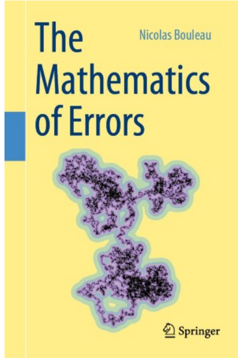


**ONLINE SUMMER SCHOOL — FREE OF CHARGE**

**For Ukrainian students** — open to the wider mathematical community

# Introduction to Stochastic Calculus and Error Theory

Error propagation with Dirichlet forms: from Gauss to N. Bouleau

**Monday 17 – Friday 21 August 2026**

Mornings **9:00–10:30** and **10:45–12:00** (Kyiv time): lectures

Afternoons **14:00–15:00** and **15:10–16:00** (Kyiv time): lecture and hands-on session

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**What the school is about.** How do errors and uncertainties propagate through a computation? Gauss answered this question for finitely many small Gaussian errors; the modern answer, due to Nicolas Bouleau following his work with Francis Hirsch on Dirichlet forms, views Dirichlet forms as tools for differential calculus compatible with probability theory: to each random quantity one attaches an operator (the "carré du champ" operator $\Gamma$) that propagates its variance. The theory works directly in infinite dimensions (Wiener, Monte-Carlo and Poisson spaces) and can characterize the errors of stochastic differential equations — something Gauss-type error calculus cannot do. The field is fully alive: a conjecture stated by Bouleau and Hirsch in 1986 (*energy image density*, EID) was settled only in October 2025.

The course follows an inter-semester course taught for three years at Télécom Paris. It is *self-contained*: stochastic calculus is one of the subjects taught, not a prerequisite — a basic undergraduate background in probability suffices (see the detailed programme and prerequisites overleaf).

**Practical information.**

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| **Format** | Online (Zoom); the link will be sent to registered participants shortly before the school begins. |
| **Schedule** | Monday 17 to Friday 21 August 2026, Kyiv time (one hour earlier in Paris). Mornings, two lectures: 9:00–10:30 and 10:45–12:00. Afternoons: lecture 14:00–15:00, then applications (exercises, simulations) 15:10–16:00. |
| **Tuition** | Free of charge. |
| **Language** | Lectures and written materials in English; questions may also be asked in French or Ukrainian (written). |
| **Audience** | Ukrainian undergraduate and Master's students in mathematics, applied mathematics, physics, engineering or related fields; PhD students, researchers, and colleagues are most welcome. |
| **Materials** | Lecture notes and exercise sheets (with solutions, including Python simulations) are distributed to participants. |

**How to register.** Send an e-mail to `victor.rabiet@telecom-paris.fr` with subject **"Summer school registration"**, indicating your name, university, and year of study, **by Friday 14 August 2026**. You will receive the Zoom link and the course materials by e-mail.

*The detailed day-by-day programme is given overleaf. Any update (schedule, materials) will be posted on this webpage.*

# Programme

### Introduction to Stochastic Calculus and Error Theory

17–21 August 2026 — mornings 9:00–12:00, afternoons 14:00–16:00 (Kyiv time)

**Monday 17 — Probabilistic foundations and Gauss-type error calculus.** A refresher on measure theory: $\sigma$-algebras, simple functions, $\pi$- and $\lambda$-systems, the Radon–Nikodym theorem; conditional expectation and conditional variance; Gauss's formulas for small errors and their connection with differential operators; classification of error calculi, examples in finite dimension.

**Tuesday 18 — Construction of Brownian motion; from error germ to error structures.** Random processes; the construction of Brownian motion via Schauder's tent functions; an intuitive introduction to error structures: orders of magnitude of errors, from the error germ $\gamma$ to the carré du champ operator $\Gamma$; simulation of random variables and of Brownian motion.

**Wednesday 19 — Stochastic calculus: Itô integral.** Quadratic variation of Brownian motion; filtrations, adaptation and martingales; construction of the Itô integral: progressively measurable processes, integration of step processes, the Itô isometry and its extension by density; Itô processes and Itô's formula, with first examples of stochastic differential calculus.

**Thursday 20 — Stochastic calculus: diffusions; semigroups and Dirichlet forms.** The Markov property, diffusions and their simulation by the Euler scheme (the Langevin equation, the Black–Scholes model); strongly continuous semigroups on a Banach space; the Ornstein–Uhlenbeck semigroup; Dirichlet forms; the Dirichlet form associated with the Langevin equation as a worked example of error propagation through an SDE.

**Friday 21 — Error structures in the general case; applications.** General definition of an error structure $(\Omega, \mathcal{A}, \mathbb{P}, \mathbb{D}, \Gamma)$: dense domain $\mathbb{D}$, functional calculus of class $\mathcal{C}^1 \cap \mathrm{Lip}$, closedness of the form (Dirichlet norm), Markovian case; images and products of error structures; applications and outlook: sensitivity of stochastic models, the EID conjecture (settled in 2025), current research on thermal fluctuations in statistical physics.

**Prerequisites and level.** The school is accessible to anyone with a basic knowledge of probability, as taught in the first two years of undergraduate studies (Bachelor's level). Stochastic calculus itself is taught from scratch — no prior knowledge of it is assumed — and the measure-theoretic notions used (conditional expectation, $\pi$- and $\lambda$-systems, Radon–Nikodym) are recalled on the first day. For a few parts (the construction of Brownian motion, the Itô integral), some familiarity with Hilbert bases and $L^2$ convergence is helpful, though not indispensable. The last day, devoted to error structures in the general case, is the most advanced one; its emphasis can be adjusted to the audience.

**Exercises and simulations.** Each afternoon session ends with one hour of applications: exercise sheets (with solutions) and Python notebooks (simulation of Brownian motion, Euler scheme for diffusions), so that participants can experiment on their own.

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**References.**


[1] N. Bouleau, F. Hirsch, *Dirichlet Forms and Analysis on Wiener Space*, De Gruyter, 1991 (where the EID conjecture is stated).

[2] N. Bouleau, *Error Calculus for Finance and Physics: The Language of Dirichlet Forms*, De Gruyter, 2003.

[3] N. Bouleau, *The Mathematics of Errors*, Springer, 2022 (French original: *Théorie des erreurs*, Spartacus-IDH & Cassini, 2019).

[4] S. Eriksson-Bique, M. Murugan, *On the energy image density conjecture of Bouleau and Hirsch*, arXiv:2510.13659 (preprint), 2025.

[5] For probability and stochastic calculus, in Ukrainian: М. В. Карташов, *Імовірність, процеси, статистика*, ВПЦ «Київський університет», Kyiv, 2008 — official PDF: `https://probability.knu.ua/userfiles/kmv/VPS_Pv.pdf`.


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