Symétrie $\mathcal{PT}$, un concept unificateur pour la physique des dispositifs à gain

H. Benisty[1*],

[1]Laboratoire Charles Fabry, IOGS, Palaiseau, France

Collaboration: A. Lupu[2], and A. Degiron[2]

[2]IEF, Univ. Paris-Sud and CNRS, Orsay, France

H. Benisty, C. Yan, A. T. Lupu, and A. Degiron,
IEEE J. Lightwave Technol., vol. 30, pp. 2675-2683, 2012.

A. Lupu, H. Benisty and A. Degiron, Optics Express **21**, 21651 (2013)

H. Benisty and M. Besbes,
JOSA. B, vol. 29, pp. 818-826, March 29 2012.

H. Benisty et al. *Opt. Express*, ***19***, *18004, 2011*

OUTLINE

➪ **$\mathcal{PT}$–symmetry, Hamiltonians, and Gain-Loss (with reminder)**

➪ **Three flavours of PT-symmetries**

- With waveguides
- With resonators
- With gratings

➪ **Application with plasmonics** (with IEF : A. Lupu, A. Degiron)

- Losses are now causing a singularity !
- Switches
- Real life ? ("healing" of smoothed singularity)

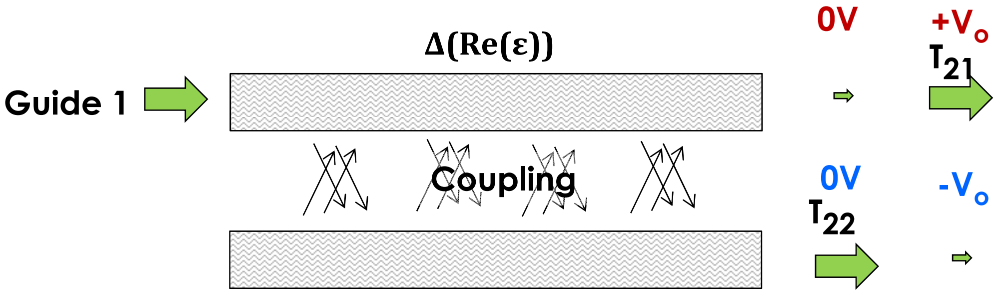
➪ **$\mathcal{PT}$–symmetry and (many) current photonic concepts**

- from coherent perfect absorbers to lasers

➪ **The brachistochrone problem (1696[!]-1990)**

- Relevance for fast Quantum computation

Reminder: ΔRe(ε) classical waveguide coupler



$$\cancel{i\hbar}\frac{\partial}{\partial z}\begin{pmatrix}A_1\\A_2\end{pmatrix} = [\boldsymbol{H}]\begin{pmatrix}A_1\\A_2\end{pmatrix}$$

0	→	1
1	→	0

$$\text{input} = \tfrac{1}{2}\begin{pmatrix}A\\A\end{pmatrix}+\tfrac{1}{2}\begin{pmatrix}A\\-A\end{pmatrix} \qquad \text{output}= \tfrac{1}{2}\begin{pmatrix}A\\A\end{pmatrix}e^{i\beta_+L}+\tfrac{1}{2}\begin{pmatrix}A\\-A\end{pmatrix}e^{i\beta_-L}$$

controlled by $(\beta_+ - \beta_-)L$

Initial motivation
generalized Quantum Mechanics

- Hermitian operator H → ♥ Real Eigenvalues
- Hermitian operator ← ♥ Real Eigenvalues

$$(\lambda - ig)(\lambda + ig) - \kappa^2 = 0$$
$$\lambda^2 = \kappa^2 - g^2$$

- Counter-example (Bender 1998)

$$\boldsymbol{H} = \begin{pmatrix}\hbar(\omega_1 + ig) & \kappa\\ \kappa & \hbar(\omega_2 - ig)\end{pmatrix}$$

$\mathcal{P}$

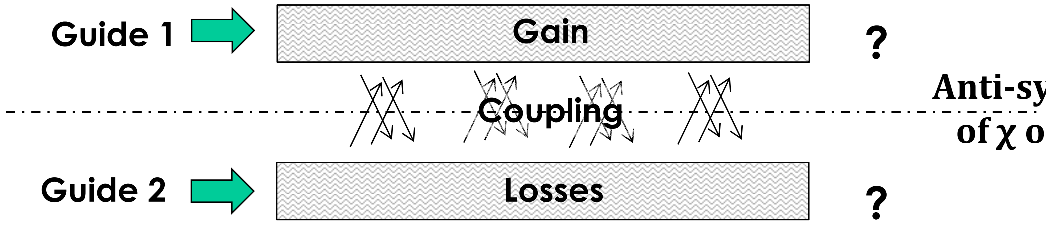
- $(\omega_1 = \omega_2 = 0)$ Real eigenvalues if : $g < \kappa$
- $\{1 \leftrightarrow 2\}$ & $\{t \leftrightarrow -t\}$ = 1 $\quad \mathcal{T}$
- $\mathcal{PT}\,H = H$

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

Coupled waveguides !

$$\not{i\hbar}\frac{\partial}{\partial z}\begin{pmatrix}A_1\\A_2\end{pmatrix} = [\boldsymbol{H}]\begin{pmatrix}A_1\\A_2\end{pmatrix}$$

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

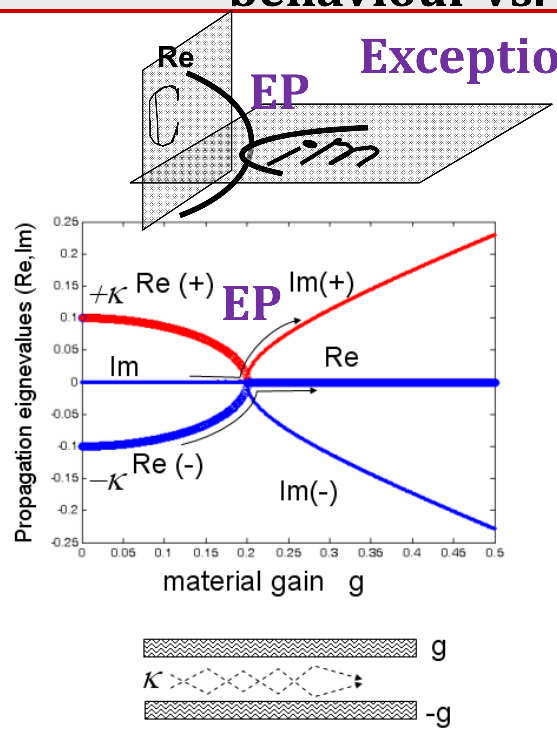


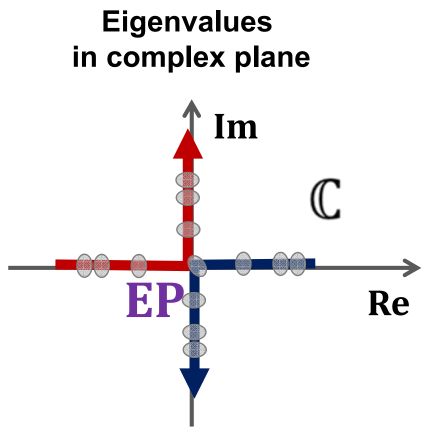
Anti-symmetry of χ or ε or g

$$\begin{pmatrix}(\boldsymbol{\beta_1}+\boldsymbol{ig}) & \boldsymbol{\kappa}\\ \boldsymbol{\kappa} & (\boldsymbol{\beta_2}-\boldsymbol{ig})\end{pmatrix}$$

Canonical eigenvalue behaviour vs. « gain-loss »

Exceptional Point



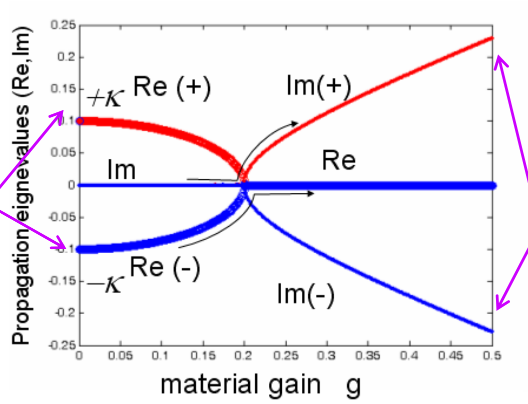


Steep derivatives of ∂(eigenvalue)/ ∂g

eigenstate behaviour vs. « gain-loss »

Symmetry breaking

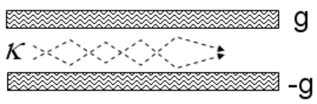
$$\frac{1}{\sqrt{2}}\begin{pmatrix} A \\ \pm A \end{pmatrix}$$



$$\begin{pmatrix} A \\ iB \end{pmatrix}$$

- Symmetry-breaking of eigenstates

$$\begin{pmatrix} iA \\ B \end{pmatrix}$$



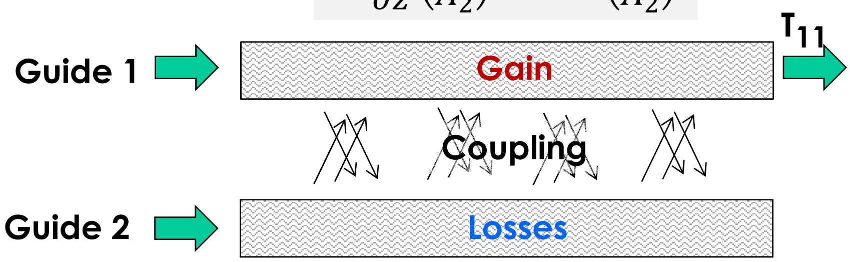
("winner-takes-all")

OUTLINE

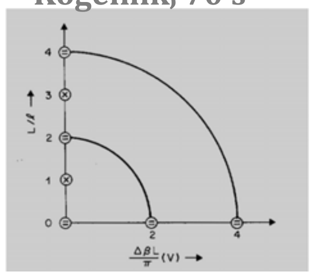
- ⇨ $\mathcal{PT}$–symmetry, Hamiltonians, and Gain-Loss
- ⇨ **Three flavours of PT-symmetries**
  - With waveguides
  - With resonators
  - With gratings
- ⇨ **Application with plasmonics** (with IEF : A. Lupu, A. Degiron)
  - Losses are now causing a singularity !
  - Switches
  - Real life ? ("healing" of smoothed singularity)
- ⇨ $\mathcal{PT}$–**symmetry and (many) current photonic concepts**
  - from coherent perfect absorbers to lasers
- ⇨ **The brachistochrone problem (1696[!]-1990)**
  - Relevance for fast Quantum computation

PT-symmetry with two waveguides

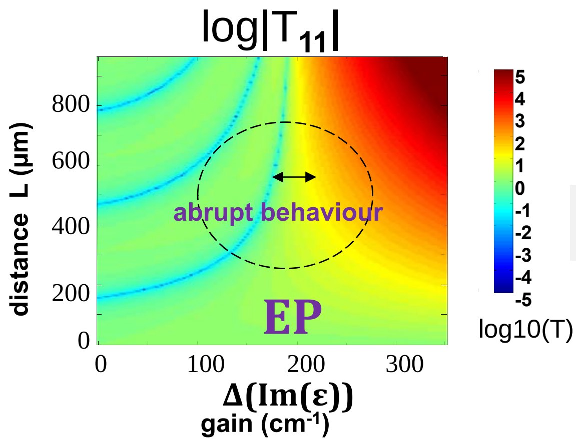
$$\cancel{i\hbar}\,\frac{\partial}{\partial z}\begin{pmatrix}A_1\\A_2\end{pmatrix} = [\boldsymbol{H}]\begin{pmatrix}A_1\\A_2\end{pmatrix}$$



Kogelnik, 70's

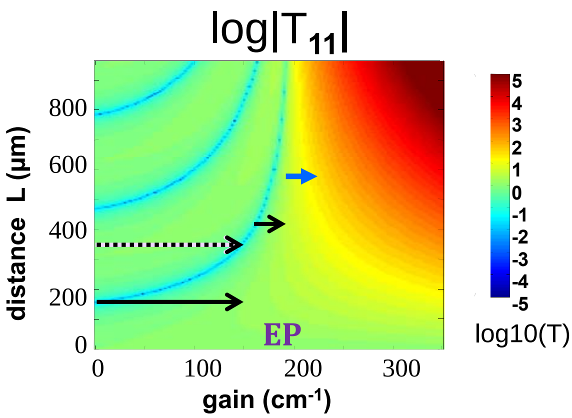


Δ(Re(ε))



In Optics : ~ Ctyroky 1996 ! Klaiman 2008; --Guo 2009; --Rüter 2010,...

Serves as switch or modulator



0→1
1→0

Differential gain enhancer

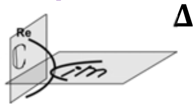
A → ***A=G**A*

large ∂*G*/ ∂*g*

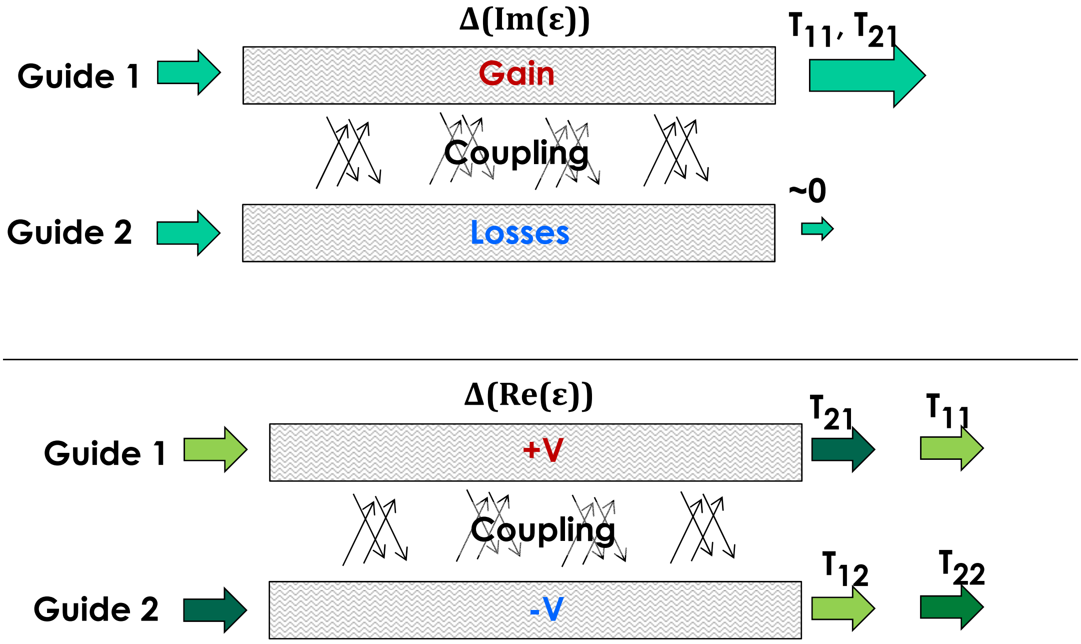
Δβ~0

Δβ~κ

Δβ~i*g*



Symmetry-breaking regime

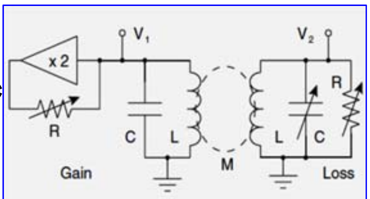


PT-symmetry in two resonators

Easier !

R*LC
+RLC

R*



R

FIG. 1. Electronic implementation of a $\mathcal{PT}$-symmetric dimer. The negative resistance gain element is provided by feedback from a voltage-doubling buffer. The coils are inductively coupled, and V_1 and V_2 provide access to the system variables.

Phys. Rev. A, vol. 84,
p. 040101, 2011.
(Kottos, Christodoules,..)

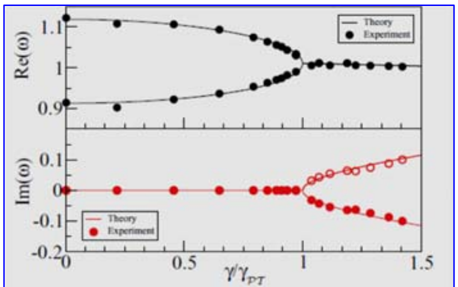
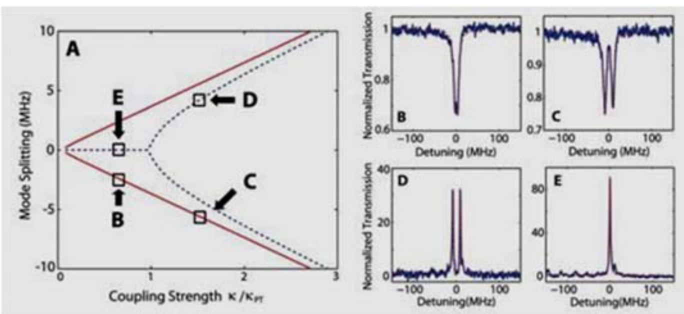
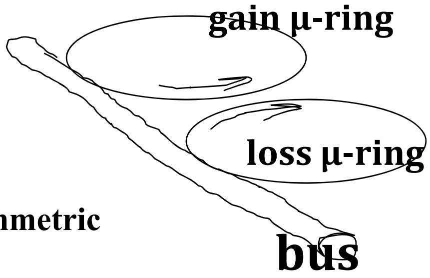


FIG. 2. (Color online) Parametric evolution of the experimentally measured eigenfrequencies vs the normalized gain and loss parameter γ/γ_{PT}. A comparison with the theoretical results of Eq. (4) indicate an excellent agreement. In all cases, we show only the Re(ω_η) > 0 eigenfrequencies. The open circles in the lower panel are reflections of the experimental data (lower curve) with respect to the Im(ω) = 0 axis.





Nonreciprocal light transmission in parity-time-symmetric whispering-gallery microcavities
Bo Peng et al. Arxiv 2013 (coor author Lan Yang @ ese.wustl.edu)

Laser studies (Wronskian attack ?)

PRL **108,** 173901 (2012)

M. Liertzer,[1,*] Li Ge,[2] A. Cerjan,[3] A. D. Stone,[3] H. E. Türeci,[2,4] and S. Rotter[1,†]

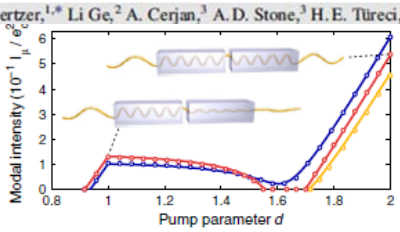
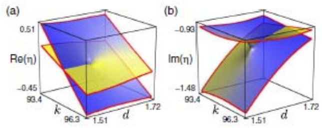
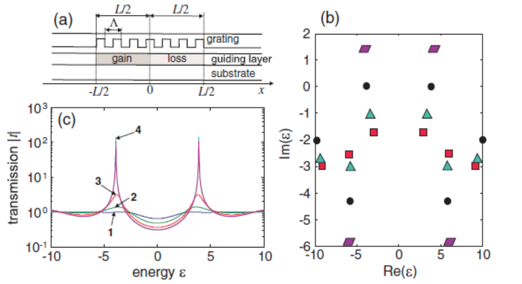


FIG. 1 (color online). Intensity output of a laser system consisting of two 1D coupled ridge lasers, each of length 100 μm with an air gap of size 10 μm and an (unpumped) index of refraction $n = 3 + 0.13i$. For $0 < d < 1$, the pump in the left ridge is linearly increased in the range $0 < D < 1.2$, and, for

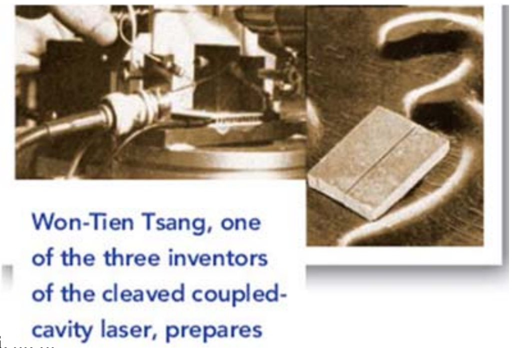


Stefano Longhi

PRL **105,** 013903 (2010) PHYSICAL REV



Non-Hermitian Dirac equation and its optical realization.—Let us consider the Dirac equation in one spatial



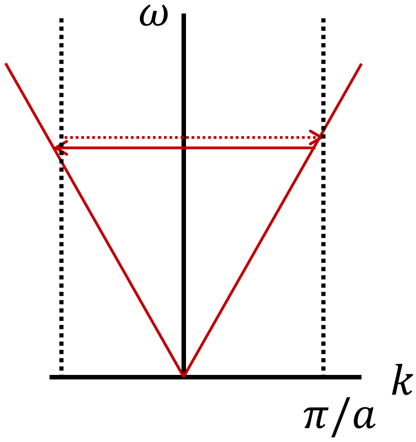
Bell Lab patent 1965
Tsang 1984
C3 laser
(Coupled cavity laser)

PT-symmetry with gratings

$$\Delta\varepsilon(x) \sim \Delta\varepsilon_1 \left[\ (\exp i\boldsymbol{G}x) \ + \ \cancel{\exp(-i\boldsymbol{G}x)} \ \right]$$

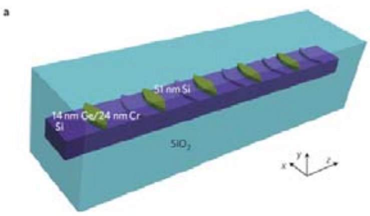
$$= \Delta\varepsilon_1 \ [\cos(kx) + i \sin(kx)]$$

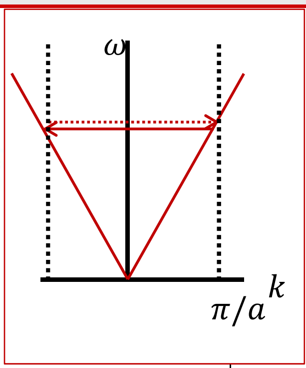
« Single sideband » grating (Fr:BLU)



PT-symmetry with gratings

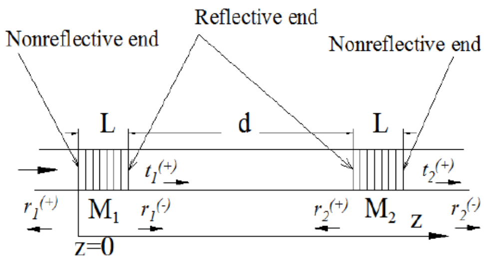
(Fr:BLU)DBR





Kulishov et al.

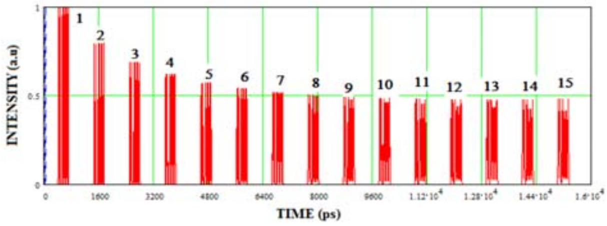
22 April 2013 | Vol. 21, No. 8 | DOI:10.1364/OE.21.009473 | OPTICS EXPRESS 9473

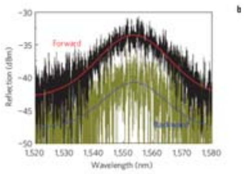


nature materials | LETTERS

Experimental demonstration of a unidirectional reflectionless parity-time metamaterial at optical frequencies

$$\Delta\varepsilon(x) \sim \Delta\varepsilon_1 \; [\cos(kx) + i(-1 + \sin(kx))]$$



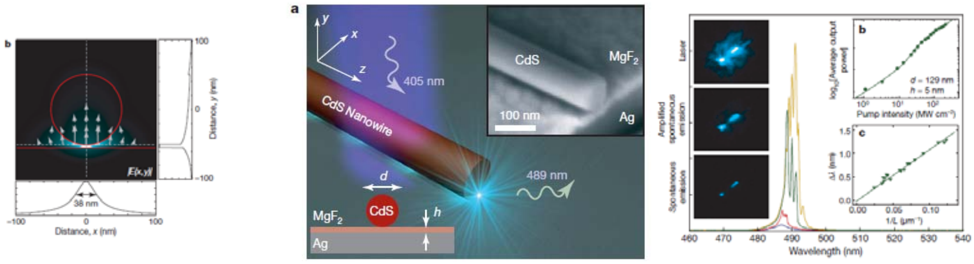


OUTLINE

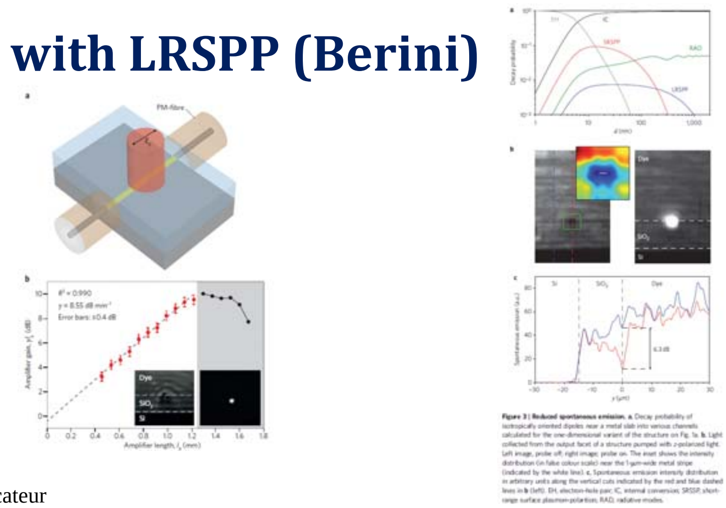
- ➩ **$\mathcal{PT}$–symmetry, Hamiltonians, and Gain-Loss**
- ➩ **Three flavours of PT-symmetries**
  - With waveguides
  - With resonators
  - With gratings
- ➩ **Application with plasmonics** (with IEF : A. Lupu, A. Degiron)
  - Losses are now causing a singularity !
  - Switches
  - Real life ? ("healing" of smoothed singularity)
- ➩ **$\mathcal{PT}$–symmetry and (many) current photonic concepts**
  - from coherent perfect absorbers to lasers
- ➩ **The brachistochrone problem (1696[!]-1990)**
  - Relevance for fast Quantum computation

CONTEXT : Gain with plasmons

- **SPASER (Stockman, Oulton with nanorods, …)**



- **Optical Amplifiers with LRSPP (Berini)**



$\mathcal{PT}$-symmetry with relaxed gain-loss balance

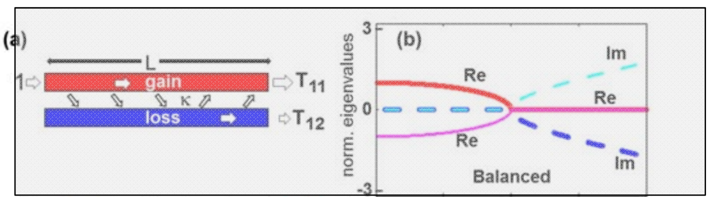
previous representation →

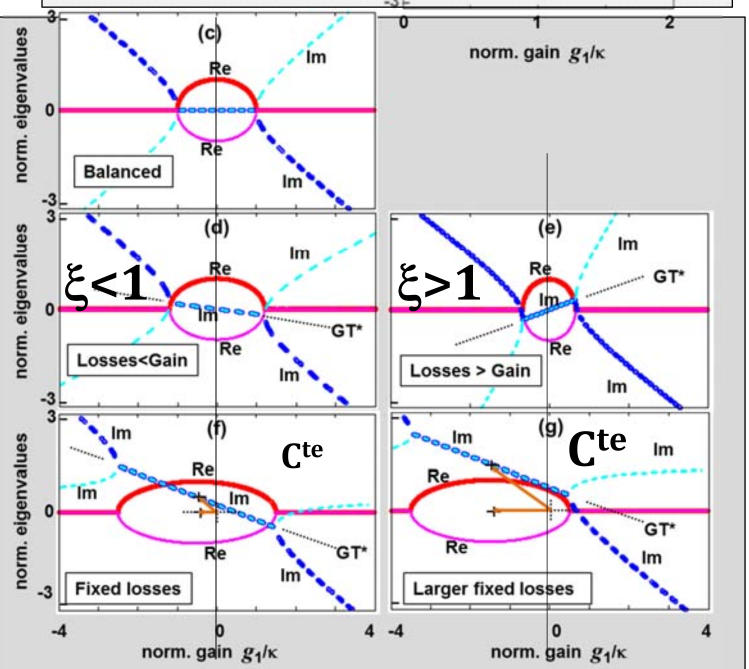
broad view

|loss| = gain

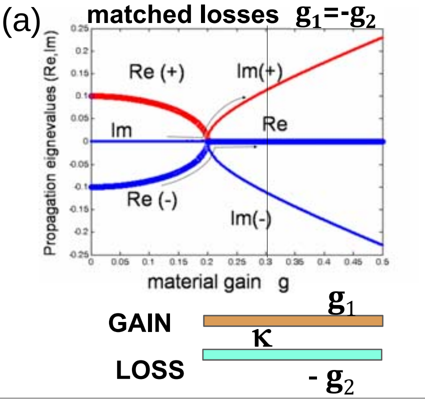
|loss| = ξ × gain

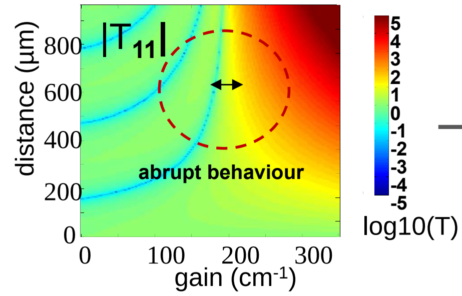
|loss| = C^{te}

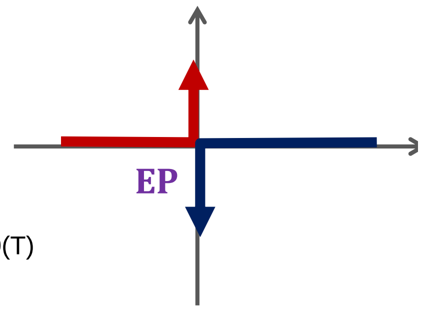


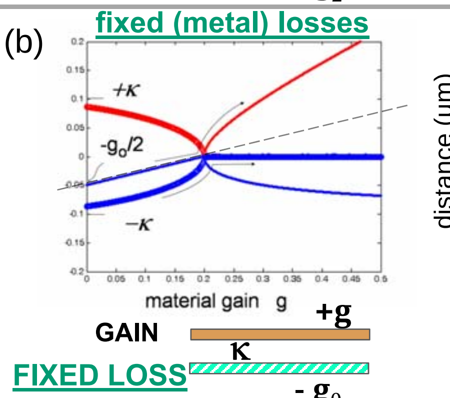


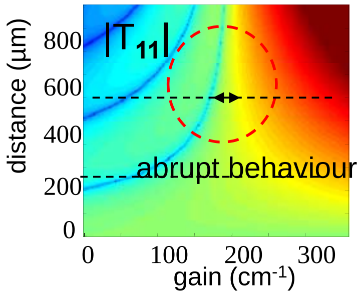
$\mathcal{PT}$-symmetry with plasmonics : fixed losses

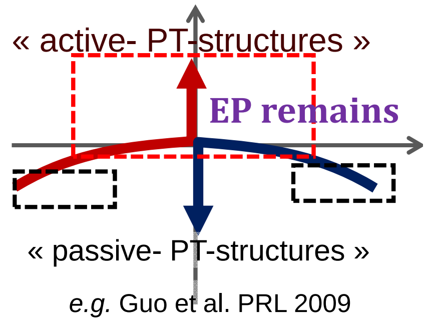








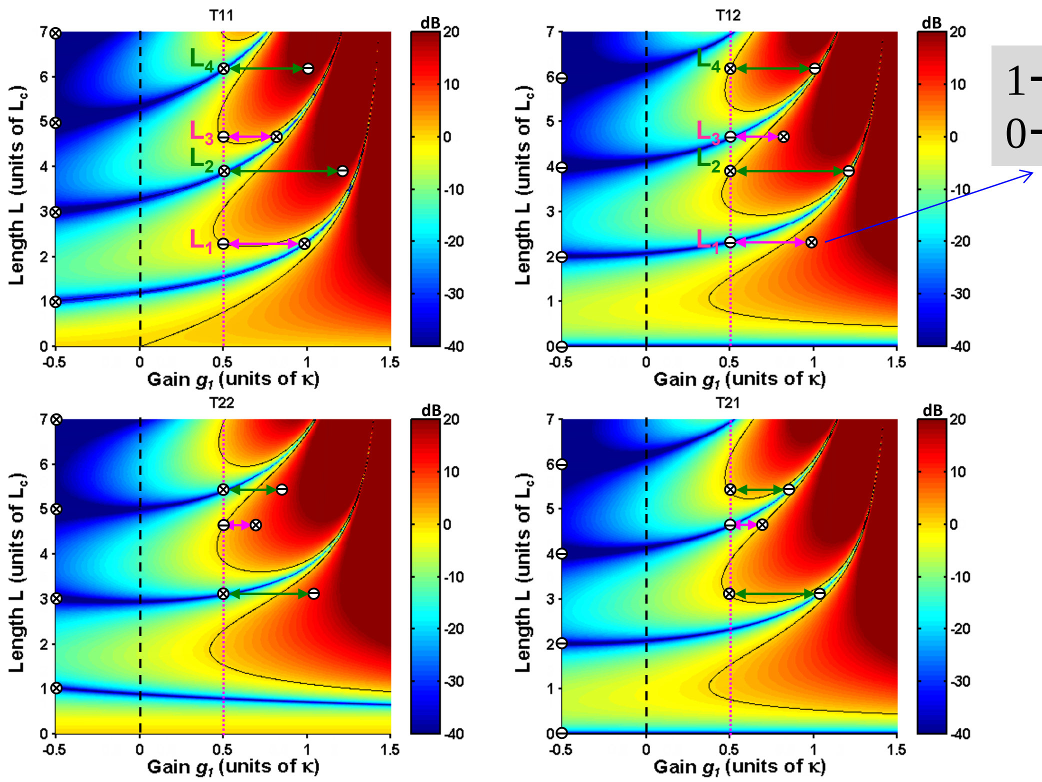




$\boxed{g_0 \text{ and } \kappa \text{ should be "matched"}}$

H. Benisty et al. *Opt. Express*, ***19***, *18004, 2011*

Whole coupler picture



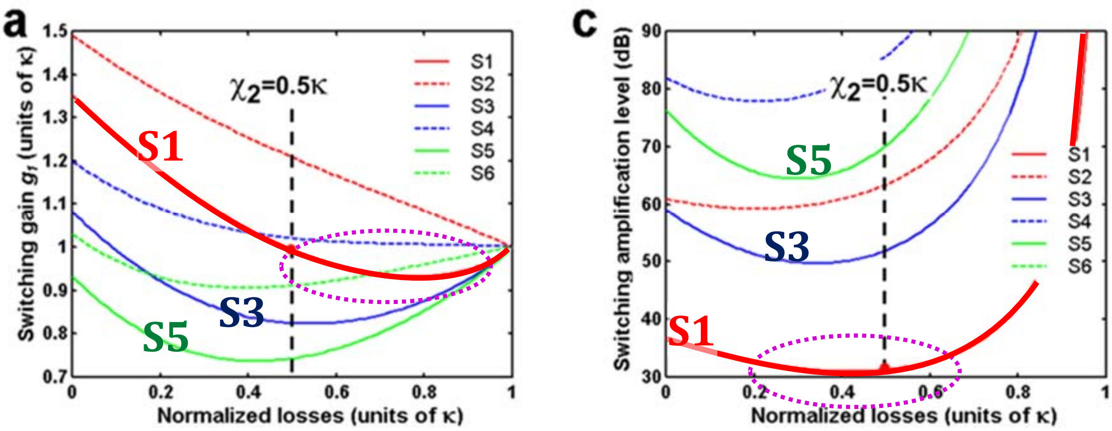
1→0
0→**A (> 1)**

MORE on SWITCHING ?

Switching using PT symmetry in plasmonic systems: positive role of the losses

Anatole Lupu,[1,2,*] Henri Benisty,[3] and Aloyse Degiron[1,2]

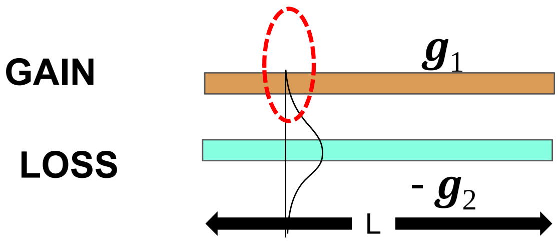
9 September 2013 | Vol. 21, No. 18 | DOI:10.1364/OE.21.021651 | OPTICS EXPRESS 21651



Existence of optimal losses, minimizing requirement for gain & amplification vs. κ

Origin of complex coupling

microscopic picture

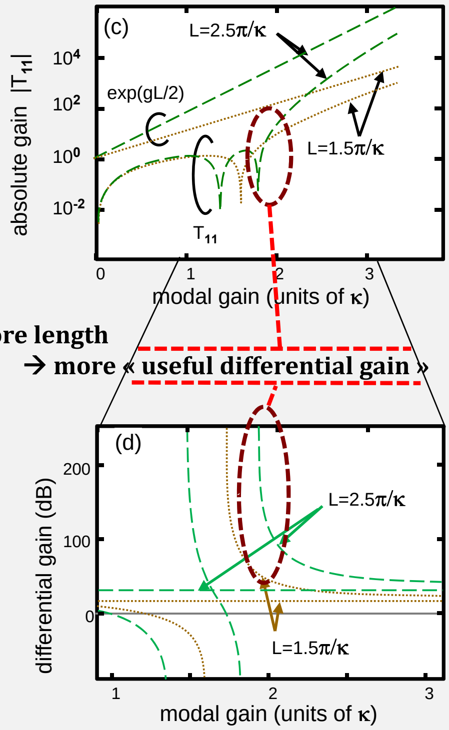


coupling constant : κ

→ coupling value κ_{21} also depends on g_1

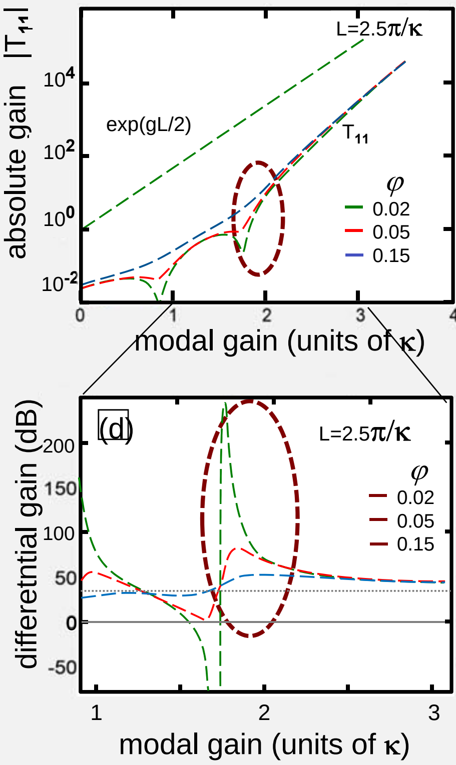
« Differential gain » : {Ideal} vs. {Fixed losses and Complex coupling}

Ideal case



more length
→ more « useful differential gain »

Fixed losses & complex coupling



$\kappa_{12}\kappa_{21} = e^{i\varphi}$

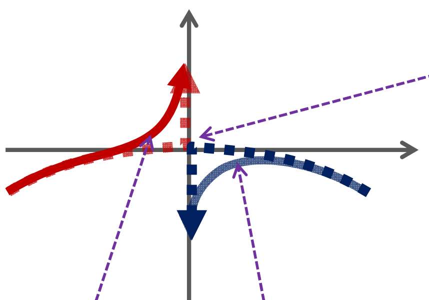
$\kappa_{12}\kappa_{21} = e^{i\varphi}$

$\mathcal{PT}$-symmetry with plasmonics

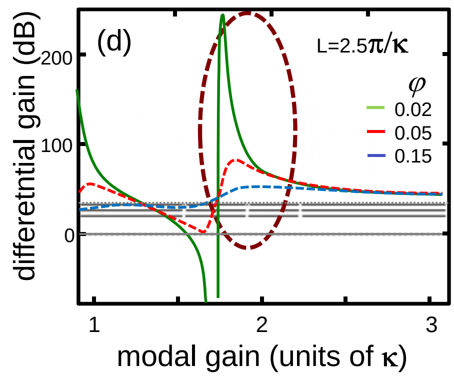
- **Exceptional point smoothed when complex phase occurs in coupling** $\kappa_{12}\kappa_{21} = e^{i(\text{phase})}$
- **Small margin to retain high differential gain** $\kappa_{12}\kappa_{21} = e^{i(\text{small phase})}$

« Active-PT-structures"

Ideal exceptional point



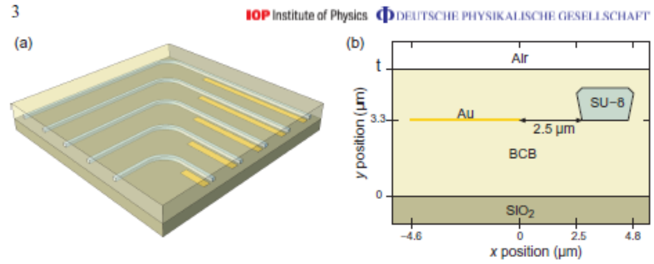
with complex phase:
EP missed
‣ smoothed singularity !
(*actual example below)

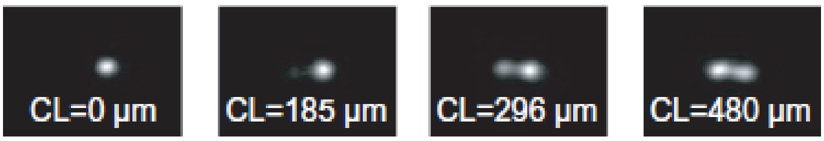


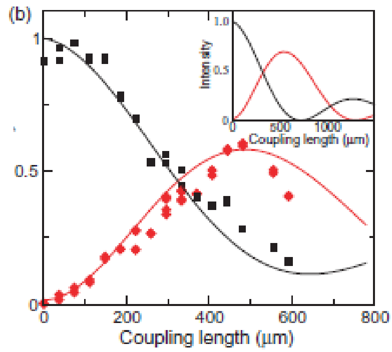
Dielectric and plasmonic waveguides

polymer + Au-LRSPP

New Journal of Physics **11** (2009) 015002   A. Degiron et al.   (@ Duke U.)

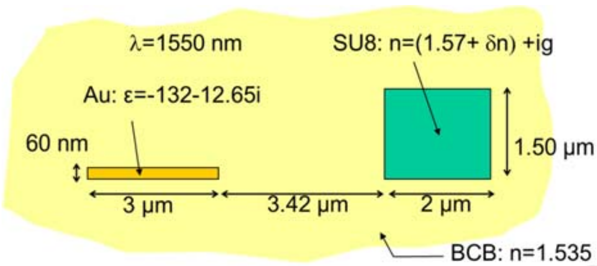


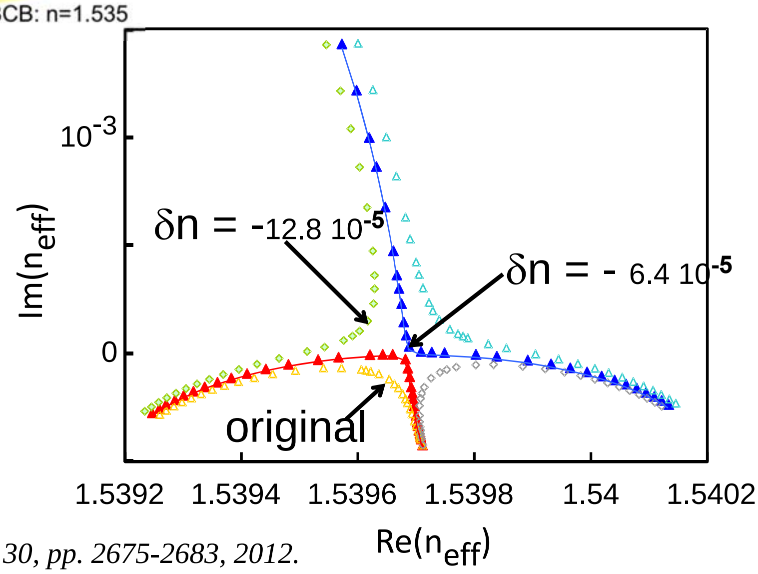




passive regime;
« Switching » based on detuning of Re(ε)

« Healing » in this realistic architecture ?

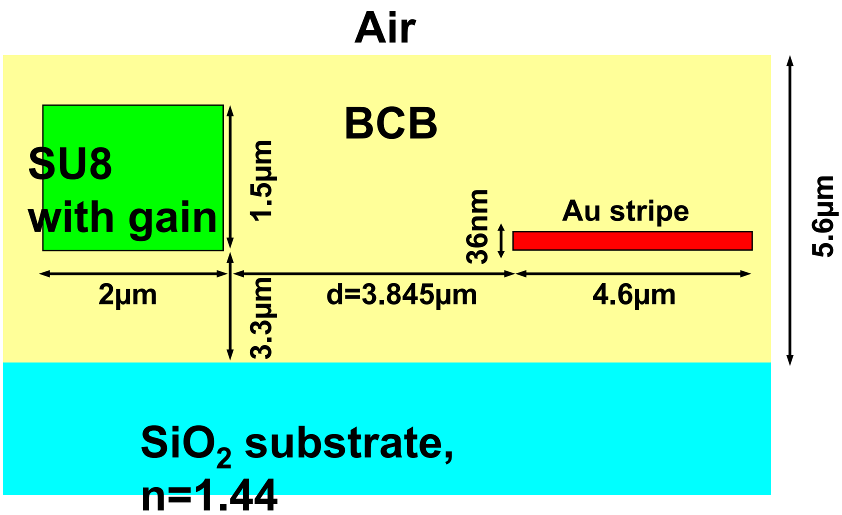


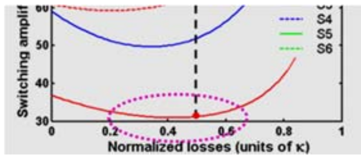


IEEE J. Lightwave Technol., vol. 30, pp. 2675-2683, 2012.

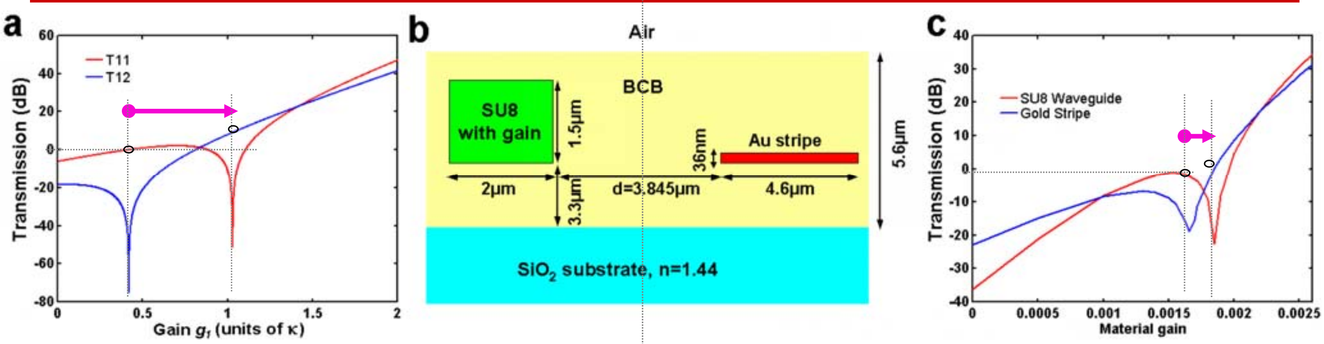
Adaptation of geometry

Targetting χ_2 =0.42κ, loss/coupling ratio





REALISTIC LRSPP + SU8 SWITCH



- **CMT**
- real κ
- **χ_2 =0.42κ, fixed losses**
- **variable g_1**

- **FEM**
- **no « healing »**

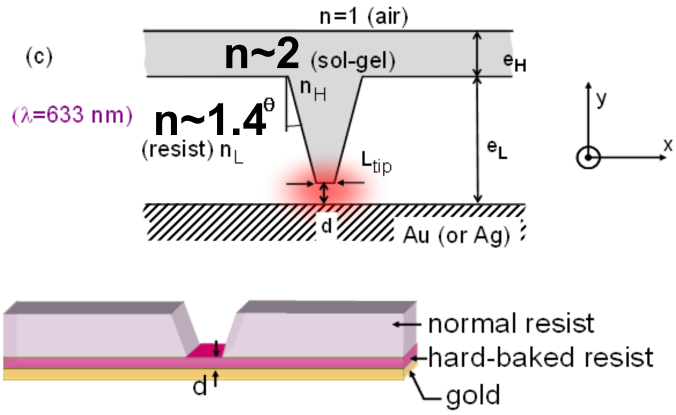
real κ

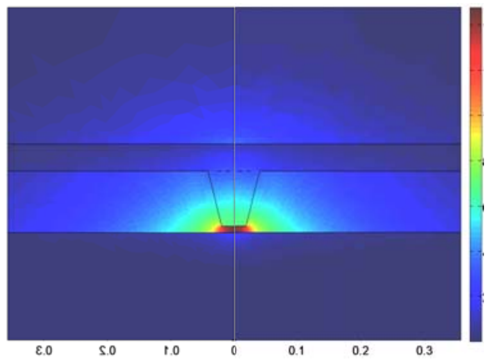
- variable Im(ε_{SU8})

imperfections...
... less gain excursion
...

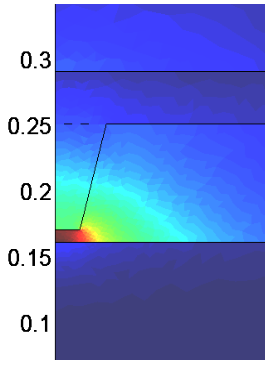
Hybrid plasmonic waveguide : the «PIROW »

PIROW: Plasmonic Inverse-Rib Optical Waveguide



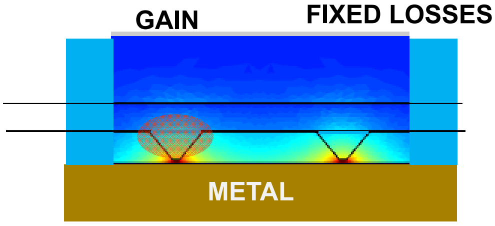


- **JAP 2010, H. Benisty and M. Besbes**
  E-field in 30-50 nm tip…
- **Like Oulton's nanorod/spaser, but deterministic**



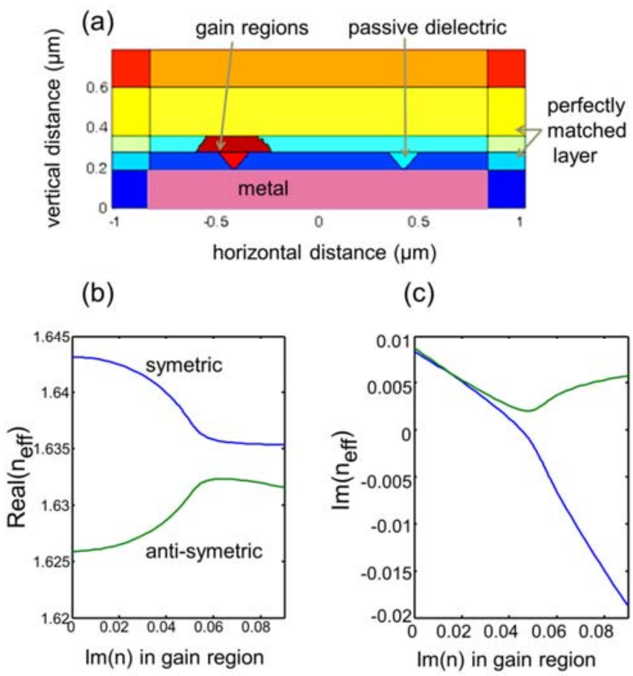
Two coupled «PIROWs »

A normal one and a gain one

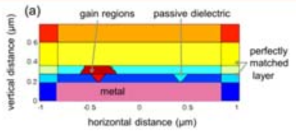


Can we have a good EP ?

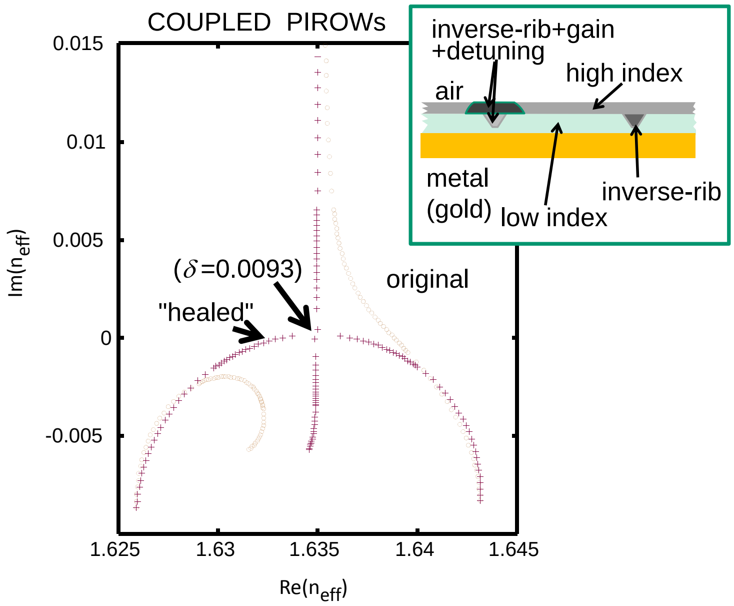
H. Benisty and M. Besbes, "Confinement and optical properties of the plasmonic inverse-rib waveguide," *JOSA. B, vol. 29, pp. 818-826, 2012.*



« Healing » of Two coupled PIROWs



$$(\tilde{n}_H) = (n_H) + ig(1 + i\alpha) \qquad \alpha=-0.174$$



« Healing » obtained here by changing both Im(ε) and Re(ε), the latter with a small factor... equivalent to detuning of waveguides with fixed Re(ε) of EP...

OUTLINE

- $\mathcal{PT}$–**symmetry, Hamiltonians, and Gain-Loss**
- **Three flavours of PT-symmetries**
  - With waveguides
  - With resonators
  - With gratings
- **Application with plasmonics** (with IEF : A. Lupu, A. Degiron)
  - Losses are now causing a singularity !
  - Switches
  - Real life ? ("healing" of smoothed singularity)
- $\mathcal{PT}$–**symmetry and (many) current photonic concepts**
  - from coherent perfect absorbers to lasers
- **The brachistochrone problem (1696[!]-1990)**
  - Relevance for fast Quantum computation

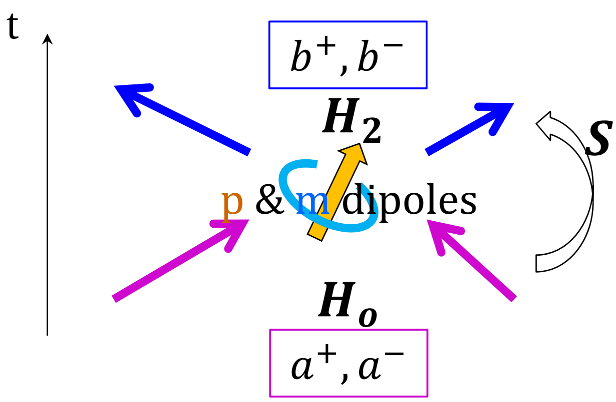
General metamaterial scattering description

Jensen Li et al. work $\gamma_i = \gamma_i^s + \gamma_i^{\mathrm{loss}}$

Kang et al. (Jensen Li)
PRA **87**, 053824 (2013)

$$\mathrm{H_n} = \begin{pmatrix} \omega_1 - i\gamma_1 & \kappa \\ \kappa & \omega_2 - i\gamma_2 \end{pmatrix} + \mathrm{n}\begin{pmatrix} i\gamma_1^s & 0 \\ 0 & i\gamma_2^s \end{pmatrix}. \quad (4)$$

n=0,1,2

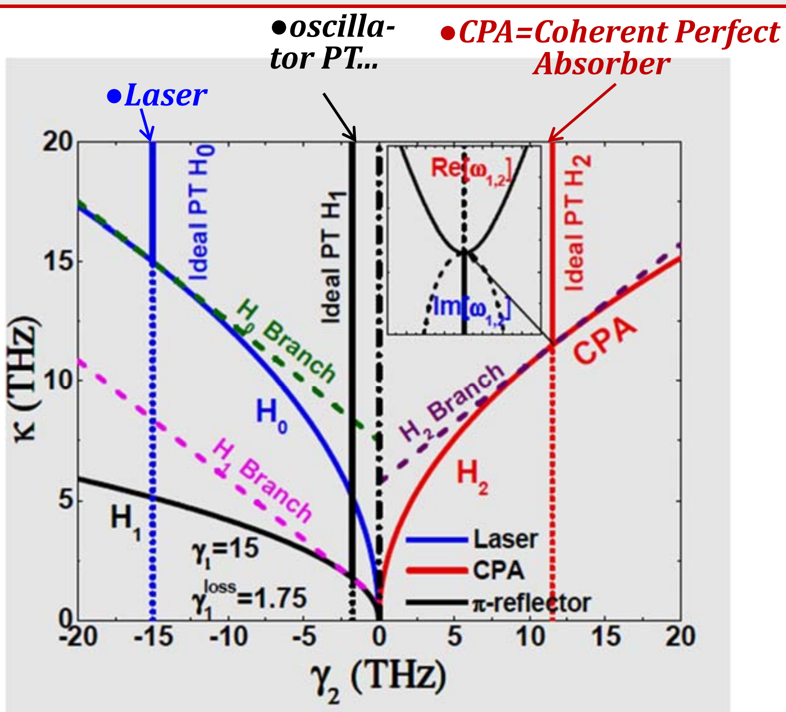


zero → CPA

$$\det(\mathbf{S}) = \frac{\det(\mathbf{H}_2 - \omega\mathbf{I})}{\det(\mathbf{H}_0 - \omega\mathbf{I})}.$$

pole → laser

Unification ? Each H_n has its PT-sym event

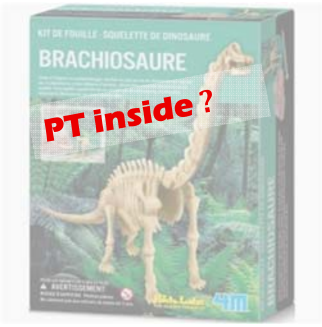


Kang et al. (Jensen Li)
PRA **87**, 053824 (2013)

Then, $\det(\mathbf{H}_1 - \omega\mathbf{I}) = 0$ implies a complete reflection $r = -1$ and $t = 0$ for one side of incidence.

●Description based on a « rich dipole » (2 coupled degrees of freedom)

The brachistochrone problem...



Johann Bernoulli

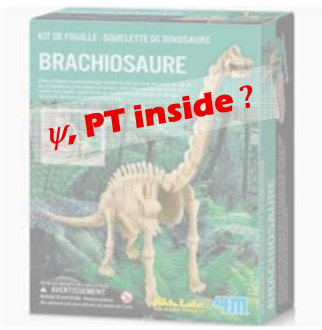
Acta Eruditorum 1696 $\frac{\sin\theta}{v} = \frac{1}{v}\frac{dx}{ds} = \frac{1}{v_m}$

Quickest path with given *g* ?

~ "Quickest Hamiltonian"

[Solution = Cycloid]

The quantum brachistochrone problem...



$\psi_A \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad H = \begin{bmatrix} s & re^{-i\theta} \\ re^{i\theta} & u \end{bmatrix} \quad$ $\psi_B \begin{pmatrix} a \\ b \end{pmatrix}$

$E_+ - E_- = \omega$

« *g* » ←→ constraint on eigenvalues

[Solution ~Rabi π/2 oscillation]

$$\tau = \pi\hbar/\omega$$

Faster than Hermitian Quantum Mechanics

Bender et al...
PRL 98, 040403 (2007)

New inner product

$$\mathcal{C}^2 = 1, \qquad [\mathcal{C}, H] = 0, \quad \text{and} \quad [\mathcal{C}, \mathcal{PT}] = 0.$$

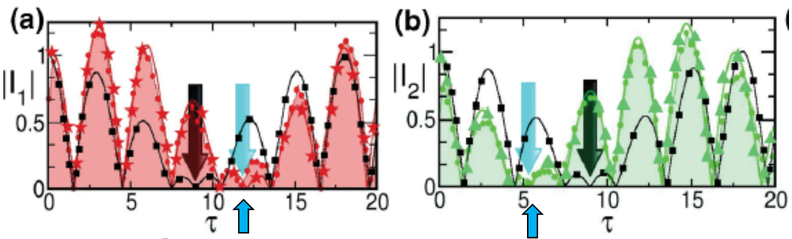
$$\mathcal{C} = \frac{1}{\cos\alpha}\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}.$$

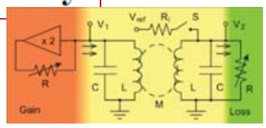
Seems to depend on H !

. If so, this would limit the applicability of a Hilbert-space wormhole to improve quantum algorithms.

PHYSICAL REVIEW A **85**, 062122 (2012)
T. Kottos, Wesleyan Univ....

Bypassing the bandwidth theorem with $\mathcal{PT}$ symmetry



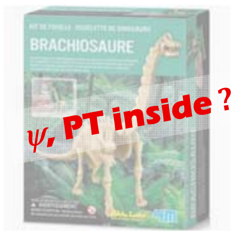


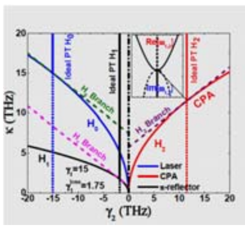
$$\tau_{\mathrm{fpt}} = \frac{1}{\delta\omega}\left[\pi \pm \arccos\left(\frac{\delta\omega^2 - \gamma^2}{\delta\omega^2 + \gamma^2}\right)\right]$$

Slower **Faster**

CONCLUSION & PERSPECTIVES

Attractive concepts from classical to quantum





Quite some potential to unify several fields using Gain & "Phase-transitions" (lasers, CPA, EIT, metamaterial ?, strong coupling ?,...)

Can be combined (...with care...) with plasmonics to yield singularity from losses !