

Analog Electronics 2

ICS905

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Dépt. COMELEC

http://perso.telecom-paristech.fr/~rodriguez/ens/cycle_master/



November 2016



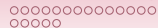
Schedule

- Radio channel characteristics ;
- Analysis and conception of the couple Tx-Rx ;
- M-QAM Modulation - theoretical analysis ;
- Distortions in M-QAM ;



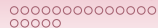
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Radio-channel characteristics

Microscopic effects

Transmitted signal :

$$s(t) = \text{Re}\{u(t) \cdot e^{j2\pi f_c t}\} = I(t) \cos(2\pi f_c t) - Q(t) \sin(2\pi f_c t)$$

f_c carrier frequency - $u(t)$ BB with B_s Hz.

Received Signal :

$$r(t) = \text{Re} \left\{ \sum_{n=0}^{N(t)} \alpha_n u(t - \tau_n(t)) \cdot e^{j\{2\pi f_c(t - \tau_n(t)) + \phi_n^D\}} \right\}$$

$$\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_n^D,$$

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Components of fading

n corresponds to a path of length

$$L_n \rightarrow \tau_n = L_n/c$$

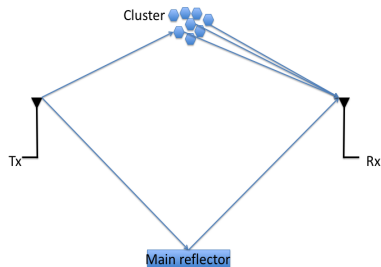
$\alpha_n(t)$ = attenuation.

$$\phi_n^D = \int 2\pi f_n^D(t) dt = \text{Doppler } f_c,$$

$$f_n^D(t) = \frac{v \cos \theta_n(t)}{\lambda},$$

$\theta_n(t)$ angle relative to the movement direction.

Paths are *solvable* if $|\tau_j - \tau_i| \gg B_s^{-1}$



Impact of *fading*

If the delay dispersion is small compared to $B_s^{-1} \sim T_s$
 \Rightarrow *narrowband fading*;

If the delay dispersion is big compared to $B_s^{-1} \sim T_s$
 \Rightarrow *wideband fading*;

Delay spread

The delay dispersion is called *Delay spread* of the channel $\rightarrow T_m$.

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Model *narrowband*

If $T_m \ll T_s$.

If τ_i represents the i th delay, so $\tau_i \leq T_m$:

$$u(t - \tau_i) \simeq u(t).$$

$$r(t) = \text{Re} \left\{ u(t) e^{j2\pi f_c t} \underbrace{\left(\sum_n \alpha_n(t) e^{-j\phi_n(t)} \right)}_{A(t)} \right\}.$$

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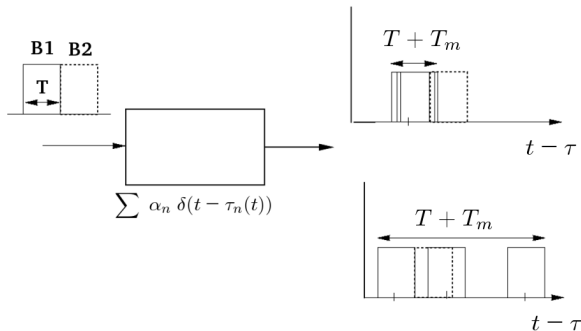
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Model *wideband*

In this case $T_m \gg T_s$.



Conclusion : if $T_m \gg T_s \rightarrow$ **ISI**

Doppler effect

This phenomenon represents the variability of the channel in time :

The mean dispersion of the frequency beside the carrier is called
Doppler spread B_D of the channel.

We call *Coherence time* of the channel, the duration of a complete cycle of dynamics induced by the Doppler effect.

$$T_c \approx 1/B_D$$

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A "Resumé"

Delay Spread - T_m

Delay spread give us a good idea of the dispersive characteristics of the channel in time.

Doppler Spread - B_D

Doppler dispersion give us a good idea of the variability of the channel.

Parameters

Mean delay spread \leftrightarrow Coherence bandwidth

$$T_m \leftrightarrow B_c$$

Coherence time \leftrightarrow Doppler dispersion

$$T_c \leftrightarrow B_D$$

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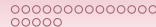
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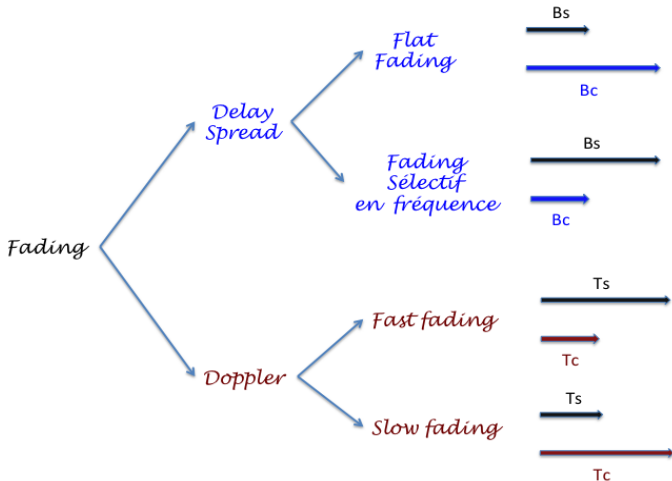
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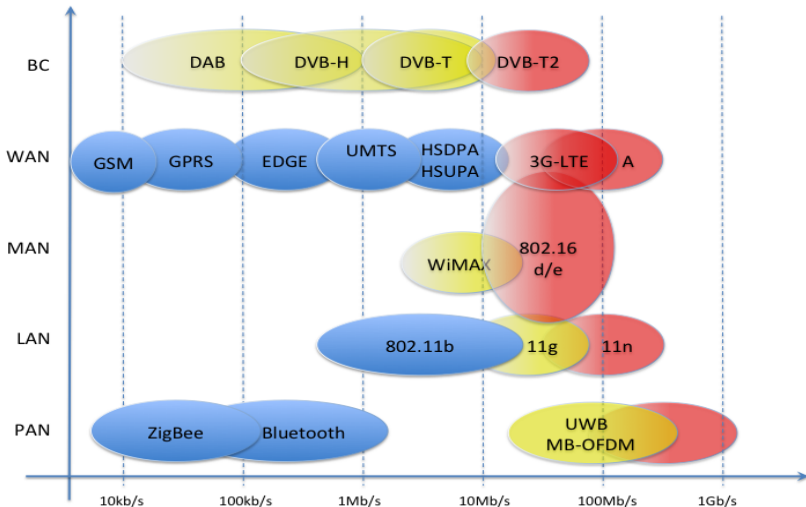
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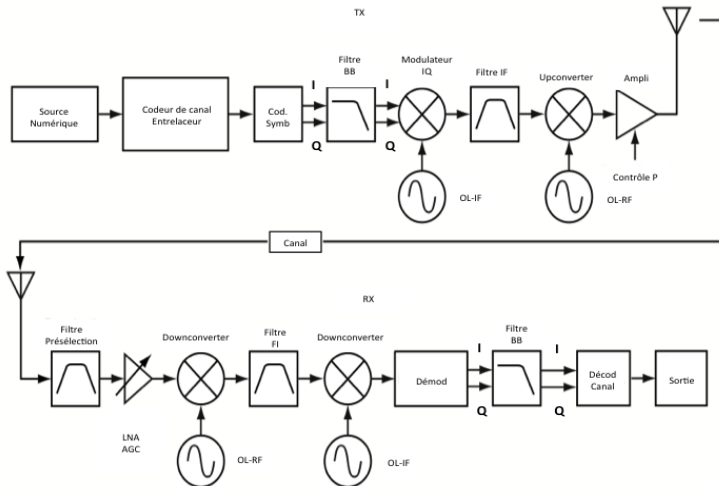
Channel models



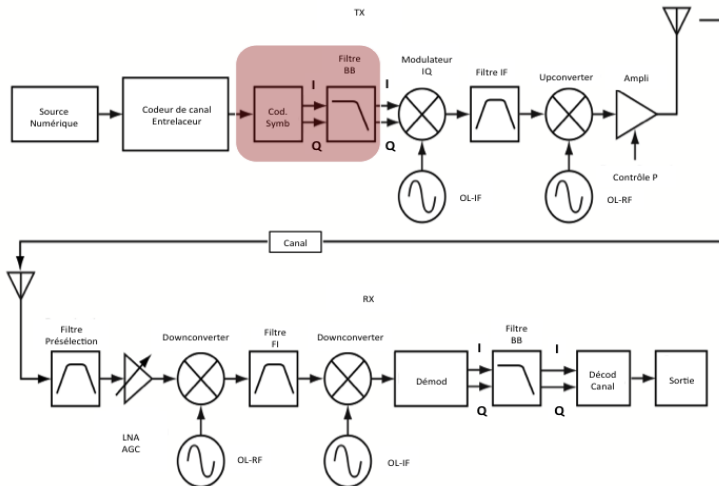
"Etat de l'art"



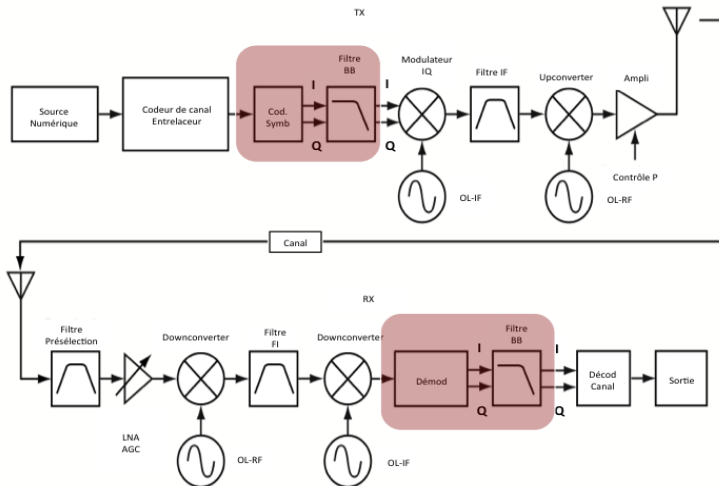
Scheme of Tx-Rx



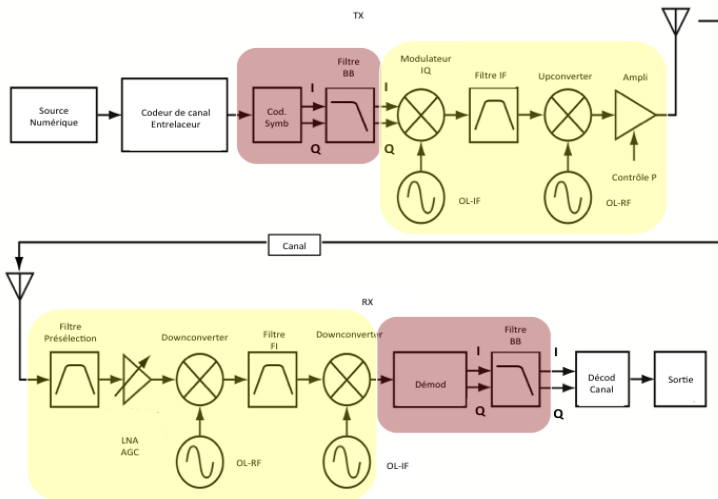
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Scheme of Tx-Rx



Scheme of Tx-Rx



Baseband Modulation

Transmitted Signal :

$$s(t) = \text{Re}\{u(t).e^{j2\pi f_c t}\} = I(t) \cos(2\pi f_c t) + j.Q(t) \sin(2\pi f_c t)$$

f_c carrier frequency - $u(t)$ baseband signal.

$$u(t) = I(t) + j.Q(t)$$

Complex representation

$I(t)$ – In – phase component

$Q(t)$ – Quadrature phase component

Baseband modulation

Digital modulation : build $u(t)$ as a functions of source(discrete) states changes.

Heuristic approach :

$\{\alpha_n\}$ state sequence of source $\rightarrow u(t)$ impulse superposition !

Example :

\mathcal{S} maxentropic binary source, $\{\alpha_n\} = \dots 0 1 1 0 0 1 \dots$

$$\{a_n\} = \dots -A \quad +A \quad +A \quad -A \quad -A \quad +A \quad \dots$$

$$u(t) = \sum_n a_n \cdot h(t - nT)$$

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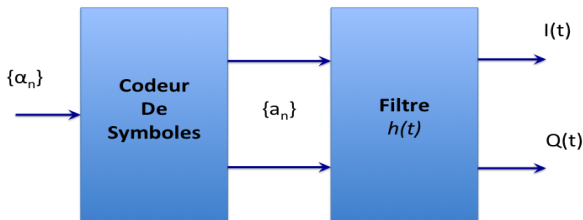
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Baseband modulation

$u(t)$ is built in two operations :

- bits \rightarrow amplitudes ;
- amplitudes \rightarrow waveforms (thanks $h(t)$!).



In this example : $I(t) = \sum_n a_n \cdot h(t - nT)$, $Q(t) = 0$.

Baseband modulation

How to choose $\{a_n\}$?

- Modulator architecture simplicity ;
- Spectral efficiency ($\eta = D/BW$) ;
- Demodulator architecture simplicity ;
- Synchronization simplicity ;
- Probability of error (P_b) ;
- Robustness to RF imperfections.

Baseband modulation

How to choose $\{h(t)\}$?

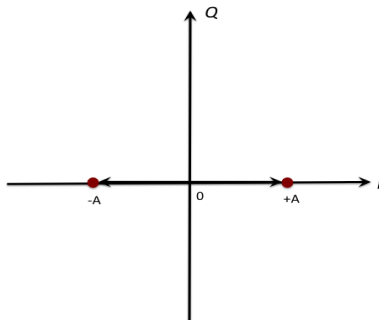
- simplicity of the BB filter ;
- Spectral efficiency ($\eta = D/BW$) ;
- ISI (Nyquist) ;
- Power amplifier performance ;
- PAPR.

Vectorial description of $\{a_n\}$

Amplitudes $\{a_n\}$ are represented by real or complex numbers

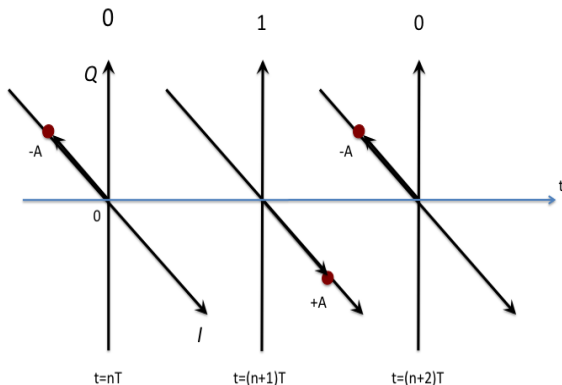
In the previous case : $0 \rightarrow -A$ $1 \rightarrow +A$

Viewed in the complex plane $I - Q$, this modulation can be represented by :



Vectorial description of $\{a_n\}$

Each « vector » carries 1 information bit
Only one complex dimension is required !



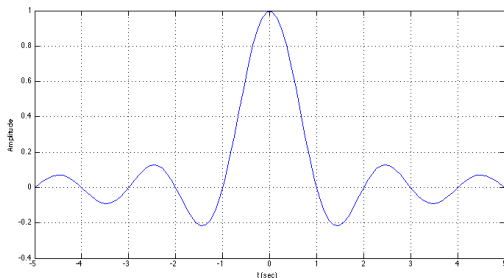
Vectorial description of $\{u(t)\}$

$h(t)$ do the “temporal link” between the discrete “vectors”.

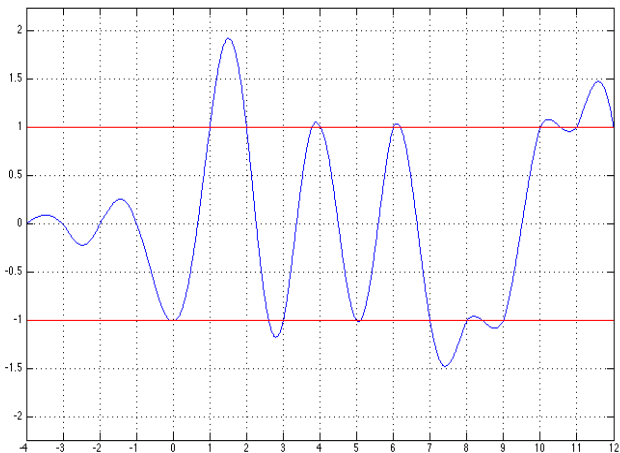
Two ways to realize $h(t)$:

- $h(t)$ limited in time $\rightarrow h(t) \neq 0 \quad t \in [0, T)$;
- $h(t)$ Nyquist.

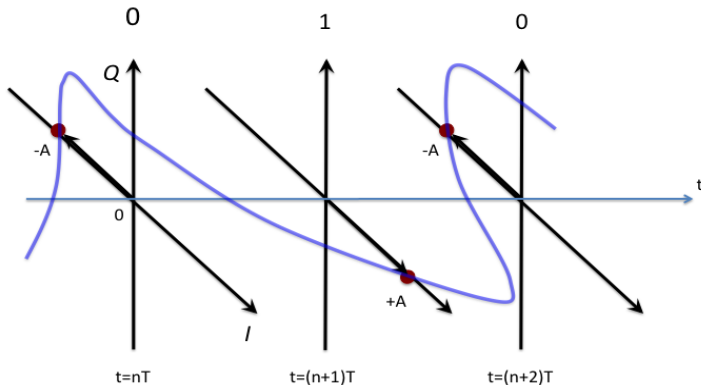
$$h(nT) = \begin{cases} 1 & \text{pour } n = 0 ; \\ 0 & \forall n \neq 0. \end{cases}$$



Vectorial description of $\{u(t)\}$



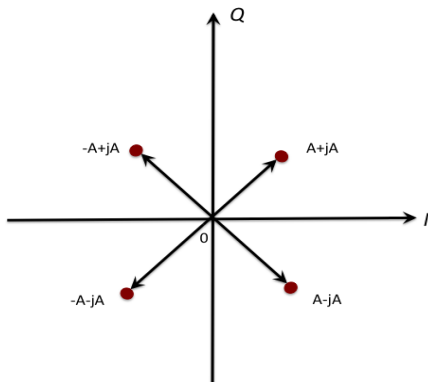
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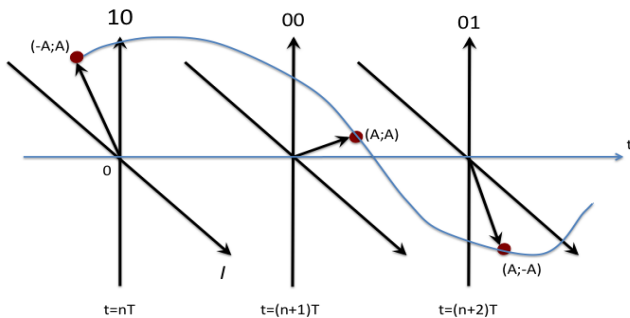
$$u(t) \in \mathcal{C}$$

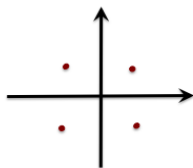
$$u(t) = \sum_n a_n \cdot h(t - nT)$$

$$a_n = \{A + jA; -A + jA; -A - jA; A - jA\}$$

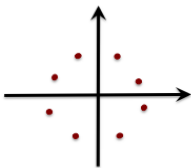


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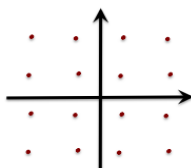


Generalization to the M^{th} order

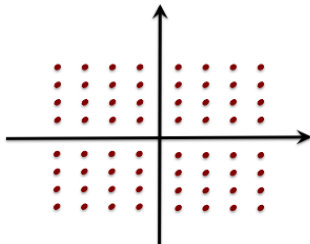
4-PSK



8-PSK



16-QAM



64-QAM

Advantages of order M

Each complex symbol represents :

$$N = \log_2 M \text{ bits}$$

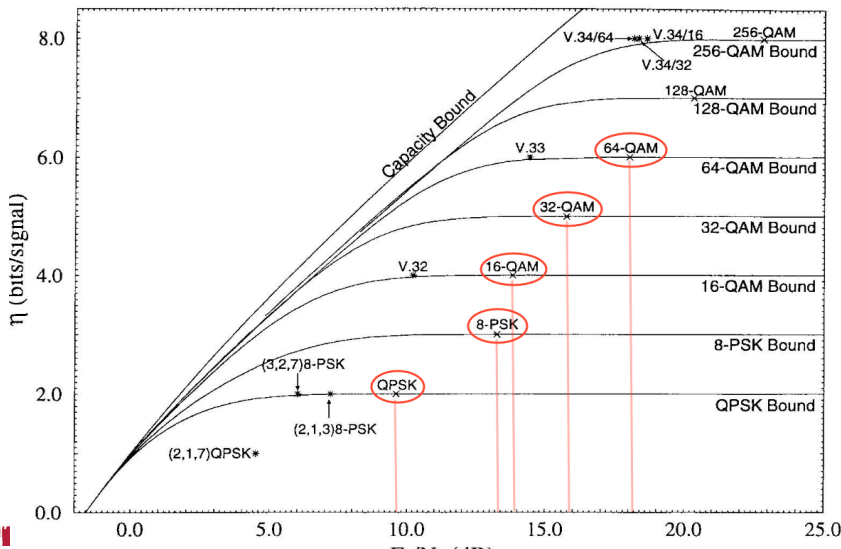
$$T_s = N.T_b$$

$$R = \frac{D}{N}$$

this means a great economy in bandwidth B_w !

Unfortunately, a significant increase in E_b/N_0
to have the same performances !

Spectral efficiency



M-QAM Modulation - theoretical analysis

Emitted signal

$$s(t) = \sum_n a_n h_t(t - nT)$$

Received signal

$$r(t) = \sum_n a_n h_{Rx}(t - nT) + b(t)$$

où

$$h_{Rx}(t) = h_t(t) * h_c(t) * h_r(t)$$

Sampled received signal

$$r(kT + \tau) = \sum_n a_n h_{Rx}(kT + \tau - nT) + b(kT + \tau)$$

M-QAM Modulation - theoretical analysis (2)

$$r(k) = \sum_n a_n h_{Rx}(k - n) + b(k)$$

$$r(k) = \underbrace{h_{Rx}(0) \cdot a_k}_{\text{symbole } k} + \underbrace{\sum_{n \neq k} a_n h_{Rx}(k - n)}_{\text{IES}} + \underbrace{b(k)}_{\text{bruit}}$$

$$\sum_k H_{Rx}\left(f + \frac{k}{T}\right) = T \cdot h_{Rx}(0)$$

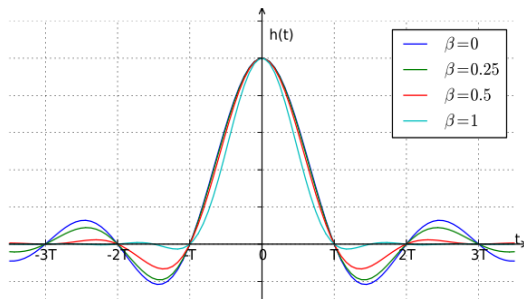
$$H_{Rx}(f) = \begin{cases} T & |f| \leq \frac{1}{2T} \\ 0 & |f| > \frac{1}{2T} \end{cases}$$

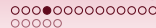
M-QAM Modulation - theoretical analysis (3)

$$H_{Rx}(f) = \begin{cases} T & 0 \leq |f| \leq \frac{1-\alpha}{2T} \\ \frac{T}{2} \left(1 - \sin\left[\frac{\pi T}{\alpha} \left(f - \frac{1}{2T}\right)\right]\right) & \frac{1-\alpha}{2T} < |f| < \frac{1+\alpha}{2T} \end{cases}$$

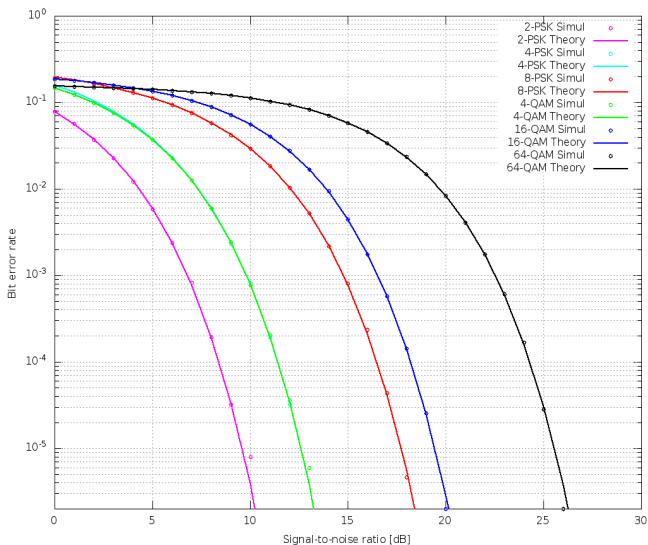
Raised cosine filter

$$h_{Rx}(t) = \frac{\cos(\alpha\pi t/T)}{1 - 4\alpha^2 t^2/T^2} \operatorname{sinc}(t/T).$$

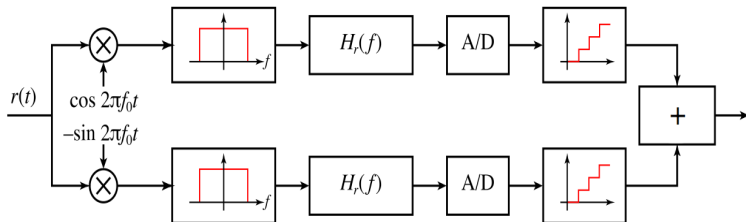
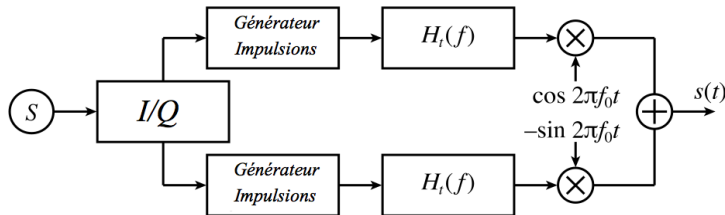




M-QAM Modulation - Performance in AWGN



M-QAM Modulation - Structure of Tx/Rx





Distortions

Distortion sources :

- channel linear distortion ;
- Tx distortions ;
- Rx distortions.

Channel distortions - M-QAM

Possible sources :

- bandwidth limitation ;
- frequency selectivity (*fading*) ;

$$\text{If } B_c \lll \frac{1}{T} \text{ ISI!}$$

ISI impact on Tx signal

At sampling times, signal is not at "RV" !

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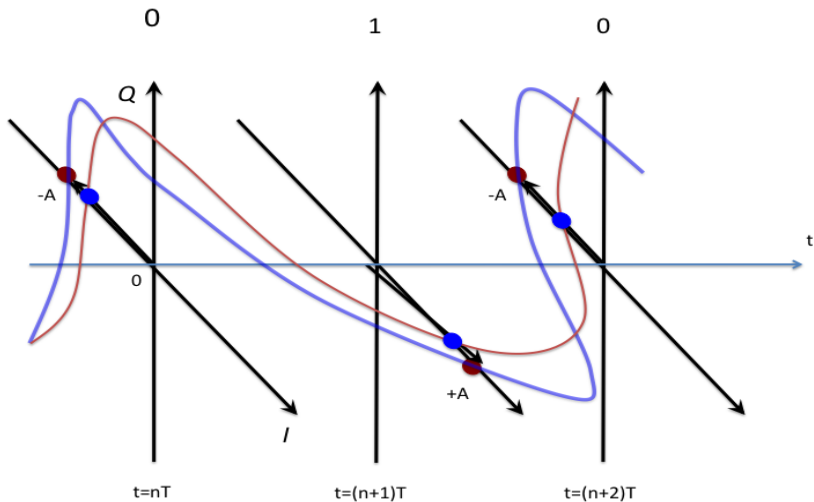
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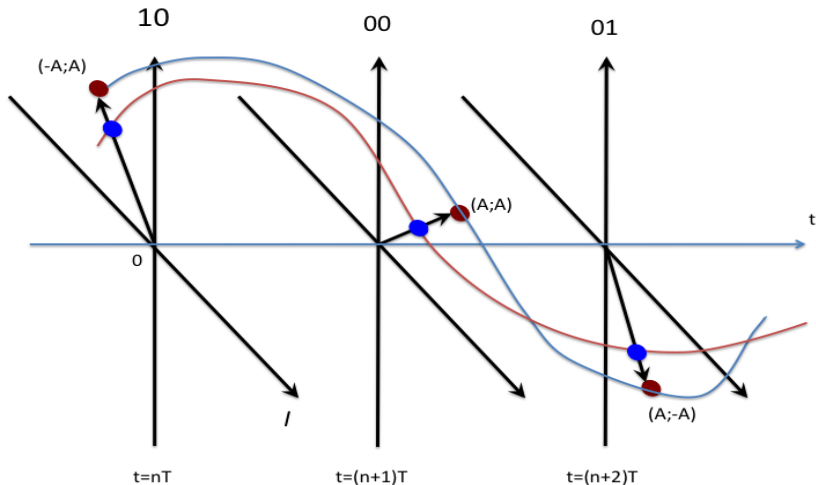
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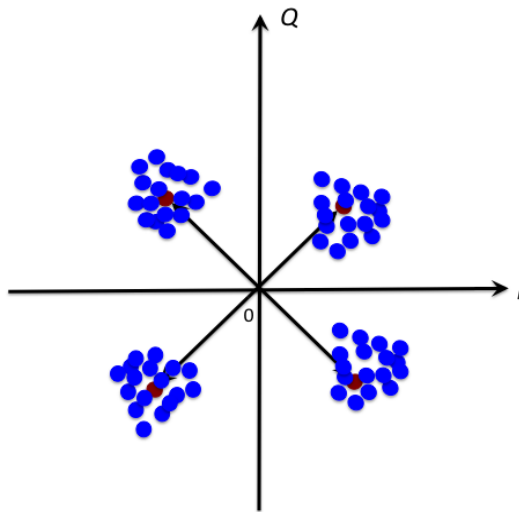
Example - BPSK



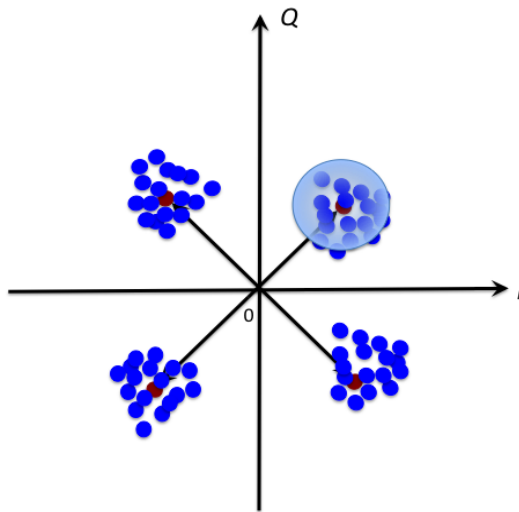
Example - QPSK



Impact of ISI - Dispersion diagrams

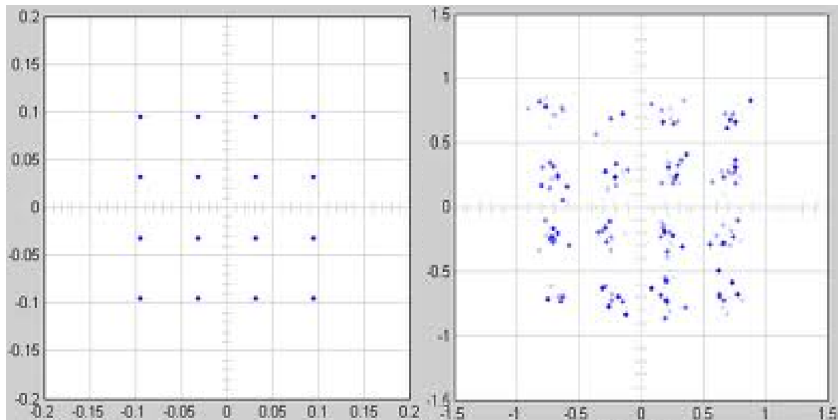


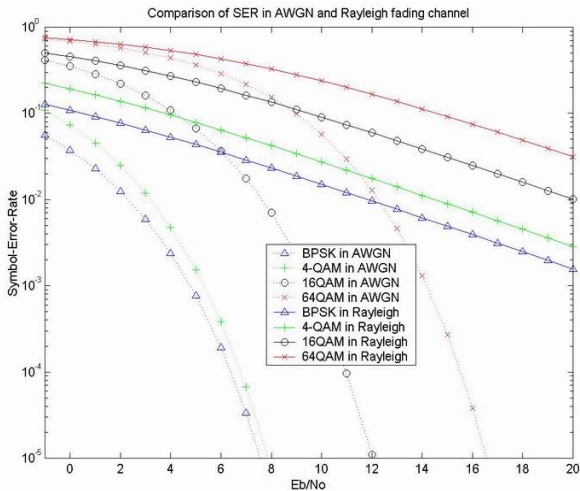
Impact of ISI - Dispersion diagrams



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Dispersion diagrams - 16-QAM



P_{sym} - M-QAM in Rayleigh channel

Equalisation

The idea consists to suppress the distortion induced by channel.

two ways to solve this problem :

- frequency domain filtering ;
- deconvolution in time (with the good IR!).

Three techniques are possible :

- linear equalization ;
- decision feedback equalization ;
- sequence estimation equalization.

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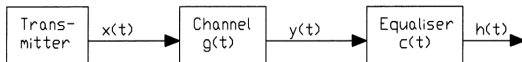
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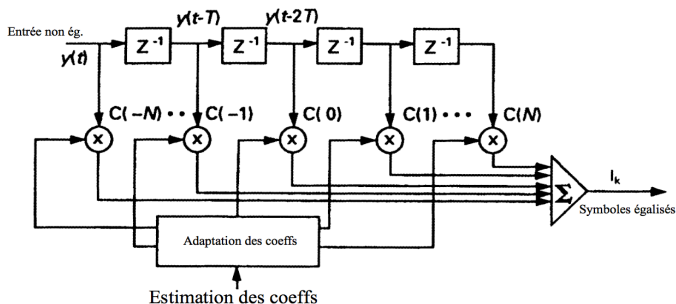
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Linear equalization

Model

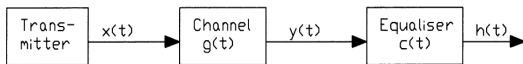


It's a linear filter with IR : $c(kT) = \sum_{k=-N}^{+N} c(k) \cdot z^{-kT}$



Zero forcing and **MSE** criteria

Model



The filtre coefficients can be calculateur as :

- suppress the ISI in and sample interval $(-N; +N) \rightarrow$ *Zero Forcing* ;
- minimise the mean error distortion \rightarrow *EQM*.

Zero forcing and **MSE** criteria

If $h(n) = y(n) * c(n)$, we call **Mean square error** ;

$$MSE = \frac{1}{h^2(0)} \sum_{k=-\infty; k \neq 0}^{+\infty} h^2(n)$$

To minimize the ISI, minimise the **MSE**

$$MSE = \epsilon = \left(\sum_{k=-\infty; k \neq 0}^{+\infty} h^2(n) \right) - h^2(0)$$

This is a quadratic fonction of the coefficients → classic problem in spectral estimation (Levinson-Durbin).

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To minimize the ISI, minimise the **MSE**

$$MSE = \epsilon = \left(\sum_{k=-\infty; k \neq 0}^{+\infty} h^2(n) \right) - h^2(0)$$

This is a quadratic fonction of the coefficients → classic problem in spectral estimation (Levinson-Durbin).

Zero forcing and **MSE** criteria

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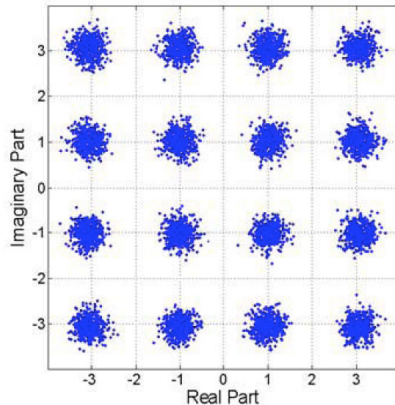
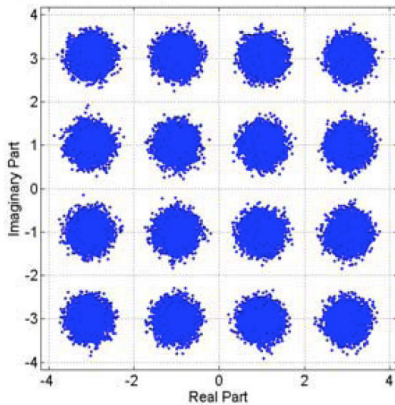
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Example MSE-16-QAM, 3 coeffs channel





The End