L'interférence dans les réseaux non filaires

Du contrôle de puissance au codage et alignement

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7 mars 2013

Séminaire Comelec

Parts

Part I

Interference in Wireless Networks

Outline of current Part

Next Frontier for Wireless Networks

Cells in cellular wireless networks are becoming **smaller and smaller** as the density of users per space unit is becoming **higher and higher**. In wireless sensor networks, the density of sensors is becoming higher and higher as well.

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Unwanted signals

Each node receives a combination of its own signal and many unwanted ones. Wireless networks become more and more **Interference Limited**.

Outline of current Part

1. Introduction
2. **In cellular systems**
3. In ad hoc networks

Cellular network

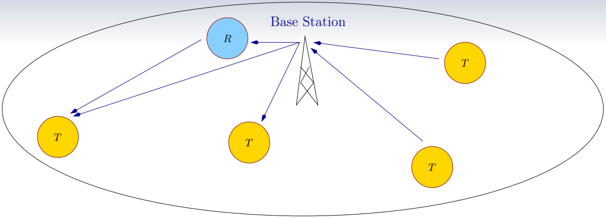

Figure: One Cell : Many Users

Cellular network

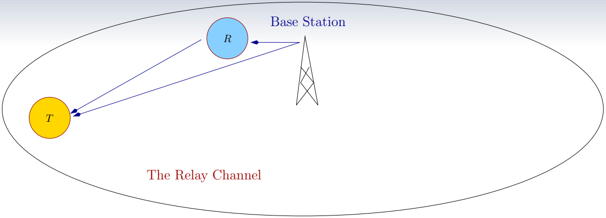

Figure: Accessing to hidden Terminals

Cellular network

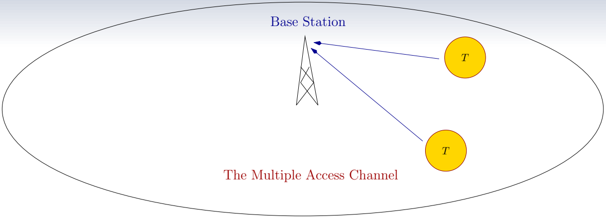

Figure: Uplink

Cellular network

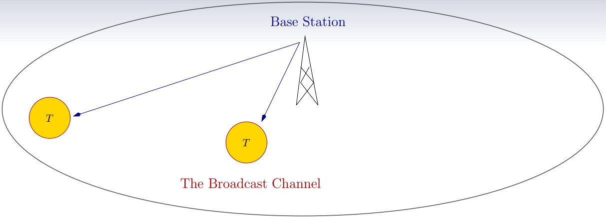

Figure: Downlink

Cellular network

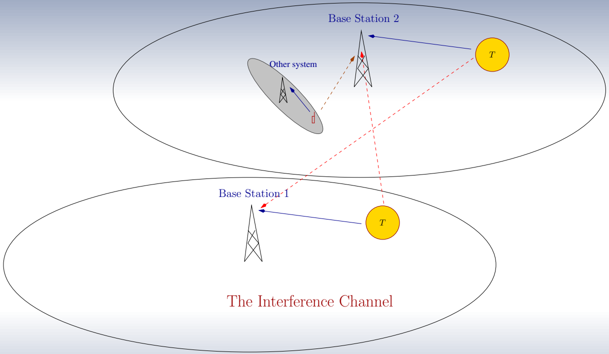

Figure: Many cells sharing the same Physical Resources

Outline of current Part

Ad Hoc and Wireless Sensor Networks

Interferences

Ad Hoc and Wireless sensor networks can experience a high level of interference between nodes when the number of nodes per area unit is high and the physical resource is scarce.

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Properties of the Wireless Medium

Main properties are

- Braodcast property.
- Superposition property.

Part II

Resource Allocation

Outline of current Part

4 **Solve the problem at the RRM level**

5 Power Control

6 Subchannel allocation

At the Physical Layer

Orthogonal Multiplexing

Transmit signals from different cells/users at different

- Subbands (FDMA)
- Time Slots (TDMA)

Interference problem is solved by **avoiding** Interference. But it is not **enough** ...

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Interference as noise

At receivers, the sum of all interfering signals is considered as **noise**. Definition of new parameters as

$$\mathsf{SINR}_i = \frac{P_i \cdot |h_{ii}|^2}{N + \sum_{j \neq i} P_j \cdot |h_{ji}|^2}$$

for user i, where P_j is the transmit power of user j, h_{ji} is the attenuation from transmit cell j to receive cell i and N is the power of noise.

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Fixed Rate

Target Data rates R_k are given $\Longrightarrow$ Target SINR, γ_k are given. **Optimization** problem:

$$\begin{cases} \min_{\vec{P}} \sum_i P_i & \\ \text{subject to} & \mathsf{SINR}_k \geq \gamma_k \\ \text{and} & P_i \leq P_{\max} \end{cases}$$

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Problem with solution

[Pischella & B., 08] This problem has solutions whenever

$$\forall k, \ \frac{\left|h_{kk}\right|^2}{\sum_{j\neq i}\left|h_{jk}\right|^2} > \gamma_k. \quad (1)$$

When (1) is not satisfied for some cell, then we say that the network is **interference-limited**. No more **degree of freedom** is available.

Rate Maximization

Same assumptions as before. Now we want to maximize a function $\varphi(R_1, \ldots, R_K)$ of the user rates.

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Resolution

[Pischella & B., 10] It is a nonconvex optimization problem that can only be solved when SINR is high enough.

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Rate Maximization

Subcarriers are not chosen when SINR falls below some threshold. Graph coloring algorithms may also be used for subcarrier allocation.

Part III

Han & Kobayashi

Outline of current Part

What is possible in Interference-Limited Networks?

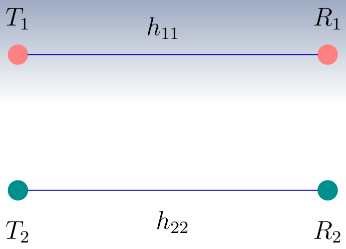

Figure: Channel Model

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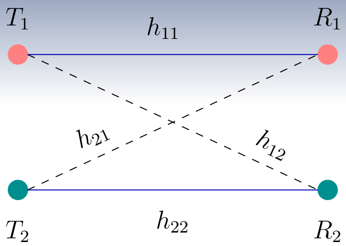

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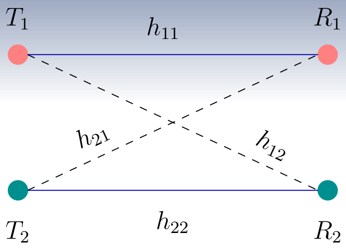

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Model of the **2-user Interference** Channel. Additive Gaussian noise is present at each receiver.

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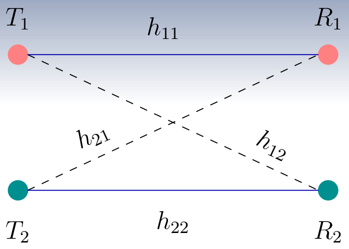

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Model of the **2-user Interference** Channel. Additive Gaussian noise is present at each receiver.

Is it better to decode interference?

When the level of interference becomes high enough, then it is better, for the receiver, to **decode** both the legitimate user and the interferers. For low level interference: still consider interference as **noise**.

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Han and Kobayashi Coding Scheme

Transmitters

Each transmitter splits the data into

- **Private** data.
- **Common** data.

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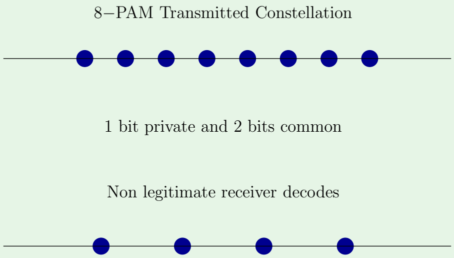
Achievable Rates

By optimizing the ratio between R_c (common) and R_p (private), significantly **higher** rates are achievable.

A toy example

PAM Constellation



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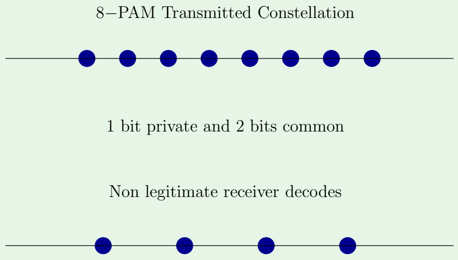

Figure: Example of an $8-$PAM constellation

- **Hierarchical** Modulation.

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Generalized Degrees of Freedom

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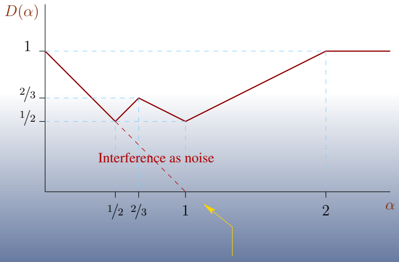

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- Generalized Degrees of freedom (*W* curve) **[Etkin, Tse & Wang, 08]**
$$\begin{cases} D(\alpha) = 1-\alpha & 0 \le \alpha \le 1/2 & \text{priv.} \\ D(\alpha) = \alpha & 1/2 \le \alpha \le 2/3 & \text{priv.} + \text{com.} \\ D(\alpha) = 1-\alpha/2 & 2/3 \le \alpha \le 1 & \text{priv.} + \text{com.} \\ D(\alpha) = \alpha/2 & 1 \le \alpha \le 2 & \text{com.} \\ D(\alpha) = 1 & \alpha \ge 2 & \text{com.} \end{cases}$$

Part IV

Interference Alignment

Alignment

Principles

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Several types of alignment have been studied among which,

1. Linear over $\mathbb{R}$ or $\mathbb{C}$. Needs **space**, **time** or **frequency** diversity.
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Main Constraint

Needs **channels knowledge** at all transmitting sides.

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10 **Linear Interference Alignment [Cadambe & Jafar, 09]**

11 Integer Interference Alignment [Jafarian & Vishwanath, 11]

12 What is really alignment?

Principle

An example

Suppose we have 3 receivers for 5 unknowns,

$$\begin{cases} y_1 = & 3x_1 + 2x_2 + 3x_3 + x_4 + 5x_5 \\ y_2 = & 2x_1 + 4x_2 + x_3 - 3x_4 + 5x_5 \\ y_3 = & 4x_1 + 3x_2 + 5x_3 + 2x_4 + 8x_5 \end{cases}$$

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In fact, the interfering beams span a $2-$D space leaving one dimension free from interference for the wanted signal,

$$\boldsymbol{H}_{*4} = \boldsymbol{H}_{*3} - \boldsymbol{H}_{*2} \text{ and } \boldsymbol{H}_{*5} = \boldsymbol{H}_{*3} + \boldsymbol{H}_{*2}.$$

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Linear **Interference alignment** allows many interfering users to communicate **simultaneously** over a small number of signalling dimensions by putting the interfering signals in a space of small dimension so that the desired signal can be projected into the **null space** of the interference.

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Feasibility of interference alignment depends on the **structure** and **probabilistic** behavior of the channel matrix.

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For instance, for the symmetric $K-$user interference channel where all transmitters have n_t antennas and all receivers, n_r antennas, interference alignment is **feasible** with probability 1 when

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Symbol extension

Beamforming across **multiple channel uses** to increase space dimension.

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This type of alignment will be studied on an example. We assume 3 pairs of transmitters/receivers. **PAM** constellations are used (transmission of **integer** symbols), i.e. $x_i \in \mathbb{Z}$.

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If $u_1 \in \mathbb{Z}_6$, $u_2 \in \mathbb{Z}_2$ and $u_3 \in \mathbb{Z}_3$, then, $u_1 = r_1 \bmod 6$, $u_2 = r_2 \bmod 2$ and $u_3 = r_3 \bmod 3$.

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Alignment

Interference is aligned on **ideals** ($6\mathbb{Z}$, $2\mathbb{Z}$ and $3\mathbb{Z}$). **Residue** Symbols are received **free** of interference.

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Linear or integer

Techniques of alignment all rely in fact on the same idea.

- **Linear:** Received signal (noiseless) belongs to a **ring** of multivariate polynomials (variables are channel gains). Interferences are put in an ideal.
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Coding

Let $\mathscr{R}$ denote the ring of all possible (noiseless) received signal. Put all interferers in an ideal $\mathscr{J}$ of $\mathscr{R}$ and encode the desired signal in the residue ring $\mathscr{R}/\mathscr{J}$.

Part V

The Compute-and-Forward tool

Outline of current Part

From Modulation and Coding ...

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What a lattice element could be

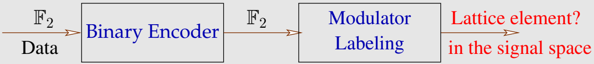


Figure: Encoder and Modulator

From Modulation and Coding ...

What a lattice element could be

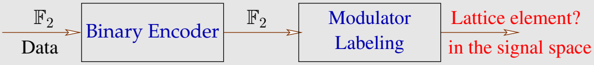


Figure: Encoder and Modulator

Requirements

- Encoder must be **linear**
- Modulation should be **PAM**, **QAM** or **HEX**
- **Labeling** (modulator) between **binary codewords** and **modulated symbols** has to respect some criteria

An example: the D_4 lattice (partition)

QAM Partition à la Ungerboeck

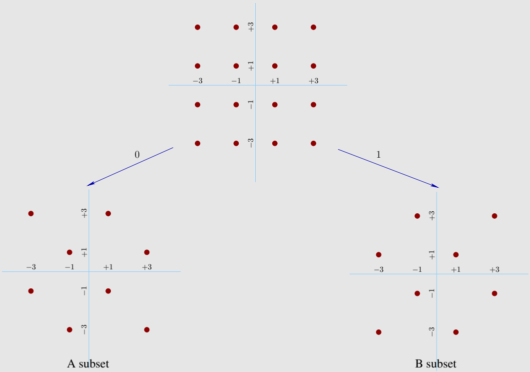
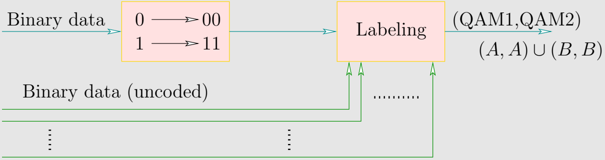


Figure: Labeling of subsets A and B

An example: the D_4 lattice (coding)

Encoder



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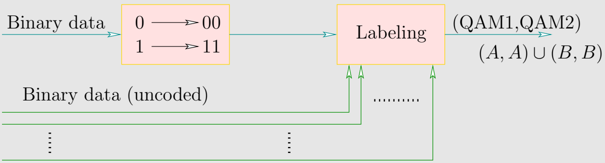

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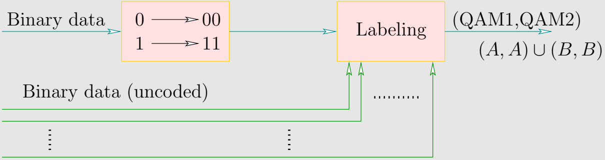

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One of the simplest examples of "Construction A"

$$D_4 = (1 + \imath)\mathbb{Z}[\imath]^2 + (2, 1)_{\mathbb{F}_2}$$

Definition

Lattice points

- An element $\boldsymbol{v}$ of $\boldsymbol{\Lambda}$ can be written as :

$$\boldsymbol{v} = a_1 \boldsymbol{v}_1 + a_2 \boldsymbol{v}_2 + \ldots + a_n \boldsymbol{v}_n, \quad a_1, a_2, \ldots, a_n \in \mathbb{Z}$$

  where $(\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n)$ is a basis of $\mathbb{R}^n$.

- The lattice $\boldsymbol{\Lambda}$ can be defined as :

$$\boldsymbol{\Lambda} = \left\{ \sum_{i=1}^{n} a_i \boldsymbol{v}_i \mid a_i \in \mathbb{Z} \right\}$$

Lattices : Generator matrix

- The set of vectors $v_1, v_2, \ldots, v_n$ is a **lattice basis**.

Definition

Matrix M whose columns are vectors $v_1, v_2, \ldots, v_n$ is a **generator matrix** of the lattice denoted Λ_M.

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Matrix $\boldsymbol{M}$ whose columns are vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ is a **generator matrix** of the lattice denoted $\Lambda_{\boldsymbol{M}}$.

- Each vector $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^\top$ in $\Lambda_{\boldsymbol{M}}$, can be written as,

$$\boxed{\boldsymbol{x} = \boldsymbol{M} \cdot \boldsymbol{z}}$$

  where $\boldsymbol{z} = (z_1, z_2, \ldots, z_n)^\top \in \mathbb{Z}^n$.

- Lattice $\Lambda_{\boldsymbol{M}}$ may be seen as the result of a linear transform applied to lattice $\mathbb{Z}^n$ (**cubic lattice**).

Lattices : Geometric properties

- The generator matrix M describes the lattice Λ_M, but it is not unique. All matrices $M \cdot T$ where T has **integer** entries and $\det T = \pm 1$ are generator matrices of Λ_M. T is called a unimodular matrix.
- $G = M^\top \cdot M$ is the **Gram matrix** of the lattice .

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Definitions

- The **fundamental parallelotope** of $\Lambda_{\boldsymbol{M}}$ is the region,

$$\mathscr{P} = \left\{ \boldsymbol{x} \in \mathbb{R}^n \mid \boldsymbol{x} = a_1 \boldsymbol{v}_1 + a_2 \boldsymbol{v}_2 + \ldots + a_n \boldsymbol{v}_n,\ 0 \leq a_i < 1,\ i = 1 \ldots n \right\}$$

- The **fundamental volume** is the volume of the fundamental parallelotope. It is denoted $\mathrm{Vol}\left(\Lambda_{\boldsymbol{M}}\right)$.
- The fundamental volume of the lattice is $\mathrm{vol}\left(\Lambda_{\boldsymbol{M}}\right) = |\det(\boldsymbol{M})|$, which is $\sqrt{\det(\boldsymbol{G})}$ either.

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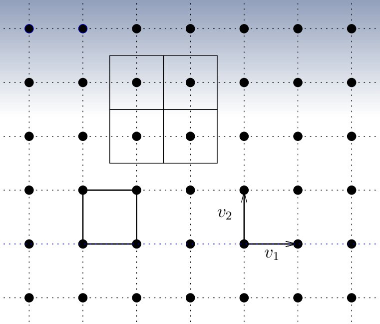
- A **bad** basis is a basis with long vectors (large orthogonality defect).
- A **good** basis (or **reduced** basis) is a basis with short vectors (small orthogonality defect).

$\mathbb{Z}^2$ lattice



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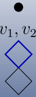

Lattice Point
Lattice Basis
Fundamental Parallelotope
Voronoi region

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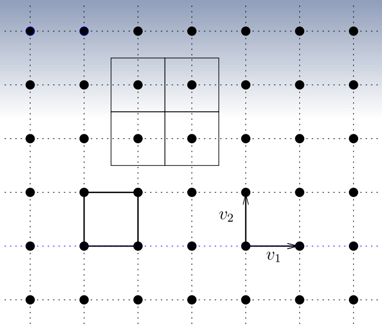


$\mathbb{Z}^2$ lattice

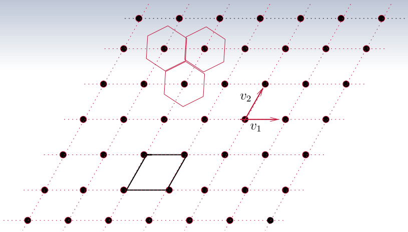
Properties

- Generator matrix is

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- A **QAM constellation** is a finite part of $\mathbb{Z}^2$.

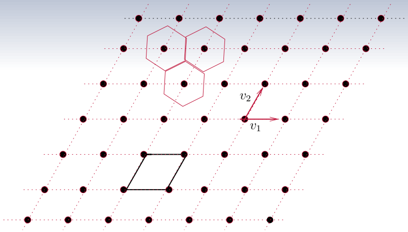
A_2 lattice



The A_2 lattice

- Lattice point
- (v_1, v_2) Lattice basis
- Fundamental parallelotope
- Voronoi region

A_2 lattice



The A_2 lattice

Properties

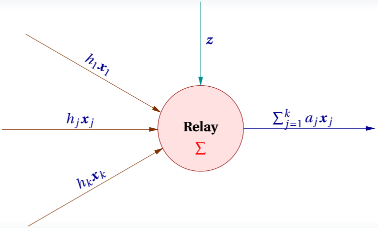
- Generator matrix is

$$M = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{bmatrix}$$

- An **HEX constellation** is a finite part of A_2, the hexagonal lattice.

Outline of current Part

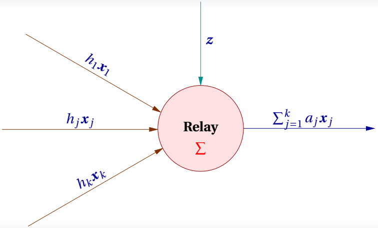
Principles


$\boldsymbol{z}$
$h_1\boldsymbol{x}_1$
$h_j\boldsymbol{x}_j$
$h_k\boldsymbol{x}_k$
Relay
Σ
$\sum_{j=1}^{k} a_j\boldsymbol{x}_j$

Principles

Relay

The Relay wants to reliably decode the result of computation $\boldsymbol{\lambda} = \sum_{j=1}^{k} a_j \boldsymbol{x}_j$. If $\boldsymbol{x}_j$ are lattice points of some integer lattice, then $\boldsymbol{\lambda}$ is also a lattice point for some lattice.



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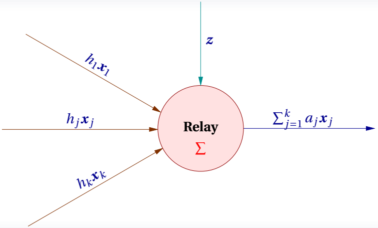

Received signal

Received signal is $\boldsymbol{y} = \sum_{j=1}^{k} h_j \boldsymbol{x}_j + \boldsymbol{z}$ where $\boldsymbol{x}_j \in \Lambda$ are lattice points, $h_j \in \mathbb{R}$ and $\boldsymbol{z}$ iid Gaussian noise. **Note that $a_j \in \mathbb{Z}$.**

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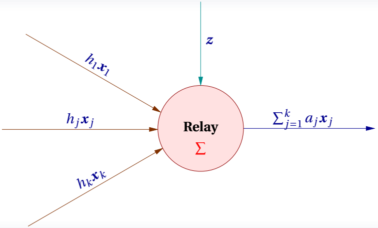

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Goal

Compute $\boldsymbol{\lambda}$ reliably.

Computation Rate

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The computation rate defined in is

$$R_{\text{comp}}(\boldsymbol{h},\boldsymbol{a}) = \log_2\left(\left(\|\boldsymbol{a}\|^2 - \frac{\mathsf{SNR}\left(\boldsymbol{h}^\top\boldsymbol{a}\right)^2}{1+\mathsf{SNR}\|\boldsymbol{h}\|^2}\right)^{-1}\right)$$

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Maximization

Maximization of R_{comp} requires to choose $\boldsymbol{a}_{\text{opt}} \in \mathbb{Z}^k]$ as **[Feng et al. 11]**,

$$\boldsymbol{a}_{\text{opt}} = \arg\min_{\boldsymbol{a}\neq\boldsymbol{0}} \boldsymbol{a}^\top\left(\boldsymbol{I} - \frac{\mathsf{SNR}}{1+\mathsf{SNR}\,\|\boldsymbol{h}\|^2}\boldsymbol{H}\right)\boldsymbol{a} = \arg\min_{\boldsymbol{a}\neq\boldsymbol{0}} \boldsymbol{a}^\top.\boldsymbol{Q}.\boldsymbol{a}$$

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Successive Minima

Find the k successive minima of Λ. **Reduce** Λ.

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Use Compute-and-Forward

- Each user transmits 2 data flows: private + common.
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Rates with common data

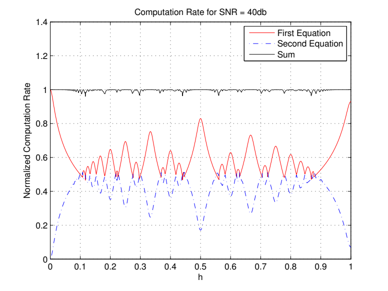

Figure: Sum Rate with Gauss Reduction

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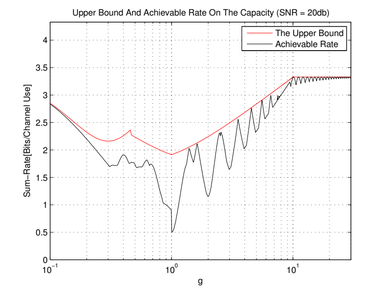

Figure: SNR = 20 dB ; Achievable Sum-Rate

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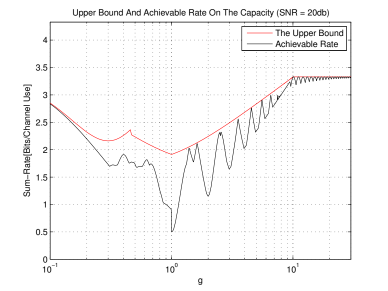

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High SNR

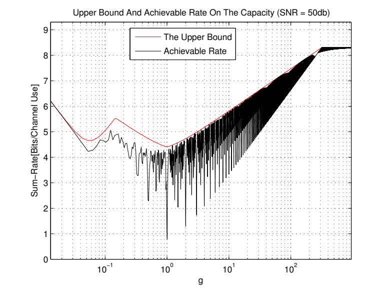

Figure: SNR = 50 dB ; Achievable Sum-Rate

Outline of current Part

From theory to practice

Linear alignment

Problems to overcome:

- **Perfect knowledge** of all channel gains at all transmitters.
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Fractal behavior

Some values of channel gains lead to performances much worse than very close other ones.

Open Problems

On the Coding+Alignment side

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Other points

- **Asynchronous Codes?**
- ...

Merci !!