum decoding delay, i.e., $\mathbb{P}(\tau - \nu \leq d) \approx 1$ for all codewords
- Natural choice: $d = O(B)$, same as delay incurred in sync case

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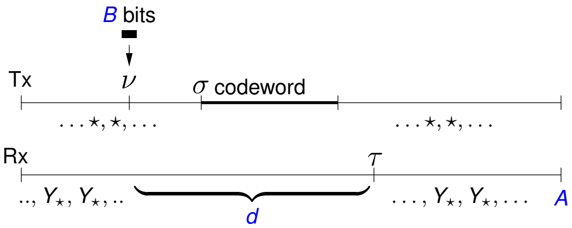
Small delay: $d = \text{sub-exp}(B)$



Theorem:

$$\mathbf{C}_{\text{asyn}}(\beta) = \frac{\mathbf{C}_{\text{syn}}}{1+\beta} \quad \text{where} \quad \beta = \frac{\log_2 A}{B}$$

i.e., Cost to transmit B bits asynchronously
$=$ cost to transmit $B + \log A$ bits synchronously
Achievable with $d = O(B)$

Why $\mathbf{C}_{\text{syn}} = (1+\beta)\mathbf{C}_{\text{asyn}}(\beta)$?

"$\geq$"

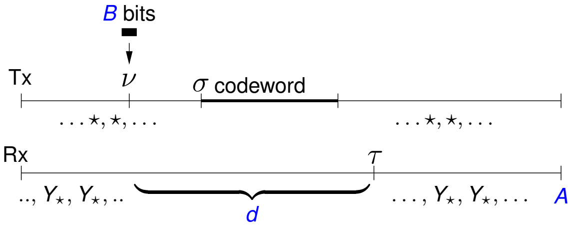
- Receiver can locate sent message within delay d
- $\Rightarrow$ additional $\log(A/d)$ bits implicitly transmitted through timing information
- $\Rightarrow$ cost to transmit B bits asynchronously is at least the cost to transmit $B + \log(A/d)$ bits synchronously
- Since $A = 2^{\beta B}$, $B + \log(A/d) \approx B + \log A$ bits
- Thus, $\mathbf{C}_{\text{syn}} \geq (1+\beta)\mathbf{C}_{\text{asyn}}(\beta)$

Why $\mathbf{C}_{\text{syn}} = (1+\beta)\mathbf{C}_{\text{asyn}}(\beta)$? (cont.)

"$\leq$"

Random uniform arrival makes asynchronous communication scheme look like a Pulse Position Modulation scheme, which is optimal for synchronous channels.

Large delay: $d = O(2^{\delta B})$, $\delta > 0$



Theorem: Cost to transmit B bits asynchronously
$=$ cost to transmit $B + \log(A/d)$ bits synchronously i.e.,

$$\mathbf{C}_{\text{asyn}}(\beta, \delta) = \mathbf{C}_{\text{asyn}}(\beta - \delta) \quad \text{where} \quad \beta = \frac{\log_2 A}{B}$$

Key for achievability: asynchronism reduction by communicating only at multiples of $2^{\delta B}$

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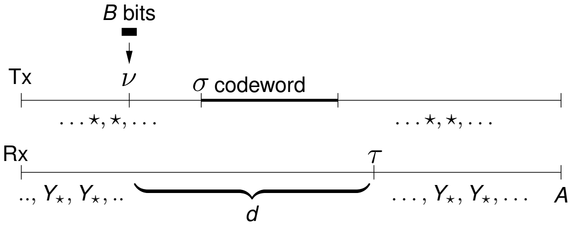
Small delay: $d = \text{sub-exp}(B)$

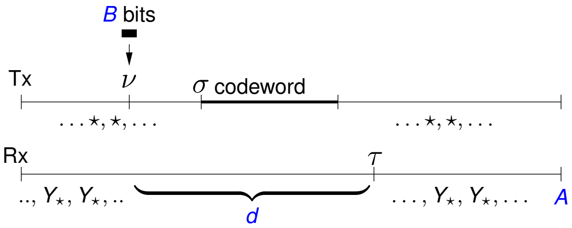


Theorem

$$\mathbf{C}_{\text{asyn}}(\beta) = \max_X \min \left\{ \frac{I(X;Y)}{\mathbb{E}(k(X))}, \frac{I(X;Y) + D(Y\|Y_\star)}{(1+\beta)\mathbb{E}(k(X))} \right\}$$

- sync term: async cannot help in increasing capacity (per unit cost)
- async term: quantifies how difficult it is for the receiver to discern a data carrying transmitted symbol from pure noise

Large delay: $d = O(2^{\delta B})$, $\delta > 0$



Theorem:

$$\mathbf{C}_{\text{asyn}}(\beta, \delta) = \mathbf{C}_{\text{asyn}}(\beta - \delta) \quad \text{for} \quad 0 < \delta < \beta$$

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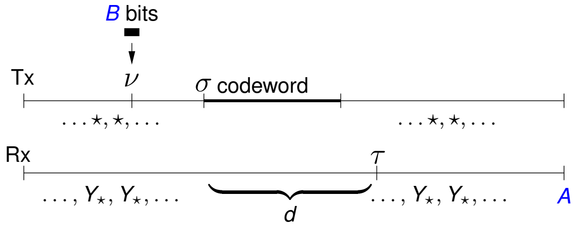
Conclusion

Most information theory research assumes continuous communication. But bursty transmission brings lots of new problems. The one we considered is just an example.

Example: Gaussian channel



- $x \to x + Z$, $Z \sim \mathcal{N}(0, N_0/2)$
- $\star = 0$
- $k(x) = x^2$

Theorem: $\mathbf{C}_{\text{asyn}}(\beta) = \dfrac{\log_2 e}{N_0(1+\beta)}$ where $\beta = \dfrac{\log_2 A}{B}$