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ASSIGNMENT 3

Exercise 1 (Conditional entropy)**.** Show that if $H(Y|X) = 0$, then Y is a function of X.

Exercise 2 (Mutual information)**.**

a. Let X be a uniform random variable over $\{1, 2, 3, 4\}$. Let

$$Y = \begin{cases} 0 \text{ if } X \text{ is odd} \\ 1 \text{ otherwise.} \end{cases} \quad Z = \begin{cases} 0 \text{ if } X \text{ is even} \\ 1 \text{ otherwise.} \end{cases}$$

Find $I(Y; Z)$.

b. We roll a fair die which has six sides. What is the mutual information between the top side and the one facing you?

Exercise 3 (Conditional mutual information)**.** Consider a sequence of n binary random variables $X_1, X_2, \cdots, X_n$. Each sequence with an even number of 1's has probability $2^{-(n-1)}$, and each sequence with an odd number of 1's has probability 0. Find the mutual informations $I(X_1; X_2)$, $I(X_2; X_3|X_1), \cdots, I(X_{n-1}; X_n|X_1, ..., X_{n-2})$.

Exercise 4 (Entropy and pairwise independence)**.** Let X, Y, Z be three binary Bernoulli($\frac{1}{2}$) random variables that are pairwise independent; that is, $I(X; Y) = I(X; Z) = I(Y; Z) = 0$.

a. Under this constraint, what is the minimum value for $H(X, Y, Z)$?

b. Give an example achieving this minimum.

Exercise 5 (Typicality)**.** To clarify the notion of typical set $A_\epsilon^{(n)}$, we will calculate the set for a simple example. Consider a sequence of i.i.d. binary random variables, $X_1, X_2, \cdots, X_n$, where the probability that $X_i = 1$ is 0.7.

a. Compute $H(X)$.

b. With $n = 8$ and $\epsilon = 0.1$, which sequences fall in the typical set $A_\epsilon^{(n)}$? What is the probability of the typical set? How many elements are there in the typical set?

Exercise 6 (AEP)**.** Let(X_i, Y_i) be i.i.d. $\sim p(x, y)$. Compute

$$\lim_{n\to\infty} \frac{1}{n} \log \frac{p(X^n)p(Y^n)}{p(X^n, Y^n)}.$$

Exercise 7 (Desintegration)**.** An entity of size 1 splits each second into two parts, with proportion of sizes having the following distribution:

$$\text{Proportion} = \begin{cases} (\frac{3}{4}, \frac{1}{4}) & \text{with probability } \frac{2}{5} \\ (\frac{2}{3}, \frac{1}{3}) & \text{with probability } \frac{3}{5} \end{cases}$$

At each time, the bigger part remains, and the smaller part will disapear. Thus, for example, a splitting in the first second may result in a part of size $\frac{3}{4}$. In the 2nd second, the size might reduce to $(\frac{3}{4}) \cdot (\frac{2}{3})$, and so on. How large, to first order in the exponent, is the remained part after n splitting?