Supplemental Material for "A Novel Mixture Model for Characterizing Human Aiming Performance Data" by Yanxi Li, Derek S. Young, Julien Gori, and Olivier Rioul

S.1: Algorithms

Algorithm 1 Block-Relaxation Method for EMG Regression

Input: $\mathbf{X}$ (matrix of predictors), $\mathbf{y}$ (response vector)
Output: Final estimate $\widehat{\boldsymbol{\psi}}$ for $\boldsymbol{\psi} = (\boldsymbol{\beta}^\top, \sigma^2, \alpha)$

1: Initialize the iteration $t = 0$; set the difference $\mathit{diff} = 1$
2: Initialize the method by selecting starting values $\boldsymbol{\psi}^{(0)} = (\boldsymbol{\beta}^{(0)\top}, \sigma^{2(0)}, \alpha^{(0)})^\top$
3: **while** $\mathit{diff} > \epsilon$ **do**
 Update $\boldsymbol{\psi}_1^{(t+1)} = \underset{\boldsymbol{\psi}_1}{\arg\max}\ \mathbf{Q}(\boldsymbol{\psi}_1; \boldsymbol{\psi}_2^{(t)})$
 Update $\boldsymbol{\psi}_2^{(t+1)} = \underset{\boldsymbol{\psi}_2}{\arg\max}\ \mathbf{Q}(\boldsymbol{\psi}_2; \boldsymbol{\psi}_1^{(t+1)})$
 Update the difference: $\mathit{diff} \longleftarrow |\mathbf{Q}(\boldsymbol{\psi}^{(t+1)}) - \mathbf{Q}(\boldsymbol{\psi}^{(t)})|$
 Update $\boldsymbol{\psi}^{(t)} \longleftarrow \boldsymbol{\psi}^{(t+1)}$
 $t \longleftarrow t + 1$
4: **end while**
5: Output $\widehat{\boldsymbol{\psi}} = \boldsymbol{\psi}^{(t)}$

Algorithm 2 ECM Algorithm for Flare Regression Model

Input: $\mathbf{X}$ (matrix of predictors), $\mathbf{y}$ (response vector)
Output: Final estimate $\widehat{\boldsymbol{\theta}}$ for $\boldsymbol{\theta} = (\lambda, \boldsymbol{\beta}^\top, \sigma^2, \alpha)^\top$

1: Initialize the iteration $t = 0$
2: Initialize the estimation by selecting starting values $\boldsymbol{\theta}^{(0)} = (\lambda^{(0)}, \boldsymbol{\beta}^{(0)\top}, \sigma^{2(0)}, \alpha^{(0)})^\top$
3: Estimate the initial hidden variable: $Z_i^{(0)} = \dfrac{\frac{\lambda^{(0)}}{2\pi\sigma^{2(0)}} \exp\left\{-\frac{1}{2\sigma^{2(0)}}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta}^{(0)})^2\right\}}{f(y_i; \mathbf{x}_i, \boldsymbol{\psi}^{(0)})}$
4: Initialize the difference $\mathit{diff} = 1$; update the objective function $m(\boldsymbol{\beta}) = \mathbf{Q}(\boldsymbol{\beta}; \boldsymbol{\theta}^{(0)})$
5: **while** $\mathit{diff} > \epsilon$ **do**
 Estimate parameters in $\boldsymbol{\theta}_2$: $\boldsymbol{\beta}^{(t+1)} = \boldsymbol{\beta}^{(t)} - \left[\frac{d^2 m}{d\boldsymbol{\beta}^2}\right]^{-1}\Bigg|_{\boldsymbol{\beta}=\boldsymbol{\beta}^{(t)}} \frac{dm}{d\boldsymbol{\beta}}\Bigg|_{\boldsymbol{\beta}=\boldsymbol{\beta}^{(t)}}$
 Set $\boldsymbol{\theta}^{(t+1/2)} = (\lambda^{(t)}, \boldsymbol{\beta}^{(t+1)\top}, \sigma^{2(t)}, \alpha^{(t)})^\top$
 Re-estimate the hidden variable: $Z_i^{(t+1/2)} = \dfrac{\frac{\lambda^{(t)}}{2\pi\sigma^{2(t)}} \exp\left\{-\frac{1}{2\sigma^{2(t)}}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta}^{(t+1)})^2\right\}}{f(y_i; \mathbf{x}_i, \boldsymbol{\theta}^{(t+1/2)})}$
 Estimate parameters in $\boldsymbol{\theta}_2$:

$$\lambda^{(t+1)} = \frac{1}{n}\sum_{i=1}^{n} Z_i^{(t+1/2)}$$

$$\sigma^{2(t+1)} = \frac{\sum_{i=1}^{n} Z_i^{(t+1/2)}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta}^{(t+1)})^2}{\sum_{i=1}^{n} Z_i^{(t+1/2)}}$$

$$\alpha^{(t+1)} = \frac{\sum_{i=1}^{n}(1 - Z_i^{(t+1/2)})}{\sum_{i=1}^{n}(1 - Z_i^{(t+1/2)})(y_i - \mathbf{x}_i^\top \boldsymbol{\beta}^{(t+1)})}$$

 Set $\boldsymbol{\theta}^{(t+1)} = (\lambda^{(t+1)}, \boldsymbol{\beta}^{(t+1)\top}, \sigma^{2(t+1)}, \alpha^{(t+1)})^\top$
 Update the difference: $\mathit{diff} \longleftarrow ||\boldsymbol{\theta}^{(t+1)} - \boldsymbol{\theta}^{(t)}||_\infty$
 Re-estimate the hidden variable: $Z_i^{(t+1)} = \dfrac{\frac{\lambda^{(t+1)}}{2\pi\sigma^{2(t+1)}} \exp\left\{-\frac{1}{2\sigma^{2(t+1)}}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta}^{(t+1)})^2\right\}}{f(y_i; \mathbf{x}_i, \boldsymbol{\theta}^{(t+1)})}$
 Update $\boldsymbol{\theta}^{(t)} \longleftarrow \boldsymbol{\theta}^{(t+1)}$, $t \longleftarrow t + 1$
6: **end while**
7: Output $\widehat{\boldsymbol{\theta}} = \boldsymbol{\theta}^{(t)}$

S.2: Proof of Part (1) in Theorem 1

Proving part (1) in Theorem 1 is equivalent to proving the logarithm of $f_X(x;\mu,\sigma,\alpha)$ is strictly concave in x, where f_X is the density function of random variable $X \sim EMG(\mu,\sigma,\alpha)$. Since $X \sim EMG(\mu,\sigma,\alpha)$, X can be expressed as $X = Y + Z$, where Y and Z are independent, Y is Gaussian with mean μ and variance σ^2, and Z is exponential with rate α. Y and Z are random variables with probability densities from the exponential family. Since the loglikelihood function in an exponential family is concave, the probability densities for Y and Z are log-concave. Then, the logarithm of $f_X(x;\mu,\sigma,\alpha)$ is strictly concave in x following the fact that the sum of two independent log-concave random variables is log-concave.

S.3: Proof of Part (2) in Theorem 1

For the EMG regression model, the density function for a single observation is:

$$f(y_i;\mathbf{x}_i,\boldsymbol{\psi}) = \frac{\alpha}{2}e^{\frac{\alpha}{2}[\alpha\sigma^2-2(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})]}\,\mathrm{erfc}\Big(\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\Big).$$

The data loglikelihood is:

$$\ell(\boldsymbol{\psi}) = n\Big(\log\frac{\alpha}{2}+\frac{\alpha^2\sigma^2}{2}\Big) - \sum_{i=1}^{n}\Big\{\alpha(y_i-\mathbf{x}_i^\top\boldsymbol{\beta}) - \log\Big[\mathrm{erfc}\Big(\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\Big)\Big]\Big\}. \tag{1}$$

Taking the second-order derivative of $\log(f_X)$ with respect to x, we have $d^2\log(f_X)/dx^2 < 0$. Also, notice that $d^2\log(f_X)/dx^2$ is the same sign as $d^2\log(\mathrm{erfc}(-x))/dx^2$, where

$$\begin{aligned}\frac{d^2\log(\mathrm{erfc}(-x))}{dx^2} &= \frac{4(-x)e^{-x^2}}{\sqrt{\pi}(2-\mathrm{erfc}(x))} - \frac{4e^{-2x^2}}{\pi(2-erfc(x))^2}\\ &= \frac{4e^{-2x^2}[\sqrt{\pi}(-x)e^{x^2}\,\mathrm{erfc}(-x)-1]}{\pi(2-\mathrm{erfc}(x))^2}.\end{aligned}$$

Hence, we have:

$$\frac{d^2\log(f_X)}{dx^2} < 0 \Rightarrow \frac{d^2\log(\mathrm{erfc}(-x))}{dx^2} < 0 \Rightarrow \sqrt{\pi}xe^{x^2}\,\mathrm{erfc}(x) < 1, \tag{2}$$

for any $x \in \mathbb{R}$.

Next, consider the EMG regression

$$y_i = \mathbf{x}_i\boldsymbol{\beta} + \epsilon_i,$$

where

$$\epsilon_i \sim EMG(0,\sigma,\alpha)$$

. Without loss of generality, we assume $\boldsymbol{\beta}^T = (\beta_1, ..., \beta_m)$. Then, the Hessian matrix of the data loglikelihood with respect to $\boldsymbol{\beta}$ is:

$$\mathbf{H}_{\boldsymbol{\beta}} = \begin{pmatrix} \frac{\partial^2\ell(\boldsymbol{\psi})}{\partial\beta_1^2} & \frac{\partial^2\ell(\boldsymbol{\psi})}{\partial\beta_1\partial\beta_2} & \cdots & \frac{\partial^2\ell(\boldsymbol{\psi})}{\partial\beta_1\partial\beta_m}\\ \vdots & \frac{\partial^2\ell(\boldsymbol{\psi})}{\partial\beta_2^2} & \cdots & \frac{\partial^2\ell(\boldsymbol{\psi})}{\partial\beta_2\partial\beta_m}\\ \vdots & \vdots & \ddots & \\ \frac{\partial^2\ell(\boldsymbol{\psi})}{\partial\beta_m\partial\beta_1} & \cdots & \cdots & \frac{\partial^2\ell(\boldsymbol{\psi})}{\partial\beta_m^2} \end{pmatrix},$$

where $\ell(\boldsymbol{\psi})$ is the data loglikelihood in (1). To calculate $\mathbf{H}_{\boldsymbol{\beta}}$, we form

$$\ell_i(\boldsymbol{\psi}) = \Big(\log\frac{\alpha}{2}+\frac{\alpha^2\sigma^2}{2}\Big) - \Big\{\alpha(y_i-\mathbf{x}_i^\top\boldsymbol{\beta}) - \log\Big[\mathrm{erfc}\Big(\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\Big)\Big]\Big\}$$

and

$$\ell(\boldsymbol{\psi}) = \sum_{i=1}^{n} \ell_i(\boldsymbol{\psi}). \tag{3}$$

Hence, we can rewrite $\mathbf{H}_{\boldsymbol{\beta}}$ as $\mathbf{H}_{\boldsymbol{\beta}} = \sum_{i=1}^{n} \mathbf{H}_i$ such that

$$\mathbf{H}_i = \begin{pmatrix} \frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \beta_1^2} & \frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \beta_1 \partial \beta_2} & \cdots & \frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \beta_1 \partial \beta_m} \\ \vdots & \frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \beta_2^2} & \cdots & \frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \beta_2 \partial \beta_m} \\ \vdots & \vdots & \ddots & \\ \frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \beta_m \partial \beta_1} & \cdots & \cdots & \frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \beta_m^2} \end{pmatrix}$$

and where

$$\frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \beta_j^2} = \frac{2x_{ij}^2 e^{-2\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]^2}\left\{\sqrt{\pi}\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]e^{\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]^2}\operatorname{erfc}\left(\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right)-1\right\}}{\pi\sigma^2\operatorname{erfc}^2\left(\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right)}$$

$$\frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \beta_j \partial \beta_k} = \frac{2x_{ij}x_{ik} e^{-2\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]^2}\left\{\sqrt{\pi}\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]e^{\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]^2}\operatorname{erfc}\left(\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right)-1\right\}}{\pi\sigma^2\operatorname{erfc}^2\left(\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right)}$$

, for $1 \leq i \leq n$, $1 \leq j \leq m$, and $j \neq k$. From the inequality in (2), we have:

$$\sqrt{\pi}\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]e^{\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]^2}\operatorname{erfc}\left(\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right)-1<0. \tag{4}$$

Now, let $\mathbf{V} = (V_1, ..., V_m)^T \in \mathbb{R}^m$. We then have

$$\mathbf{V}^T\mathbf{H}_i\mathbf{V} = \sum_{j=1}^{n}\sum_{k=1}^{n} V_j V_k \frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \beta_j \partial \beta_k}$$

. For an arbitrary fixed i, let

$$q_i = \frac{2e^{-2\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]^2}\left\{\sqrt{\pi}\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]e^{\left[\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right]^2}\operatorname{erfc}\left(\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right)-1\right\}}{\pi\sigma^2\operatorname{erfc}^2\left(\frac{\alpha\sigma^2-(y_i-\mathbf{x}_i^\top\boldsymbol{\beta})}{\sqrt{2}\sigma}\right)}. \tag{5}$$

Then,

$$\mathbf{V}^\top\mathbf{H}_i\mathbf{V} = \sum_{j=1}^{n}\sum_{k=1}^{n} V_j V_k x_{ij} x_{ik} q_i = q_i\Big(\sum_{j=1}^{n} V_j x_{ij}\Big)^2$$

From the inequality in (4), we have $q_i < 0$. Hence, $\mathbf{V}^\top\mathbf{H}_i\mathbf{V} < 0 \Rightarrow \mathbf{H}_i$ is negative-definite. Then, $l_i(\boldsymbol{\psi})$ is strictly concave in $\boldsymbol{\beta}$ for every $1 \leq i \leq n$. As a trivial consequence, $l(\boldsymbol{\psi})$ is strictly concave in $\boldsymbol{\beta}$.

S.4: Proof of Part (3) in Theorem 1

Similarly, we have:

$$\frac{\partial^2 \ell(\boldsymbol{\psi})}{\partial \sigma^2} = \sum_{i=1}^{n} \frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \sigma^2}$$

. For an arbitrary fixed i,

$$\frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \sigma^2} = \sigma^2 - \frac{1}{\alpha^2} + \sigma^4(*),$$

where q_i is defined as above. Hence, $q_i < 0$. Then, $\alpha\sigma < 1$ will be a sufficient condition for each $\frac{\partial^2 \ell_i(\boldsymbol{\psi})}{\partial \sigma^2} < 0$. Hence, it is a sufficient condition for $\frac{\partial^2 \ell(\boldsymbol{\psi})}{\partial \sigma^2} < 0$.

S.5: Technical Details for the Derivation of Fisher Information by Louis's Method

Note that

$$\mathbf{I}(\boldsymbol{\theta}) = -\mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left(\frac{\partial^2 \ell_c(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^\top}\right) - \mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left[\left(\frac{\partial \ell_c(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)\left(\frac{\partial \ell_c(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^\top\right] + \mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left(\frac{\partial \ell_c(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)\mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left(\frac{\partial \ell_c(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^\top. \tag{6}$$

Rewrite

$$\ell_c(\boldsymbol{\theta}) = \sum_{i=1}^{n} \ell_i(\boldsymbol{\theta}; y_i, \mathbf{x}_i, Z_i)$$

, where

$$\begin{aligned}\ell_i(\boldsymbol{\theta}; y_i, \mathbf{x}_i, Z_i) =& Z_i \log\left(\frac{\lambda}{\sqrt{2\pi\sigma^2}} \exp\Big\{-\frac{1}{2\sigma^2}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta})^2\Big\}\right) \\ &+ (1 - Z_i)\log\Big[(1-\lambda)\alpha \exp\Big(-\alpha(y_i - \mathbf{x}_i^\top \boldsymbol{\beta}) I\Big\{(y_i - \mathbf{x}_i^\top \boldsymbol{\beta}) > 0\Big\}\Big)\Big]\end{aligned}$$

As Louis [1] points out, when $y_1, \ldots, y_n$ are independent but not necessarily identical distributed, we have:

$$\mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left(\frac{\partial^2 \ell_c(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^\top}\right) = \sum_{i=1}^{n} \mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left(\frac{\partial^2 \ell_i(\boldsymbol{\psi}; y_i, \mathbf{x}_i, Z_i)}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^\top}\Big|_{\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}}\right)$$

$$\begin{aligned}\mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left[\left(\frac{\partial \ell_c(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)\left(\frac{\partial \ell_c(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^\top\right] =& \sum_{i=1}^{n} \mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left[\left(\frac{\partial \ell_i(\boldsymbol{\theta}; y_i, \mathbf{x}_i, Z_i)}{\partial \boldsymbol{\theta}}\right)\left(\frac{\ell_i(\boldsymbol{\theta}; y_i, \mathbf{x}_i, Z_i)}{\partial \boldsymbol{\theta}}\right)^\top\right] + \\ & 2\sum_{i<j}^{n}\left[\mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left(\frac{\partial \ell_i(\boldsymbol{\theta}; y_i, \mathbf{x}_i, Z_i)}{\partial \boldsymbol{\theta}}\right)\right]\left[\mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left(\frac{\partial \ell_j(\boldsymbol{\theta}; y_j, \mathbf{x}_j, Z_j)}{\partial \boldsymbol{\theta}}\right)\right]^\top\end{aligned}$$

$$\mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left(\frac{\partial \ell_c(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right) = \sum_{i=1}^{n} \mathbb{E}_{\widehat{\boldsymbol{\theta}}}\left(\frac{\partial \ell_i(\boldsymbol{\theta}; y_i, \mathbf{x}_i, Z_i)}{\partial \boldsymbol{\theta}}\right)$$

.

For any $1 \le i \le n$, $j \in \{1, 2\}$, let

$$\delta(Z_i, j) = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Denote $\mathbb{E}_{\boldsymbol{\theta}}\big[\delta(Z_i, j = 1)\big] = \gamma_{i1}(\boldsymbol{\theta})$, and $\mathbb{E}_{\boldsymbol{\theta}}\big[\delta(Z_i, j = 2)\big] = \gamma_{i2}(\boldsymbol{\theta})$. We then have

$$\gamma_{i1}(\boldsymbol{\theta}) = \frac{\frac{\lambda}{\sqrt{2\pi\sigma^2}} \exp\Big\{-\frac{1}{\sigma^2}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta})^2\Big\}}{f(y_i; x_i, \boldsymbol{\theta})}$$

$$\gamma_{i2}(\boldsymbol{\theta}) = \frac{(1-\lambda)\alpha\Big\{-\alpha(y_i - \mathbf{x}_i^\top\boldsymbol{\beta})\Big\}I\Big\{(y_i - \mathbf{x}_i^\top\boldsymbol{\beta}) > 0\Big\}}{f(y_i; x_i, \boldsymbol{\theta})}$$

.

Next we need to calculate $\mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^\top}\Big)$, $\mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\Big)$, and $\mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\Big)\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\Big)^\top\Big]$ to form $\mathbf{I}(\boldsymbol{\theta})$. Still, without loss of generality, we assume $\boldsymbol{\beta}^\top = (\beta_1, ..., \beta_m)$. To form $\mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^\top}\Big)$, we calculate:

$$\mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^\top}\Big) = \begin{pmatrix} \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\lambda^2}\Big) & \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\lambda\partial\sigma^2}\Big) & \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\lambda\partial\alpha}\Big) & \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\lambda\partial\beta_1}\Big) & \dots & \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\lambda\partial\beta_m}\Big) \\ \vdots & \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{(\partial\sigma^2)^2}\Big) & \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\sigma^2\partial\alpha}\Big) & \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\sigma^2\partial\beta_1}\Big) & \dots & \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\sigma^2\partial\beta_m}\Big) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\lambda\partial\beta_m}\Big) & \dots & \dots & \dots & \dots & \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial^2 \ell_i}{\partial\beta_m^2}\Big) \end{pmatrix}$$

, where, for $1 \le i \le n$,

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\lambda^2}\right) = -\left(\frac{\gamma_{i1}}{\lambda^2} + \frac{\gamma_{i2}}{(1-\lambda^2)}\right)$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{(\partial\sigma^2)^2}\right) = \gamma_{i1}\left[\frac{1}{2\sigma^4} - \frac{(y_i - \mathbf{x}_i^\top\boldsymbol{\beta})^2}{\sigma^6}\right]$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\alpha^2}\right) = -\frac{\gamma_{i2}}{\alpha^2}$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\beta_j^2}\right) = -\gamma_{i1}\frac{x_{ij}^2}{\sigma^2}, 1 \le j \le m$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\lambda\partial\sigma^2}\right) = \mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\lambda\partial\alpha}\right) = \mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\lambda\partial\beta_1}\right) = ... = \mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\lambda\partial\beta_m}\right) = \mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\alpha\partial\sigma^2}\right) = 0$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\sigma^2\partial\beta_j}\right) = -\gamma_{i1}\left[\frac{\mathbf{x}_{ij}(y_i - \mathbf{x}_i^\top\boldsymbol{\beta})}{\sigma^4}\right], 1 \le j \le m$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\alpha\partial\beta_j}\right) = \gamma_{i2}x_{ij}, 1 \le j \le m$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial^2 \ell_i}{\partial\beta_j\beta_k}\right) = -\gamma_{i1}\left(\frac{x_{ij}x_{jk}}{\sigma^2},\right), 1 \le j < k \le m$$

for $1 \le i \le n$.

To form $\mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\Big)$, we have:

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right) = \begin{pmatrix} \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \lambda}\Big) \\ \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \sigma}\Big) \\ \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \alpha}\Big) \\ \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \beta_1}\Big) \\ \vdots \\ \mathbb{E}_{\boldsymbol{\theta}}\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \beta_m}\Big) \end{pmatrix}$$

, where, for $1 \le i \le n$,

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial \ell_i}{\partial \lambda}\right) = \frac{1}{\lambda}\gamma_{i1} - \frac{1}{1-\lambda}\gamma_{i2}$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial \ell_i}{\partial \sigma^2}\right) = \gamma_{i1}\left[-\frac{1}{2\sigma^2} + \frac{(y_i - \mathbf{x}_i^\top \boldsymbol{\beta})^2}{2\sigma^4}\right]$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial \ell_i}{\partial \alpha}\right) = \gamma_{i2}\left[\frac{1}{\alpha} - (y_i - \mathbf{x}_i^\top \boldsymbol{\beta})\right]$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left(\frac{\partial \ell_i}{\partial \beta_j}\right) = \gamma_{i1}\left[\frac{1}{\sigma^2}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta})x_{ij}\right] + \gamma_{i2}\Bigg(\alpha x_{ij}\Bigg), 1 \le j \le m$$

for $1 \le i \le n$

To form $\mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\Big)\Big(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\Big)^\top\Big]$, we have:

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)\left(\frac{\partial \ell_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^\top\right] = \mathbb{E}_{\boldsymbol{\theta}}\begin{pmatrix} \Big(\frac{\partial \ell_i}{\partial \lambda}\Big)^2 & \Big(\frac{\partial \ell_i}{\partial \lambda}\Big)\Big(\frac{\partial \ell_i}{\partial \sigma^2}\Big) & \Big(\frac{\partial \ell_i}{\partial \lambda}\Big)\Big(\frac{\partial \ell_i}{\partial \alpha}\Big) & \Big(\frac{\partial \ell_i}{\partial \lambda}\Big)\Big(\frac{\partial \ell_i}{\partial \beta_1}\Big) & \dots & \Big(\frac{\partial \ell_i}{\partial \lambda}\Big)\Big(\frac{\partial \ell_i}{\partial \beta_m}\Big) \\ \vdots & \Big(\frac{\partial \ell_i}{\partial \sigma^2}\Big)^2 & \Big(\frac{\partial \ell_i}{\partial \alpha}\Big)\Big(\frac{\partial \ell_i}{\partial \sigma^2}\Big) & \Big(\frac{\partial \ell_i}{\partial \sigma^2}\Big)\Big(\frac{\partial \ell_i}{\partial \beta_1}\Big) & \dots & \Big(\frac{\partial \ell_i}{\partial \sigma^2}\Big)\Big(\frac{\partial \ell_i}{\partial \beta_m}\Big) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \Big(\frac{\partial \ell_i}{\partial \lambda}\Big)\Big(\frac{\partial \ell_i}{\partial \beta_m}\Big) & \dots & \dots & \dots & \dots & \Big(\frac{\partial \ell_i}{\partial \beta_m}\Big)^2 \end{pmatrix} =$$

$$\begin{pmatrix} \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i}{\partial \lambda}\Big)^2\Big] & \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i}{\partial \lambda}\Big)\Big(\frac{\partial \ell_i}{\partial \sigma^2}\Big)\Big] & \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i}{\partial \lambda}\Big)\Big(\frac{\partial \ell_i}{\partial \alpha}\Big)\Big] & \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i}{\partial \lambda}\Big)\Big(\frac{\partial \ell_i}{\partial \beta_1}\Big)\Big] & \dots & \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i}{\partial \lambda}\Big)\Big(\frac{\partial \ell_i}{\partial \beta_m}\Big)\Big] \\ \vdots & \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i}{\partial \sigma^2}\Big)^2\Big] & \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial l_i}{\partial \alpha}\Big)\Big(\frac{\partial \ell_i}{\partial \sigma^2}\Big)\Big] & \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i}{\partial \sigma^2}\Big)\Big(\frac{\partial \ell_i}{\partial \beta_1}\Big)\Big] & \dots & \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i}{\partial \sigma^2}\Big)\Big(\frac{\partial \ell_i}{\partial \beta_m}\Big)\Big] \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i}{\partial \lambda}\Big)\Big(\frac{\partial \ell_i}{\partial \beta_m}\Big)\Big] & \dots & \dots & \dots & \dots & \mathbb{E}_{\boldsymbol{\theta}}\Big[\Big(\frac{\partial \ell_i}{\partial \beta_m}\Big)^2\Big] \end{pmatrix}$$

, where, for $1 \leq i \leq n$,

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \lambda}\right)^2\right] = \frac{1}{\lambda^2}\gamma_{i1} + \frac{1}{(1-\lambda)^2}\gamma_{i2}$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \sigma^2}\right)^2\right] = \gamma_{i1}\left[-\frac{1}{2\sigma^2} + \frac{(y_i - \mathbf{x}_i^\top \boldsymbol{\beta})^2}{2\sigma^4}\right]^2$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \alpha}\right)^2\right] = \gamma_{i2}\left[\frac{1}{\alpha} - (y_i - \mathbf{x}_i^\top \boldsymbol{\beta})\right]^2$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \beta_j}\right)^2\right] = \gamma_{i1}\left[\frac{1}{\sigma^2}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta})x_{ij}\right]^2 + \gamma_{i2}\left(\alpha x_{ij}\right)^2, 1 \leq j \leq m$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \lambda}\right)\left(\frac{\partial \ell_i}{\partial \sigma^2}\right)\right] = \frac{1}{\lambda}\gamma_{i1}\left[-\frac{1}{2\sigma^2} + \frac{(y_i - \mathbf{x}_i^\top \boldsymbol{\beta})^2}{2\sigma^4}\right]$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \lambda}\right)\left(\frac{\partial \ell_i}{\partial \alpha}\right)\right] = -\frac{1}{1-\lambda}\gamma_{i2}\left[\frac{1}{\alpha} - (y_i - \mathbf{x}_i^\top \boldsymbol{\beta})\right]$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \lambda}\right)\left(\frac{\partial \ell_i}{\partial \beta_j}\right)\right] = \frac{1}{\lambda}\gamma_{i1}\left[\frac{1}{\sigma^2}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta})x_{ij}\right] - \frac{1}{1-\lambda}\gamma_{i2}\left(\alpha x_{ij}\right), 1 \leq j \leq m$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \alpha}\right)\left(\frac{\partial \ell_i}{\partial \sigma^2}\right)\right] = 0$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \sigma^2}\right)\left(\frac{\partial \ell_i}{\partial \beta_j}\right)\right] = \gamma_{i1}\left[-\frac{1}{2\sigma^2} + \frac{(y_i - \mathbf{x_i}^\top \boldsymbol{\beta})^2}{2\sigma^4}\right]\left[\frac{1}{\sigma^2}(y_i - \mathbf{x_i}^\top \boldsymbol{\beta})x_{ij}\right]$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \alpha}\right)\left(\frac{\partial \ell_i}{\partial \beta_j}\right)\right] = \gamma_{i2}\left[\frac{1}{\alpha}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta})\right]\left(\alpha x_{ij}\right), 1 \leq j \leq m$$

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\left(\frac{\partial \ell_i}{\partial \beta_j}\right)\left(\frac{\partial \ell_i}{\partial \beta_k}\right)\right] = \gamma_{i1}\frac{1}{\sigma^4}(y_i - \mathbf{x}_i^\top \boldsymbol{\beta})^2 x_{ij}x_{ik} + \gamma_{i2}\alpha^2 x_{ij}x_{ik}, j, k \in [1, m]$$

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S.6: Tables

Table 1: Counts of estimation results with the lowest BIC values given by each candidate model. Where **n** indicates the sample size for each generated sample; **flare** indicates the flare regression model; **EMG** indicates the EMG regression model; **linear** indicates the ordnary linear regression model; **regmix** indicates the mixture of linear regressions with two components model; and **B** is Monte Carlo samples for each simulation setting.

Simulation Setting	n	flare	EMG	linear	regmix	B
M1	100	998	2	0	0	1000
M1	500	1000	0	0	0	1000
M1	1000	1000	0	0	0	1000
M2	100	875	123	0	2	1000
M2	500	1000	0	0	0	1000
M2	1000	1000	0	0	0	1000
M3	100	133	848	0	19	1000
M3	500	60	933	0	7	1000
M3	1000	22	978	0	0	1000
M4	150	994	0	0	6	1000
M4	500	1000	0	0	0	1000
M4	1000	844	156	0	0	1000
M5	100	980	19	1	0	1000
M5	500	987	12	0	1	1000
M5	1000	991	9	0	0	1000
M6	100	768	213	19	0	1000
M6	500	999	1	0	0	1000
M6	1000	1000	0	0	9	1000
M7	100	1000	0	0	0	1000
M7	500	1000	0	0	0	1000
M7	1000	1000	0	0	0	1000
M8	100	994	4	0	2	1000
M8	500	1000	0	0	0	1000
M8	1000	1000	0	0	0	1000
M9	100	369	612	0	19	1000
M9	500	674	314	0	12	1000
M9	1000	800	189	0	11	1000
M10	100	977	0	0	23	1000
M10	500	1000	0	0	0	1000
M10	1000	1000	0	0	0	1000
M11	100	893	15	0	92	1000
M11	500	992	0	0	8	1000
M11	1000	967	0	0	33	1000
M12	100	611	200	15	174	1000
M12	500	960	2	0	38	1000
M12	1000	978	0	0	22	1000

Table 2: Root-mean-squared-errors (RMSEs) of each parameter estimates, after fitting the flare regression model to data generated under simulation settings M1–M6.

Simulation Setting	**n**	$\boldsymbol{\lambda}$	$\boldsymbol{\beta_0}$ (intercept)	$\boldsymbol{\beta_1}$	$\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$
M1	100	0.054	0.178	0.018	0.144	0.006
M1	500	0.024	0.057	0.008	0.035	0.003
M1	1000	0.017	0.047	0.005	0.025	0.002
M2	100	0.105	0.401	0.020	0.297	0.022
M2	500	0.059	0.171	0.008	0.063	0.012
M2	1000	0.055	0.164	0.005	0.055	0.010
M3	100	0.319	0.498	0.017	0.241	0.124
M3	500	0.285	0.418	0.007	0.142	0.111
M3	1000	0.283	0.411	0.005	0.134	0.111
M4	100	0.026	0.045	0.008	0.033	0.017
M4	500	0.014	0.024	0.004	0.017	0.008
M4	1000	0.010	0.018	0.002	0.012	0.005
M5	100	0.035	0.055	0.010	0.041	0.094
M5	500	0.016	0.025	0.004	0.018	0.027
M5	1000	0.011	0.018	0.003	0.013	0.019
M6	100	0.044	0.060	0.010	0.044	0.790
M6	500	0.022	0.030	0.004	0.020	0.093
M6	1000	0.016	0.025	0.003	0.014	0.064

Table 3: Mean biases of each parameter estimates, after fitting the flare regression model to data generated under simulation settings M1–M6.

Simulation Setting	**n**	$\boldsymbol{\lambda}$	$\boldsymbol{\beta_0}$ (intercept)	$\boldsymbol{\beta_1}$	$\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$
M1	100	0.008	0.043	−0.001	−0.008	6.731e-04
M1	500	0.004	0.035	0.0002	0.003	−5.229e-05
M1	1000	0.004	0.034	−0.0004	0.005	−2.126e-04
M2	100	0.071	0.221	−0.001	0.076	−7.209e-03
M2	500	0.053	0.161	0.0002	0.044	−7.485e-03
M2	1000	0.052	0.158	−0.0002	0.045	−7.885e-03
M3	100	0.303	0.445	−0.001	0.147	−1.100e-01
M3	500	0.283	0.410	0.0001	0.127	−1.084e-01
M3	1000	0.281	0.407	−0.0001	0.126	−1.094e-01
M4	100	−1.217e-03	0.004	−1.460e-04	−0.003	0.004
M4	500	−4.636e-05	0.001	−2.084e-04	−0.001	0.001
M4	1000	3.344e-04	0.001	−1.035e-04	−0.0004	0.001
M5	100	−3.765e-03	0.003	−5.797e-05	−0.008	0.024
M5	500	−3.160e-05	0.005	2.285e-04	−0.0001	0.002
M5	1000	6.574e-04	0.005	−1.613e-04	0.0005	0.0002
M6	100	−2.185e-03	0.017	−2.459e-04	−0.005	0.089
M6	500	5.373e-03	0.017	2.455e-04	0.001	−0.005
M6	1000	6.398e-03	0.018	−9.835e-05	0.003	−0.013

Table 4: RMSEs of each parameter estimates, after fitting the flare regression model to data generated under simulation settings M7–M12.

Simulation Setting	**n**	$\boldsymbol{\lambda}$	$\boldsymbol{\beta_0}$ (intercept)	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$
M7	100	0.052	0.081	0.0138	0.0139	0.062	0.006
M7	500	0.023	0.036	0.0058	0.0056	0.026	0.003
M7	1000	0.016	0.026	0.0040	0.0042	0.018	0.002
M8	100	0.062	0.093	0.0093	0.0094	0.188	0.031
M8	500	0.030	0.042	0.0040	0.0042	0.164	0.014
M8	1000	0.023	0.037	0.0027	0.0028	0.161	0.010
M9	100	0.175	0.257	0.0144	0.0140	0.123	0.103
M9	500	0.154	0.215	0.0058	0.0062	0.071	0.086
M9	1000	0.153	0.213	0.0041	0.0040	0.067	0.085
M10	100	0.032	0.052	0.0094	0.0092	0.040	0.019
M10	500	0.014	0.024	0.0041	0.0043	0.018	0.006
M10	1000	0.010	0.016	0.0029	0.0028	0.013	0.004
M11	100	0.038	0.060	0.0098	0.0096	0.046	0.220
M11	500	0.018	0.027	0.0041	0.0042	0.019	0.034
M11	1000	0.013	0.065	0.0035	0.0069	0.105	0.188
M12	100	0.046	0.061	0.0098	0.0095	0.053	0.744
M12	500	0.027	0.08	0.0046	0.0046	0.092	0.3136
M12	1000	0.021	0.067	0.0032	0.0030	0.082	0.2974

Table 5: Mean biases of each parameter estimates, after fitting the flare regression model to data generated under simulation settings M7–M12.

Simulation Setting	**n**	$\boldsymbol{\lambda}$	$\boldsymbol{\beta_0}$ (intercept)	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$
M7	100	−0.001	0.012	−7.076e-04	−5.356e-04	−0.017	9.541e-04
M7	500	0.002	0.011	−2.624e-04	2.469e-04	−0.002	1.267e-04
M7	1000	0.002	0.011	−2.151e-04	−1.068e-04	−0.001	−4.33e-05
M8	100	0.012	0.032	7.379e-05	1.778e-04	−0.170	9.094e-05
M8	500	0.013	0.033	8.452e-05	−1.821e-04	−0.163	−3.094e-03
M8	1000	0.013	0.032	9.580e-05	8.652e-05	−0.163	−3.612e-03
M9	100	0.159	0.225	−2.393e-04	−7.881e-04	0.057	−7.762e-02
M9	500	0.150	0.210	−8.385e-05	−1.758e-04	0.058	−8.128e-02
M9	1000	0.151	0.210	1.5738e-04	7.421e-05	0.061	−8.313e-02
M10	100	−0.001	0.003	−5.838e-04	2.421e-04	−0.009	0.005
M10	500	−0.0007	0.0004	3.589e-05	3.892e-05	−0.002	0.001
M10	1000	−0.0002	0.001	−1.988e-04	−7.716e-05	−0.0004	0.001
M11	100	−0.003	0.006	−3.587e-04	1.099e-04	−0.013	0.040
M11	500	0.0001	0.007	5.174e-05	1.764e-05	−0.002	0.004
M11	1000	0.0006	0.010	−2.289e-04	1.829e-04	0.006	0.072
M12	100	−0.0006	0.021	−5.21e-04	1.214e-04	−0.005	0.112
M12	500	0.008	0.028	−3.182e-05	9.468e-05	0.017	0.521
M12	1000	0.009	0.027	−2.617e-04	−9.009e-05	0.015	0.416

Table 6: Table of classification MCAs, each was calculated from $B = 1000$ Monte Carlo samples, featuring settings for two different cut-off probabilities after fitting the flare regression model to data generated under simulation settings M1–M12

Simulation Setting	**n**	B	p^*	MCA	Percentage
M1	300	1000	0.5	287.612	95.9%
M2	300	1000	0.5	266.527	88.9%
M3	300	1000	0.5	194.331	64.8%
M4	300	1000	0.5	297.394	99.1%
M5	300	1000	0,5	292.971	97.7%
M6	300	1000	0.5	283.936	94.6%
M7	300	1000	0.5	291.535	97.2%
M8	300	1000	0.5	267.818	89.3%
M9	300	1000	0.5	225.710	75.2%
M10	300	1000	0.5	298.022	99.3%
M11	300	1000	0.5	291.34	97.1%
M12	300	1000	0.5	282.912	94.3%
M1	300	1000	0.85	277.085	92.4%
M2	300	1000	0.85	256.829	85.6
M3	300	1000	0.85	242.02	80.7%
M4	300	1000	0.85	296.63	98.9%
M5	300	1000	0.85	290.175	96.7%
M6	300	1000	0.85	275.803	91.2%
M7	300	1000	0.85	287.361	95.8%
M8	300	1000	0.85	236.014	78.7%
M9	300	1000	0.85	230.563	76.9%
M10	300	1000	0.85	297.387	99.1%
M11	300	1000	0.85	287.196	95.7%
M12	300	1000	0.85	271.790	90.6%

Table 7: Root-mean-squared-errors (RMSEs) of each parameter estimates, after fitting the flare regression model to data generated under simulation settings M1–M12, but contaminated with outliers

Simulation Setting	**n**	Outlier Percentage	$\boldsymbol{\lambda}$	$\boldsymbol{\beta_0}$ (intercept)	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$
M1	200	1%	0.038	0.086	0.012	NA	0.055	0.008
M1	200	5%	0.032	0.112	0.014	NA	0.068	0.022
M1	200	10%	0.029	0.134	0.010	NA	0.077	0.028
M2	200	1%	0.091	0.313	0.013	NA	0.231	0.034
M2	200	5%	0.148	0.604	0.018	NA	0.512	0.082
M2	200	10%	0.199	0.884	0.020	NA	0.768	0.111
M3	200	1%	0.290	0.414	0.011	NA	0.163	0.172
M3	200	5%	0.368	0.571	0.014	NA	0.325	0.326
M3	200	10%	0.372	0.645	0.012	NA	0.391	0.384
M4	200	1%	0.022	0.037	0.006	NA	0.027	0.023
M4	200	5%	0.044	0.040	0.007	NA	0.029	0.038
M4	200	10%	0.088	0.038	0.007	NA	0.029	0.041
M5	200	1%	0.025	0.043	0.007	NA	0.032	0.082
M5	200	5%	0.036	0.047	0.007	NA	0.030	0.127
M5	200	10%	0.077	0.053	0.007	NA	0.033	0.139
M6	200	1%	0.039	0.052	0.007	NA	0.035	0.288
M6	200	5%	0.022	0.059	0.008	NA	0.035	0.403
M6	200	10%	0.052	0.068	0.007	NA	0.042	0.429
M7	200	1%	0.036	0.055	0.008	0.008	0.041	0.008
M7	200	5%	0.036	0.062	0.009	0.009	0.045	0.019
M7	200	10%	0.046	0.070	0.009	0.011	0.055	0.025
M8	200	1%	0.061	0.114	0.010	0.010	0.056	0.049
M8	200	5%	0.071	0.195	0.010	0.012	0.144	0.104
M8	200	10%	0.068	0.227	0.013	0.012	0.170	0.135
M9	200	1%	0.190	0.254	0.008	0.010	0.115	0.189
M9	200	5%	0.235	0.345	0.011	0.011	0.224	0.335
M9	200	10%	0.217	0.371	0.013	0.012	0.230	0.388
M10	200	1%	0.025	0.037	0.006	0.006	0.0264	0.017
M10	200	5%	0.046	0.037	0.006	0.007	0.025	0.030
M10	200	10%	0.088	0.029	0.006	0.007	0.028	0.033
M11	200	1%	0.022	0.037	0.007	0.007	0.029	0.105
M11	200	5%	0.033	0.048	0.008	0.008	0.034	0.158
M11	200	10%	0.076	0.051	0.006	0.008	0.034	0.168
M12	200	1%	0.032	0.065	0.007	0.007	0.038	0.306
M12	200	5%	0.021	0.060	0.008	0.009	0.036	0.397
M12	200	10%	0.051	0.067	0.006	0.008	0.048	0.431

Table 8: Mean biases of each parameter estimates, after fitting the flare regression model to data generated under simulation settings M1–M12, but contaminated with outliers

Simulation Setting	**n**	Outlier Percentage	$\boldsymbol{\lambda}$	$\boldsymbol{\beta_0}$ (intercept)	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$
M1	200	1%	4.934e-03	3.879e-02	-4.645e-05	NA	7.964e-04	-5.737e-03
M1	200	5%	0.006	0.081	0.003	NA	0.022	-0.022
M1	200	10%	-0.008	0.097	-0.001	NA	0.027	-0.028
M2	200	1%	0.066	0.199	-0.0003	NA	0.073	-0.028
M2	200	5%	0.112	0.361	-0.003	NA	0.223	-0.079
M2	200	10%	0.160	0.636	0.004	NA	0.485	-0.110
M3	200	1%	2.836e-01	3.96e-01	-7.536e-05	NA	1.230e-01	-1.653e-01
M3	200	5%	0.353	0.533	-0.003	NA	0.260	-0.321
M3	200	10%	0.362	0.603	0.003	NA	0.314	-0.382
M4	200	1%	-6.760e-03	7.400e-04	4.279e-05	NA	4.165e-04	-2.017e-02
M4	200	5%	-0.040	0.004	0.0005	NA	0.003	-0.038
M4	200	10%	-0.087	0.001	-0.002	NA	0.006	-0.041
M5	200	1%	0.002	0.012	-0.0003	NA	0.003	-0.073
M5	200	5%	-0.029	0.022	0.0005	NA	0.008	-0.126
M5	200	10%	-0.074	0.027	-0.0004	NA	0.0103	-0.139
M6	200	1%	0.023	0.033	-0.0003	NA	0.011	-0.262
M6	200	5%	-0.0001	0.043	0.0008	NA	0.018	-0.401
M6	200	10%	-0.048	0.046	-0.0002	NA	0.018	-0.428
M7	200	1%	-0.0002	0.020	-0.0002	0.0009	-0.004	-0.006
M7	200	5%	-0.011	0.031	0.0003	0.0002	0.004	-0.018
M7	200	10%	-0.030	0.038	-0.001	0.0004	0.017	-0.025
M8	200	1%	0.040	0.095	-0.0007	-0.0001	0.017	-0.041
M8	200	5%	0.050	0.135	0.0004	0.0003	0.057	-0.101
M8	200	10%	0.048	0.181	0.001	-0.0002	0.094	-0.134
M9	200	1%	0.182	0.245	0.0002	0.002	0.091	-0.174
M9	200	5%	0.223	0.310	0.0004	0.001	0.165	-0.331
M9	200	10%	0.205	0.339	0.003	-0.0004	0.177	-0.386
M10	200	1%	-0.009	-0.003	0.0002	-0.0007	-0.001	-0.016
M10	200	5%	-0.039	0.005	-0.0006	0.0009	0.004	-0.030
M10	200	10%	-0.086	0.002	-0.0006	0.0008	0.003	-0.033
M11	200	1%	0.007	0.018	0.001	-0.002	0.010	-0.095
M11	200	5%	-0.027	0.024	0.0010	0.0008	0.009	-0.150
M11	200	10%	-0.073	0.031	-0.0004	0.001	0.015	-0.168
M12	200	1%	0.023	0.038	-0.001	0.0007	0.007	-0.295
M12	200	5%	-6.278e-03	3.946e-02	9.884e-04	3.623e-06	1.441e-02	-3.951e-01
M12	200	10%	-0.046	0.054	0.0001	0.003	0.030	-0.430

Table 9: Root-mean-squared-errors (RMSEs) of each parameter estimates, after fitting the flare regression model to data generated under simulation settings M13–M18

Simulation Setting	**n**	$\boldsymbol{\lambda}$	$\boldsymbol{\beta_0}$ (intercept)	$\boldsymbol{\beta_1}$	$\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$
M13	200	0.040	0.130	0.022	0.166	0.006
M14	200	0.0340	0.111	0.018	0.078	0.006
M15	200	0.038	0.112	0.0173	0.077	0.006
M16	200	0.449	0.785	0.019	0.199	0.647
M17	200	0.414	0.712	0.017	0.215	0.904
M18	200	0.407	0.703	0.016	0.191	0.843

Table 10: Mean biases of each parameter estimates, after fitting the flare regression model to data generated under simulation settings M13–M18

Simulation Setting	**n**	$\boldsymbol{\lambda}$	$\boldsymbol{\beta_0}$ (intercept)	$\boldsymbol{\beta_1}$	$\boldsymbol{\sigma}$	$\boldsymbol{\alpha}$
M13	200	0.001	0.037	0.0003	-0.071	0.0004
M14	200	2.602e-03	4.186e-02	6.460e-06	-7.446e-03	3.867e-04
M15	200	0.004	0.042	-0.0008	-0.003	0.0006
M16	200	0.445	0.770	0.0007	0.041	-0.126
M17	200	0.411	0.696	0.0004	0.139	-0.081
M18	200	4.038e-01	6.892e-01	-6.435e-05	1.321e-01	-1.159e-01

Table 11: BIC values calculated from candidate models fitted to truncated data with cut-off threshold $T = 40s$. Smallest BIC values marked as **bold**.

User	BIC (flare)	BIC (EMG)	BIC (linear)	BIC (regmix)
User 1	**75002.97**	76845.59	136629.60	82028.67
User 2	4879.58	**4849.93**	9498.89	5449.34
User 3	169557.40	**168779.80**	311671.60	184768.50
User 4	11720.44	**11407.23**	18551.67	12889.54
User 5	**49223.03**	50254.93	100294.80	56083.53
User 6	**48229.38**	49406.38	88479.89	51929.93
User 7	**6649.05**	6900.26	12037.95	7086.26
User 8	**13288.47**	14054.39	28380.59	14626.85
User 9	24222.51	**23743.64**	40882.83	26475.89
User 10	**60323.56**	64736.94	148034.40	68268.92
User 11	15609.90	**15436.00**	25559.78	16874.98
User 12	**54383.37**	54517.25	92754.52	60019.01
User 13	**12645.21**	12869.95	21007.73	13821.76
User 14	45066.85	**43951.10**	73991.75	50287.50
User 15	**65867.15**	67398.71	118929.30	70509.74
User 16	**20860.78**	21389.94	43109.30	23209.32
User 17	**1438.73**	1462.06	2189.52	1453.66
User 18	11302.8	**11126.11**	19703.64	12540.03
User 19	**97815.6**	100936	173747.10	105075.80
User 20	**13709.56**	14003.65	22340.00	14299.33
User 21	**2446.62**	2525.58	4892.08	2707.50
User 22	8712.722	**8573.871**	12685.28	9257.19
User 23	**19504.54**	19682.34	31664.97	21396.08
User 24	**22468.24**	22685.65	41990.04	24906.90

Table 12: Parameter estimates obtained from the flare regression model fitted to truncated data with cut-off threshold $T = 40s$. Where (**s.e.**) indicates the estimated standard error in the parentheses.

User	$\widehat{\boldsymbol{\lambda}}$(**s.e.**)	$\widehat{\boldsymbol{\beta_0}}$(**s.e.**)	$\widehat{\boldsymbol{\beta_1}}$(**s.e.**)	$\widehat{\boldsymbol{\sigma}}$(**s.e.**)	$\widehat{\boldsymbol{\alpha}}$(**s.e.**)
User 1	0.74(0.042)	0.49(0.044)	0.17(0.011)	0.35(0.026)	0.56(0.085)
User 2	0.73(0.035)	0.36(0.030)	0.15(0.0081)	0.28(0.019)	0.86(0.12)
User 3	0.76(0.038)	0.45(0.042)	0.16(0.0095)	0.34(0.025)	0.66(0.092)
User 4	0.65(0.043)	0.50(0.047)	0.18(0.0086)	0.31(0.032)	0.76(0.062)
User 5	0.72(0.036)	0.20(0.023)	0.19(0.0095)	0.26(0.021)	0.86(0.10)
User 6	0.80(0.040)	0.58(0.043)	0.29(0.017)	0.38(0.029)	0.56(0.10)
User 7	0.82(0.031)	0.64(0.049)	0.14(0.014)	0.44(0.029)	0.44(0.080)
User 8	0.80(0.031)	0.45(0.035)	0.11(0.0090)	0.30(0.018)	0.70(0.11)
User 9	0.73(0.032)	0.52(0.038)	0.10(0.0087)	0.29(0.023)	0.88(0.089)
User 10	0.78(0.041)	0.33(0.032)	0.20(0.011)	0.28(0.023)	0.65(0.13)
User 11	0.72(0.036)	0.37(0.032)	0.15(0.0084)	0.31(0.024)	0.77(0.075)
User 12	0.70(0.035)	0.45(0.028)	0.14(0.0070)	0.30(0.023)	0.74(0.076)
User 13	0.71(0.033)	0.53(0.055)	0.15(0.013)	0.41(0.025)	0.50(0.058)
User 14	0.65(0.043)	0.34(0.046)	0.17(0.011)	0.30(0.033)	0.75(0.074)
User 15	0.79(0.04)	0.52(0.05)	0.14(0.01)	0.39(0.03)	0.50(0.09)
User 16	0.79(0.04)	0.33(0.03)	0.14(0.01)	0.30(0.02)	0.75(0.14)
User 17	0.86(0.03)	0.78(0.03)	0.07(0.01)	0.36(0.02)	0.75(0.09)
User 18	0.75(0.03)	0.35(0.04)	0.15(0.01)	0.32(0.02)	0.88(0.10)
User 19	0.79(0.03)	0.85(0.04)	0.17(0.01)	0.36(0.02)	0.62(0.09)
User 20	0.86(0.03)	0.70(0.04)	0.09(0.01)	0.39(0.02)	0.71(0.13)
User 21	0.77(0.03)	0.42(0.04)	0.11(0.01)	0.27(0.02)	0.83(0.12)
User 22	0.74(0.04)	0.60(0.05)	0.20(0.01)	0.48(0.03)	0.54(0.06)
User 23	0.67(0.03)	0.32(0.03)	0.16(0.01)	0.34(0.02)	0.59(0.05)
User 24	0.75(0.03)	0.47(0.03)	0.14(0.01)	0.32(0.02)	0.75(0.10)

Table 13: Parameter estimates obtained from the EMG regression model fitted to truncated data with cut-off threshold $T = 40s$.

User	$\widehat{\boldsymbol{\beta_0}}$**(s.e.)**	$\widehat{\boldsymbol{\beta_1}}$**(s.e.)**	$\widehat{\boldsymbol{\sigma}}$**(s.e.)**	$\widehat{\boldsymbol{\alpha}}$**(s.e.)**
User 1	0.14(0.042)	0.14(0.013)	0.16(0.018)	1.09(0.067)
User 2	0.087(0.036)	0.12(0.010)	0.15(0.020)	1.54(0.096)
User 3	0.12(0.034)	0.13(0.012)	0.14(0.017)	1.28(0.078)
User 4	0.14(0.029)	0.17(0.0068)	0.14(0.012)	1.19(0.068)
User 5	−0.067(0.030)	0.17(0.016)	0.13(0.011)	1.56(0.099)
User 6	0.21(0.10)	0.24(0.017)	0.18(0.019)	1.20(0.093)
User 7	0.26(0.092)	0.080(0.016)	0.20(0.017)	1.05(0.092)
User 8	0.18(0.033)	0.077(0.011)	0.16(0.010)	1.56(0.11)
User 9	0.22(0.030)	0.084(0.0090)	0.15(0.015)	1.54(0.10)
User 10	0.050(0.071)	0.17(0.014)	0.15(0.019)	1.47(0.11)
User 11	0.054(0.052)	0.13(0.011)	0.15(0.20)	1.35(0.069)
User 12	0.12(0.071)	0.12(0.0063)	0.16(0.015)	1.29(0.070)
User 13	0.14(0.040)	0.11(0.0099)	0.19(0.019)	0.90(0.060)
User 14	0.05(0.038)	0.13(0.011)	0.13(0.019)	1.19(0.056)
User 15	0.17(0.04)	0.10(0.01)	0.17(0.02)	1.08(0.06)
User 16	0.05(0.06)	0.11(0.01)	0.16(0.01)	1.54(0.12)
User 17	0.47(0.02)	0.03(0.01)	0.21(0.01)	1.71(0.10)
User 18	0.06(0.04)	0.12(0.01)	0.17(0.01)	1.56(0.09)
User 19	0.46(0.03)	0.15(0.01)	0.21(0.01)	1.32(0.09)
User 20	0.36(0.07)	0.07(0.01)	0.24(0.01)	1.63(0.09)
User 21	0.15(0.03)	0.09(0.01)	0.15(0.01)	1.65(0.11)
User 22	0.09(0.03)	0.17(0.01)	0.21(0.02)	0.95(0.06)
User 23	−0.01(0.04)	0.13(0.01)	0.16(0.02)	1.01(0.05)
User 24	0.15(0.07)	0.12(0.01)	0.17(0.01)	1.40(0.09)

Table 14: Parameter estimates obtained from the ordinary linear regression model fitted to truncated data with cut-off threshold $T = 40s$.

User	$\widehat{\boldsymbol{\beta_0}}$**(s.e.)**	$\widehat{\boldsymbol{\beta_1}}$**(s.e.)**	$\widehat{\boldsymbol{\sigma}}$**(s.e.)**
User 1	0.96(0.14)	0.17(0.034)	1.62(0.32)
User 2	0.57(0.087)	0.17(0.023)	1.08(0.20)
User 3	0.79(0.12)	0.16(0.029)	1.29(0.28)
User 4	0.98(0.13)	0.17(0.029)	1.18(0.18)
User 5	0.33(0.085)	0.24(0.022)	1.05(0.19)
User 6	0.98(0.13)	0.27(0.048)	1.46(0.25)
User 7	1.10(0.15)	0.12(0.037)	1.77(0.33)
User 8	0.73(0.099)	0.11(0.026)	1.14(0.23)
User 9	0.82(0.078)	0.10(0.019)	0.92(0.16)
User 10	0.64(0.12)	0.21(0.044)	1.50(0.35)
User 11	0.54(0.077)	0.19(0.019)	1.028(0.13)
User 12	0.72(0.074)	0.17(0.020)	1.16(0.22)
User 13	0.75(0.17)	0.22(0.044)	1.88(0.32)
User 14	0.72(0.12)	0.18(0.030)	1.23(0.23)
User 15	0.94(0.14)	0.14(0.03)	1.62(0.32)
User 16	0.70(0.11)	0.11(0.03)	1.16(0.32)
User 17	0.96(0.05)	0.06(0.01)	0.77(0.07)
User 18	0.50(0.09)	0.17(0.03)	0.97(0.30)
User 19	1.17(0.09)	0.17(0.02)	1.26(0.22)
User 20	0.88(0.08)	0.09(0.02)	0.92(0.16)
User 21	0.81(0.12)	0.08(0.02)	0.97(0.15)
User 22	0.99(0.13)	0.21(0.03)	1.50(0.24)
User 23	0.58(0.15)	0.23(0.03)	1.50(0.25)
User 24	0.66(0.10)	0.18(0.02)	1.21(0.26)

Table 15: Parameter estimates obtained from the mixture of two linear regressions model fitted to truncated data with cut-off threshold $T = 40s$.

User	$\widehat{\boldsymbol{\lambda}}$**(s.e.)**	$\widehat{\boldsymbol{\beta_{10}}}$**(s.e.)**	$\widehat{\boldsymbol{\beta_{11}}}$**(s.e.)**	$\widehat{\boldsymbol{\beta_{20}}}$**(s.e.)**	$\widehat{\boldsymbol{\beta_{21}}}$**(s.e.)**	$\widehat{\boldsymbol{\sigma_1}}$**(s.e.)**	$\widehat{\boldsymbol{\sigma_2}}$**(s.e.)**
User 1	0.89(0.20)	0.57(0.87)	0.19(0.065)	4.48(1.76)	−0.068(0.32)	0.48(0.95)	3.79(1.32)
User 2	0.93(0.21)	0.39(0.87)	0.17(0.065)	3.46(1.76)	−0.032(0.32)	0.42(0.95)	3.08(1.32)
User 3	0.91(0.22)	0.51(1.01)	0.17(0.094)	3.59(1.63)	0.02(0.30)	0.44(0.68)	3.18(1.33)
User 4	0.89(0.18)	0.64(0.56)	0.19(0.039)	3.57(1.27)	0.0043(0.21)	0.49(0.53)	2.46(1.01)
User 5	0.12(0.19)	2.13(0.41)	0.18(0.051)	0.22(1.11)	0.21(0.20)	2.30(0.46)	0.34(0.97)
User 6	0.91(0.18)	0.63(0.75)	0.30(0.092)	4.49(1.48)	−0.076(0.39)	0.47(0.76)	3.72(1.25)
User 7	0.91(0.20)	0.66(1.25)	0.16(0.14)	5.79(1.98)	−0.36(0.30)	0.52(0.95)	4.57(1.61)
User 8	0.90(0.24)	0.46(0.89)	0.12(0.086)	3.64(1.37)	−0.18(0.21)	0.35(0.68)	2.82(1.12)
User 9	0.86(0.20)	0.56(0.47)	0.11(0.043)	2.42(0.92)	0.0090(0.15)	0.36(0.27)	1.78(0.89)
User 10	0.93(0.14)	0.40(0.60)	0.21(0.048)	3.67(1.36)	0.12(0.38)	0.39(0.59)	4.43(1.40)
User 11	0.85(0.20)	0.40(0.41)	0.16(0.042)	1.94(0.74)	0.21(0.11)	0.38(0.41)	1.91(0.63)
User 12	0.84(0.21)	0.52(0.42)	0.15(0.027)	2.38(0.89)	0.15(0.10)	0.39(0.37)	2.20(0.94)
User 13	0.88(0.22)	0.58(0.71)	0.17(0.17)	3.24(1.52)	0.31(0.35)	0.58(1.29)	4.20(1.58)
User 14	0.15(0.27)	2.86(0.99)	0.046(0.11)	0.42(1.29)	0.18(0.19)	2.38(0.72)	0.44(1.081)
User 15	0.91(0.26)	0.56(1.34)	0.16(0.14)	4.96(2.00)	−0.13(0.29)	0.50(0.83)	4.05(1.61)
User 16	0.91(0.22)	0.38(0.91)	0.15(0.12)	3.73(1.67)	−0.21(0.29)	0.38(0.54)	2.98(1.34)
User 17	0.78(0.22)	0.52(0.71)	0.17(0.13)	2.50(0.88)	−0.21(0.16)	0.27(0.34)	1.23(0.41)
User 18	0.89(0.24)	0.38(0.41)	0.16(0.06)	2.06(0.71)	0.15(0.31)	0.40(0.44)	2.24(1.79)
User 19	0.90(0.24)	0.92(0.90)	0.17(0.11)	3.73(1.46)	0.07(0.23)	0.45(0.80)	3.03(1.31)
User 20	0.92(0.27)	0.68(0.91)	0.11(0.15)	3.68(1.38)	−0.21(0.43)	0.42(0.62)	2.33(1.20)
User 21	0.90(0.25)	0.50(0.97)	0.11(0.11)	3.91(1.81)	−0.25(0.27)	0.34(0.47)	2.11(0.70)
User 22	0.12(0.24)	3.20(0.80)	0.24(0.09)	0.68(1.30)	0.21(0.29)	3.08(0.77)	0.60(1.23)
User 23	0.82(0.20)	0.37(1.65)	0.18(0.21)	2.63(1.51)	0.20(0.23)	0.45(0.85)	2.73(1.00)
User 24	0.91(0.21)	0.52(0.98)	0.16(0.13)	3.11(1.93)	0.09(0.35)	0.44(0.84)	3.15(1.44)

Table 16: System running time (in seconds) for candidate models fitted to truncated data with cut-off threshold $T = 40s$.

User	Sample Size	flare	EMG	linear	regmix
User 1	35894	17.765	459.582	0.011	22.494
User 2	3161	1.296	27.047	0.002	1.337
User 3	92932	47.234	1015.530	0.032	66.863
User 4	5835	2.088	29.440	0.002	5.685
User 5	34238	18.182	214.214	0.011	22.829
User 6	24649	8.596	179.505	0.008	12.506
User 7	3023	0.660	23.920	0.002	1.460
User 8	9122	3.294	108.063	0.074	3.775
User 9	15344	8.501	137.606	0.004	9.024
User 10	40560	20.915	245.800	0.007	20.616
User 11	8824	3.996	79.704	0.072	6.715
User 12	29578	13.429	307.304	0.011	21.331
User 13	5118	1.528	46.568	0.080	6.863
User 14	22724	11.902	213.853	0.085	18.979
User 15	31222	14.045	355.395	0.009	16.238
User 16	13772	5.098	147.767	0.083	6.966
User 17	941	0.028	5.852	0.002	0.615
User 18	7075	2.489	65.856	0.003	5.671
User 19	52676	20.801	662.411	0.018	26.243
User 20	8371	0.139	81.317	0.004	7.890
User 21	1759	0.464	10.850	0.004	1.072
User 22	3471	0.772	23.618	0.002	2.437
User 23	8669	3.468	84.152	0.003	6.001
User 24	13053	5.715	123.239	0.004	8.888

Table I: System running time (in seconds)

S.7: Figures

Figure 1

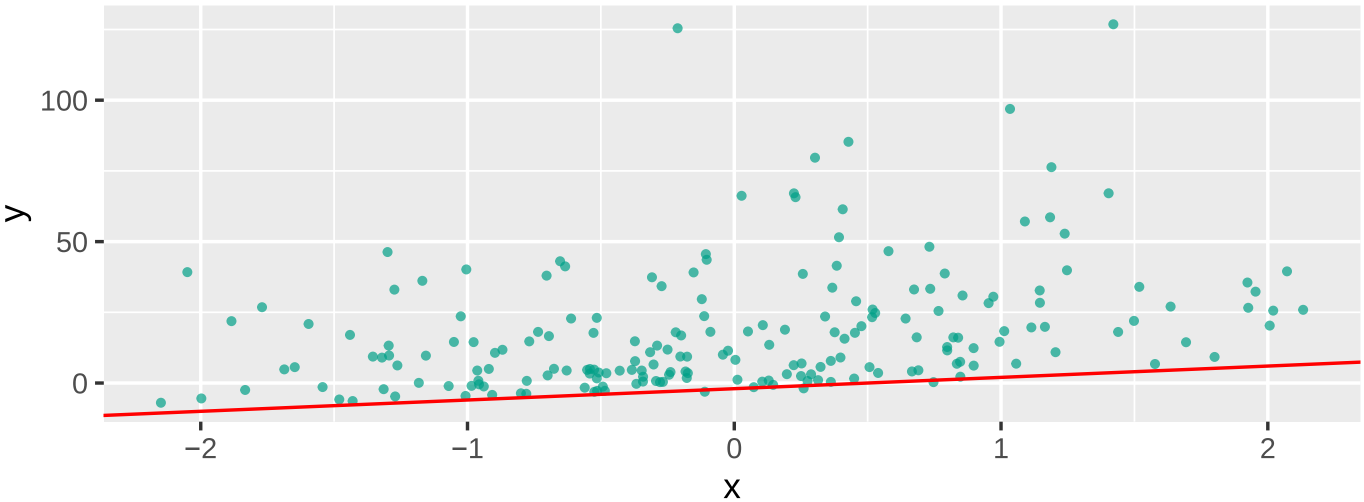
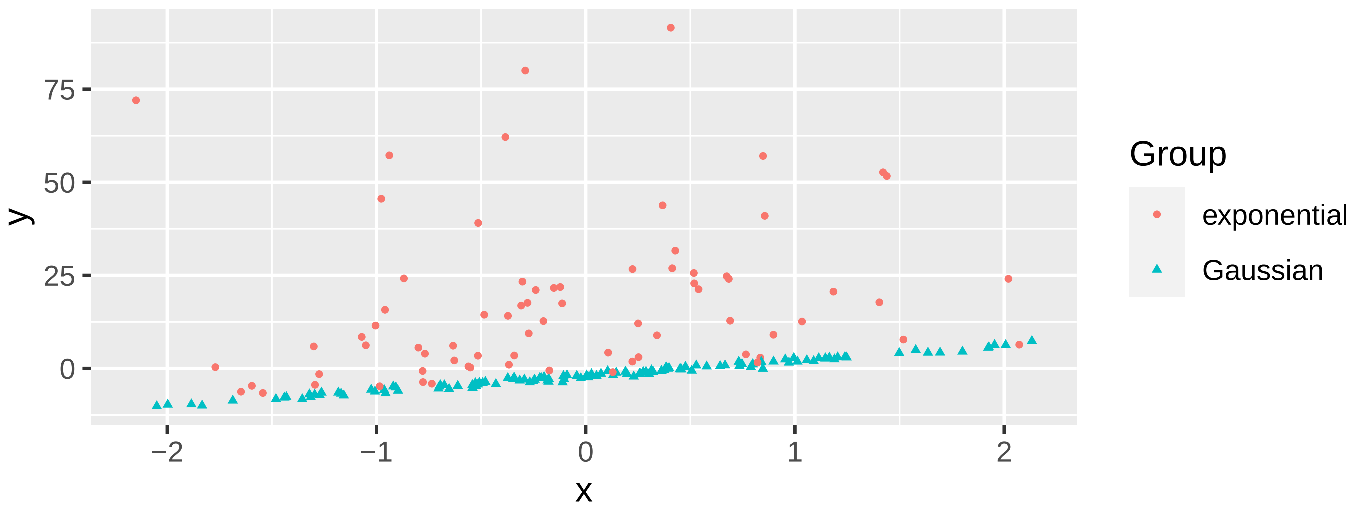
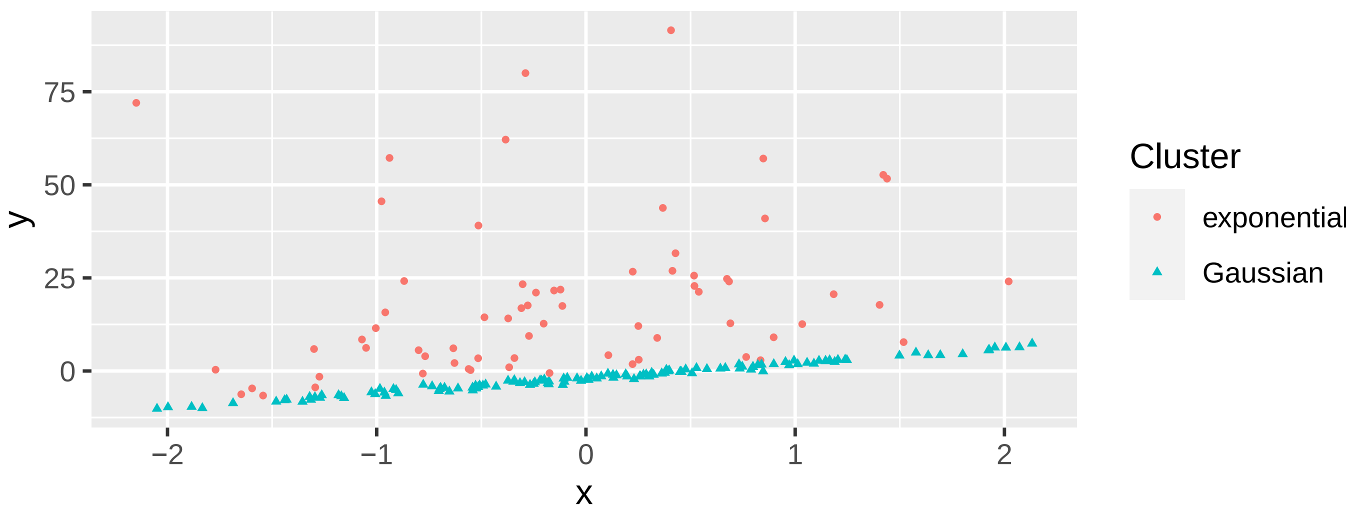


Figure 1: Scatter plot of a simulated data follows the EMG regression model (n=200, $x \sim N(0,1)$, $\beta = (-2,4)$, $\sigma = 0.5$, and $\alpha = 0.05$).

Figure 2



(a)



(b)

Figure 2: (a) Scatter plot of the simulated data assuming a flare regression model with two components labelled (n=200, $x \sim N(0,1)$, $\lambda = 0.6$ $\beta = (-2,4)$, $\sigma = 0.5$, and $\alpha = 0.05$) and (b) scatter plot of the same simulated data with clusters labelled (cut-off probability equals to 0.8)

Figure 3

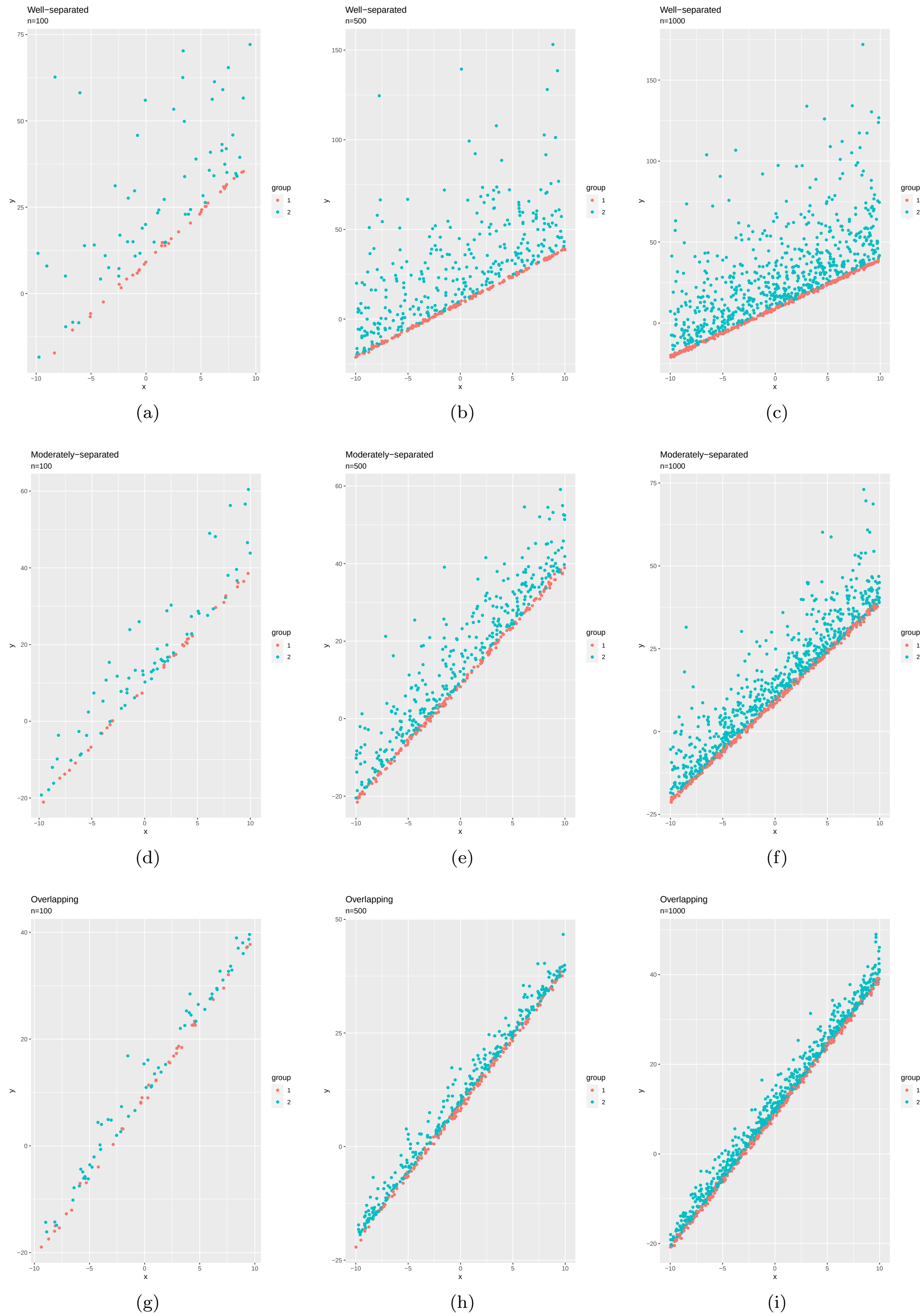


Figure 3: Scatterplots for data generated from simulation settings M1–M3 (group 1 indicates data simulated from the Gaussian part, group 2 indicates data simulated from the exponential part).

Figure 4

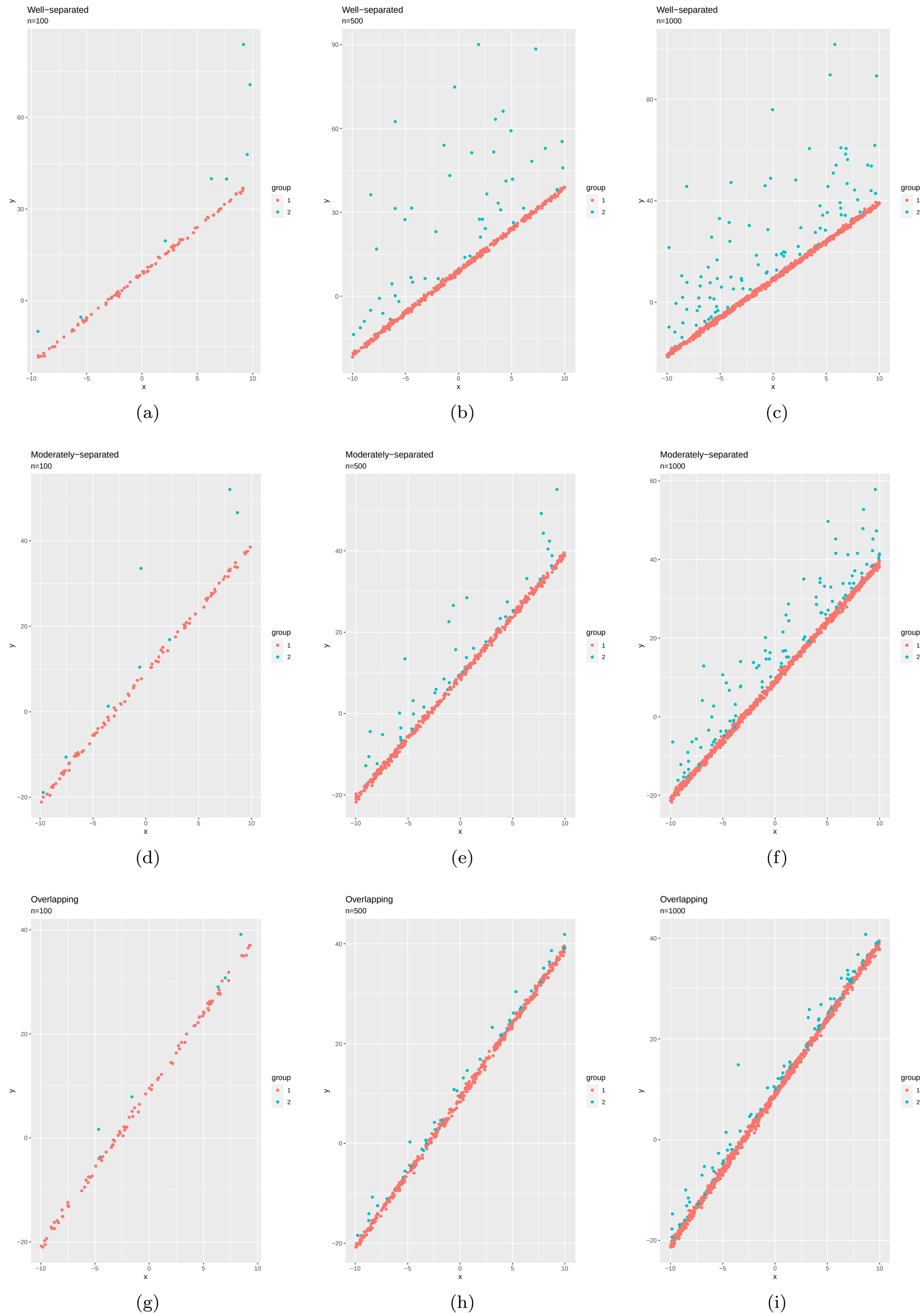


Figure 4: Scatterplots for data generated from simulation settings M4–M6 (group 1 indicates data simulated from the Gaussian part, group 2 indicates data simulated from the exponential part).

Figure 5

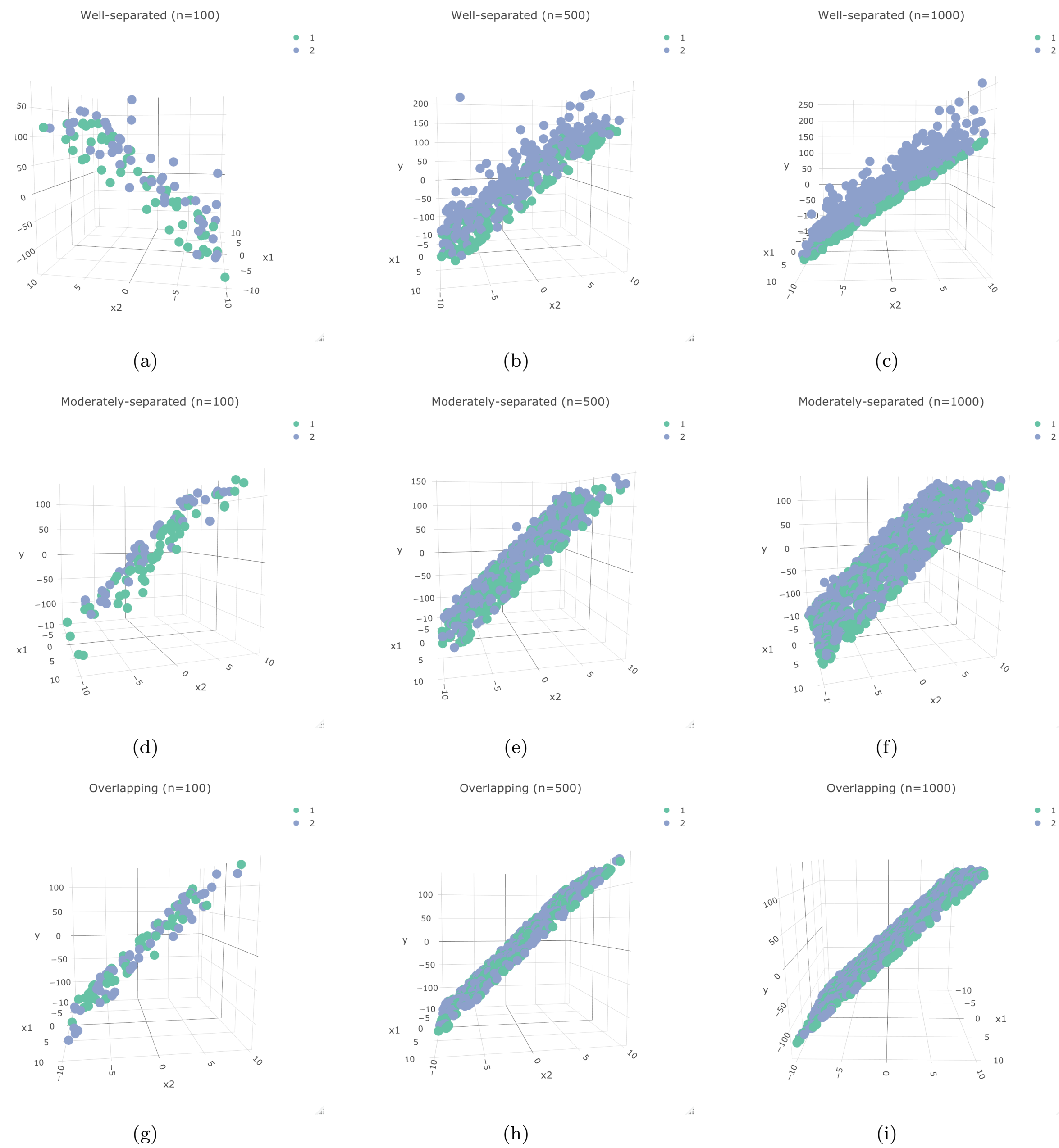


Figure 5: Scatterplots for data generated from simulation settings M7–M9 (group 1 indicates data simulated from the Gaussian part, group 2 indicates data simulated from the exponential part).

Figure 6

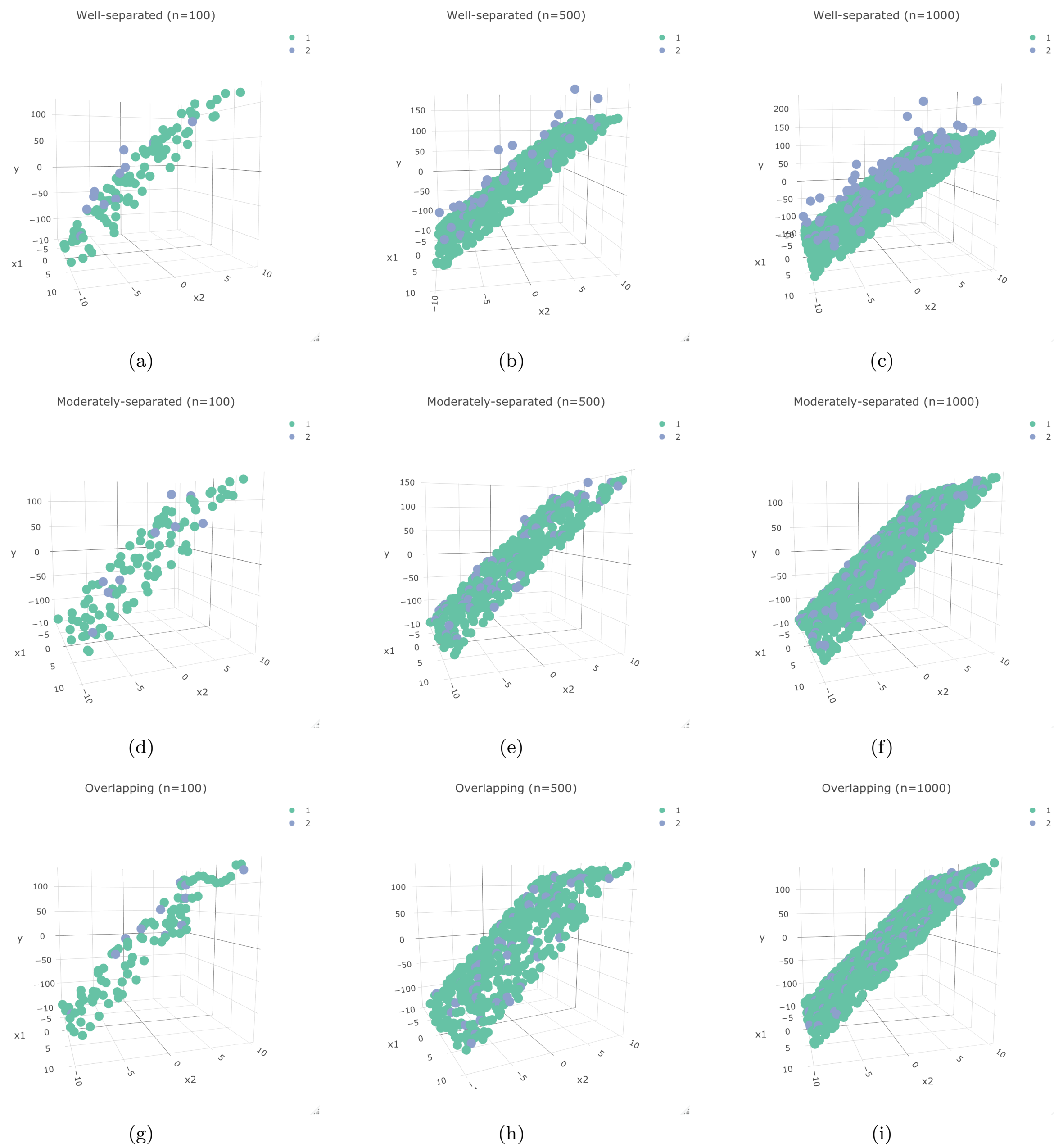
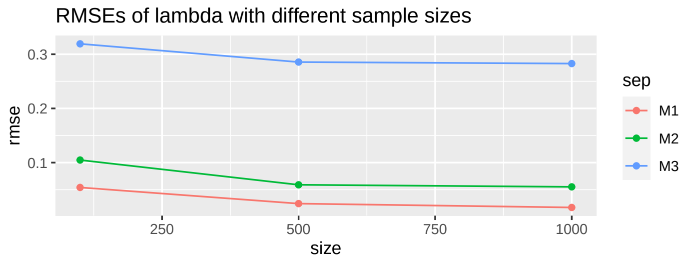


Figure 6: Scatterplots for data generated from simulation settings M10–M12 (group 1 indicates data simulated from the Gaussian part, group 2 indicates data simulated from the exponential part).

Figure 7



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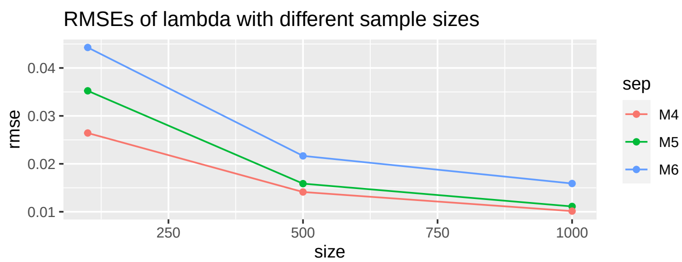

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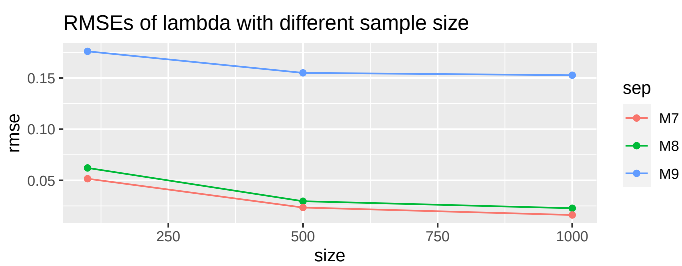
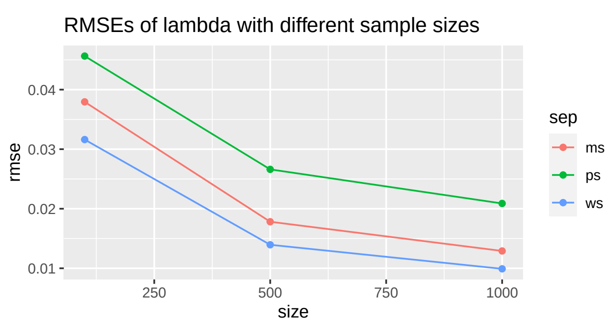


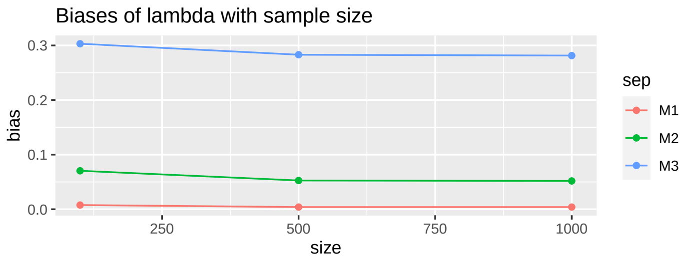
(c)



(d)

Figure 7: Comparing RMSEs of parameter estimations for λ (the mixing proportion)

Figure 8



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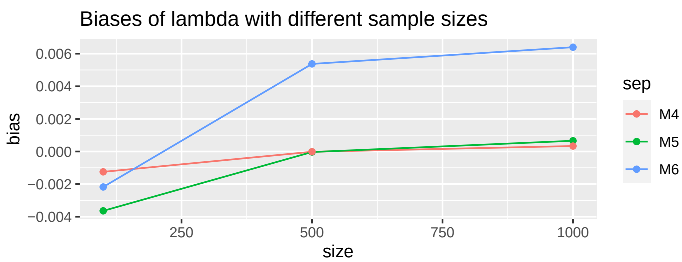

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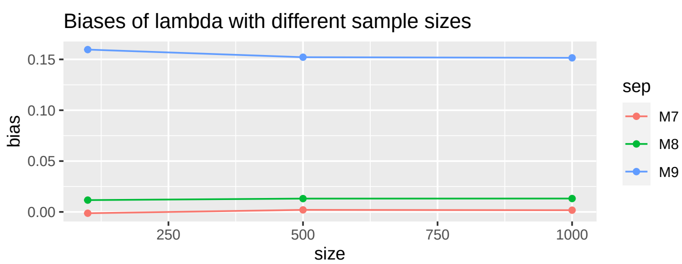

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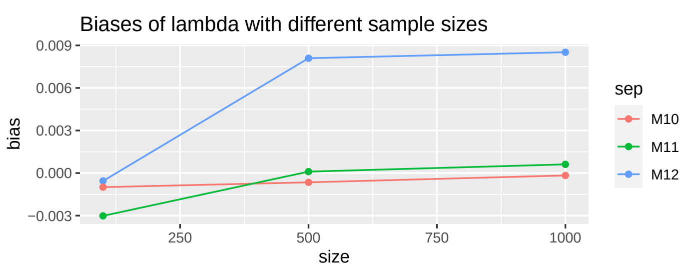
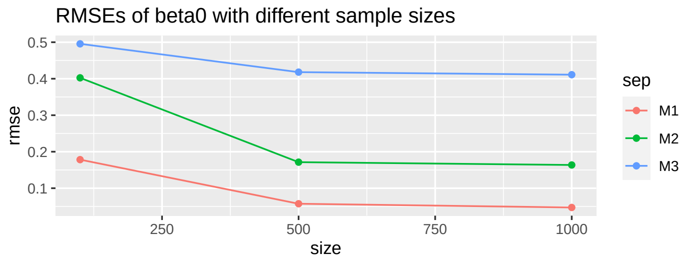


(d)

Figure 8: Comparing RMSEs of parameter estimations for λ (the mixing proportion)

Figure 9



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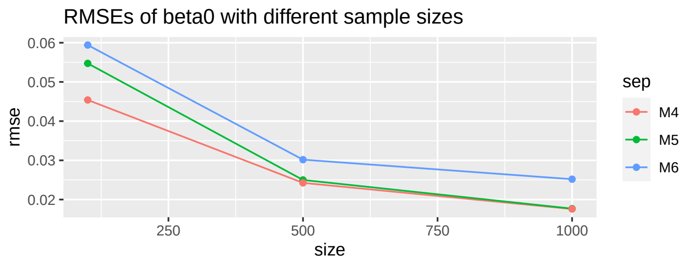

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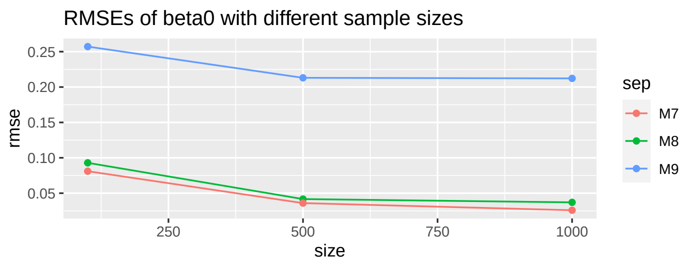

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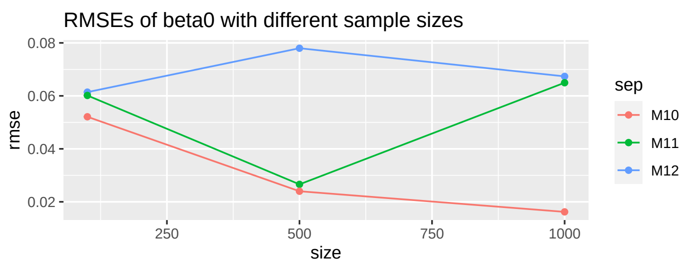
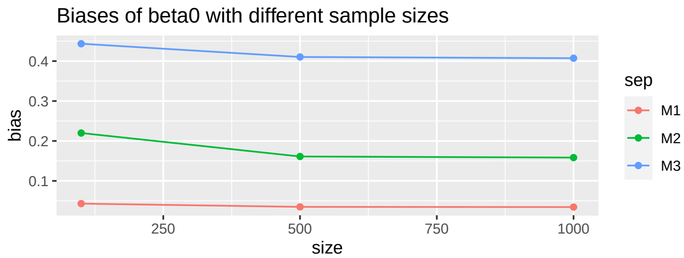


(d)

Figure 9: Comparing RMSEs of parameter estimations for β_0 (the intercept)

Figure 10



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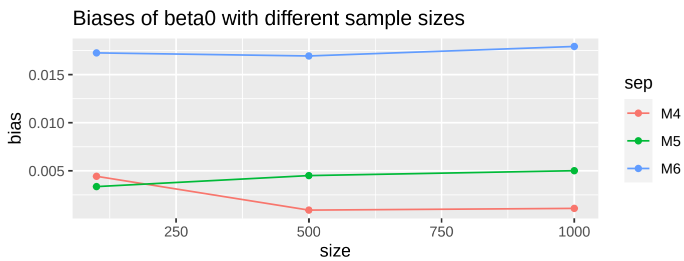

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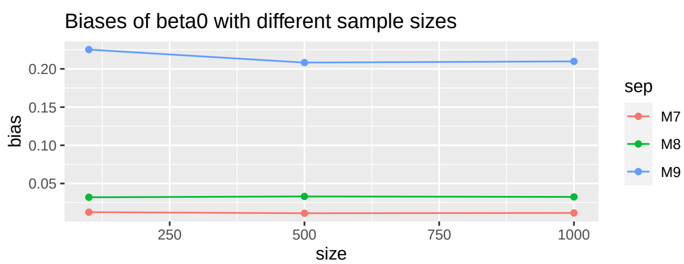

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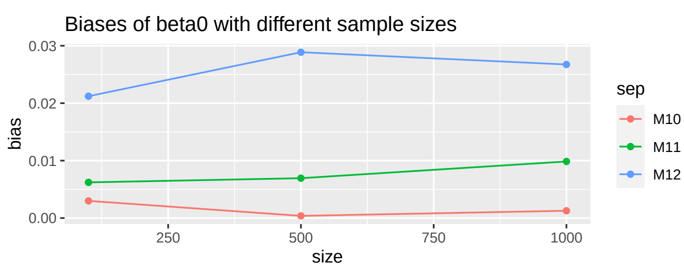
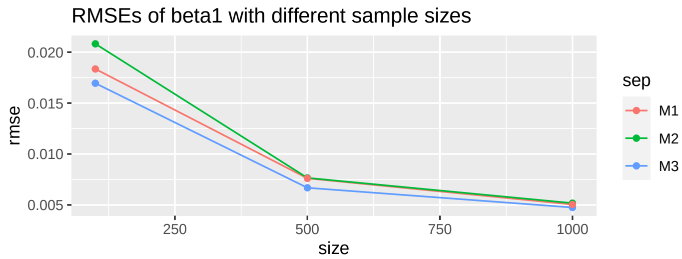


(d)

Figure 10: Comparing biases of parameter estimations for β_0 (the intercept)

Figure 11



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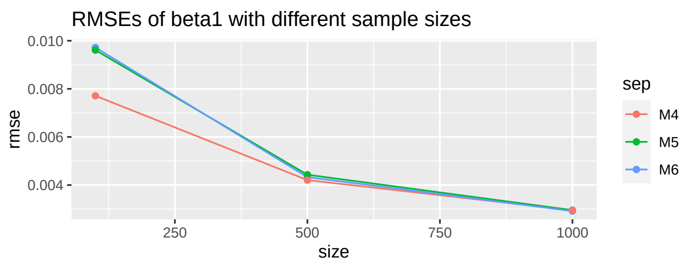

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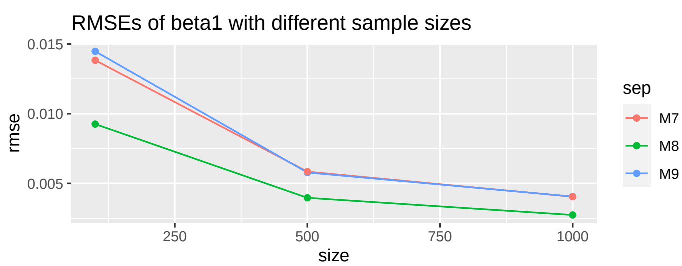

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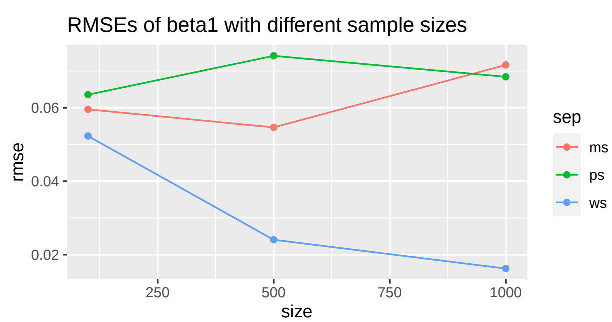
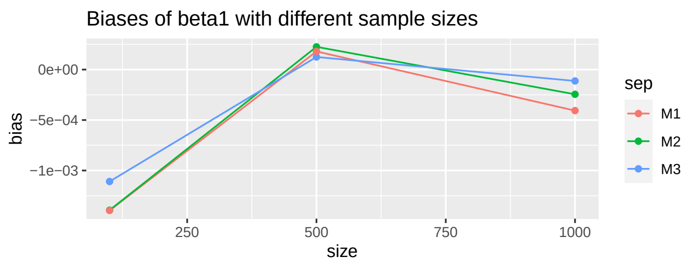


(d)

Figure 11: Comparing RMSEs of parameter estimations for β_1 (regression coefficient 1)

Figure 12



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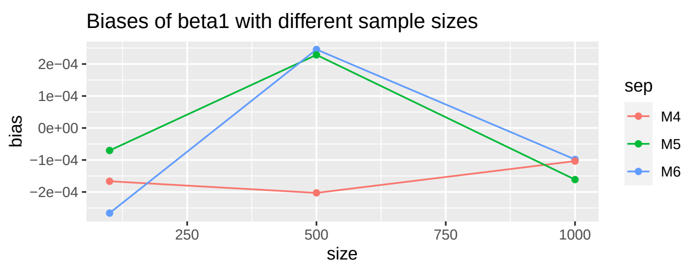

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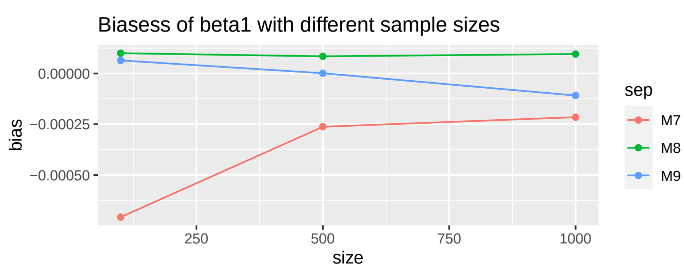

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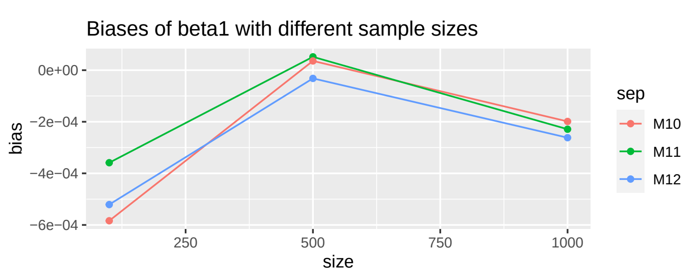
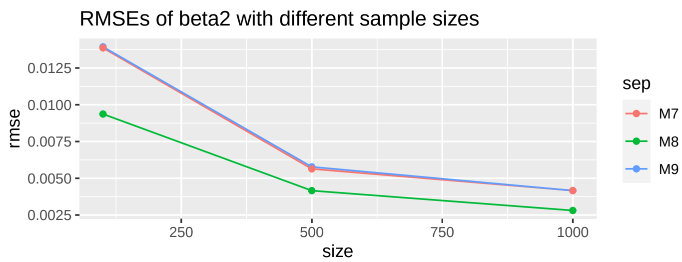


(d)

Figure 12: Comparing biases of parameter estimations for β_1 (regression coefficient 1)

Figure 13



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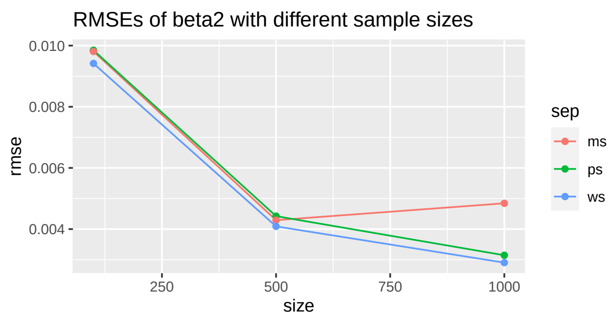
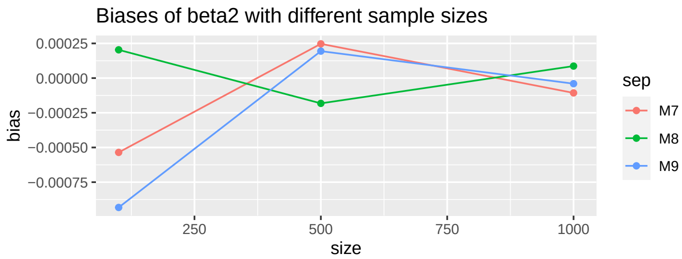


(b)

Figure 13: Comparing RMSEs of parameter estimations for β_2 (regression coefficient 2)

Figure 14



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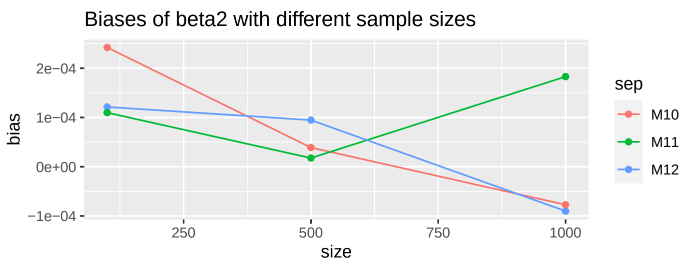

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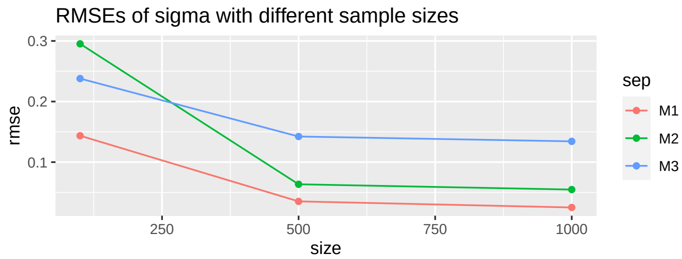
Figure 14: Comparing biases of parameter estimations for β_2 (regression coefficient 2)

Figure 15



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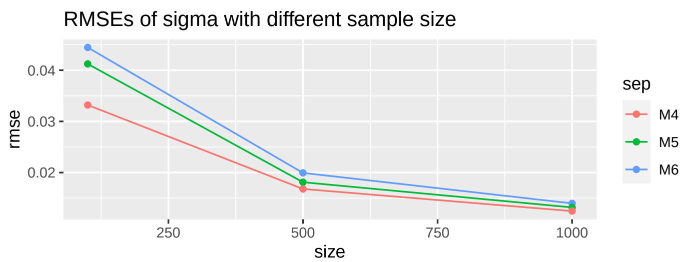

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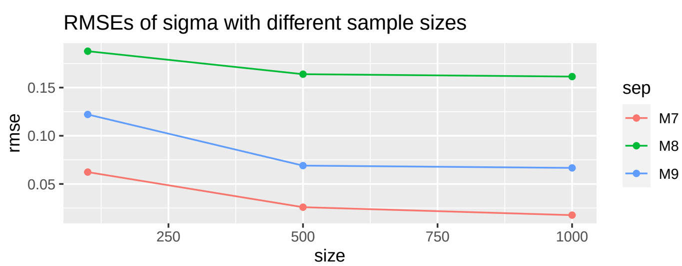

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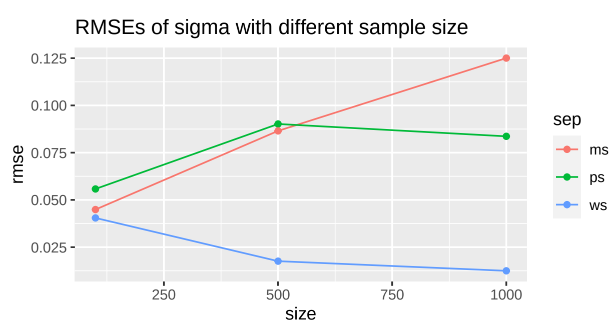
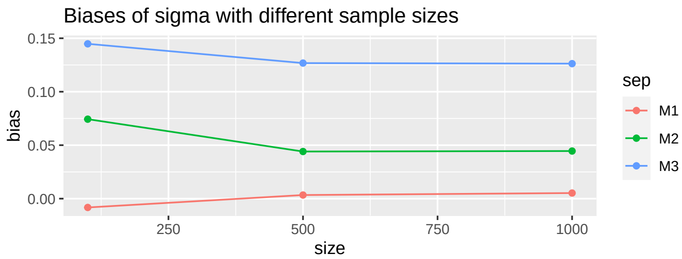


(d)

Figure 15: Comparing RMSEs of parameter estimations for σ (standard deviation for the Gaussian component)

Figure 16



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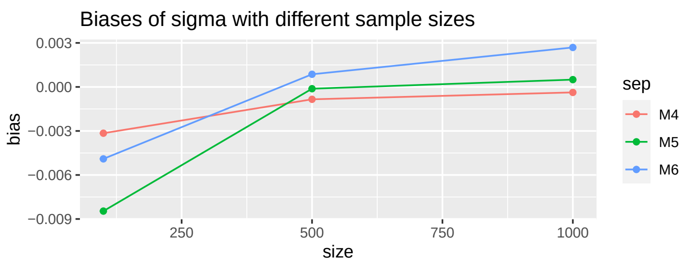

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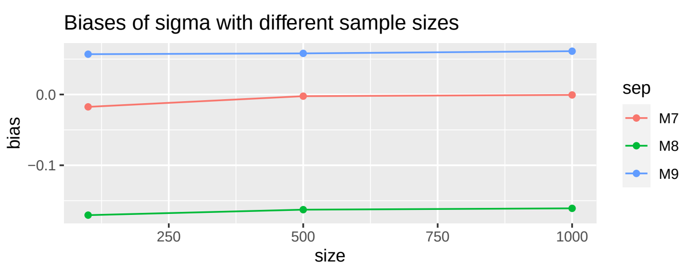

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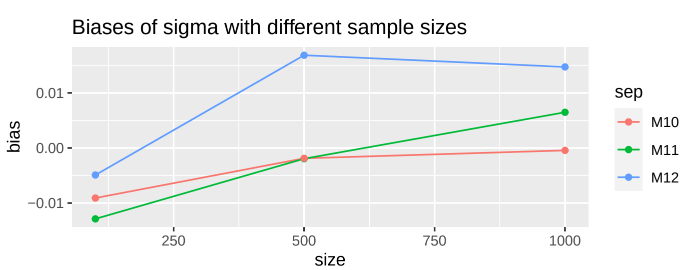
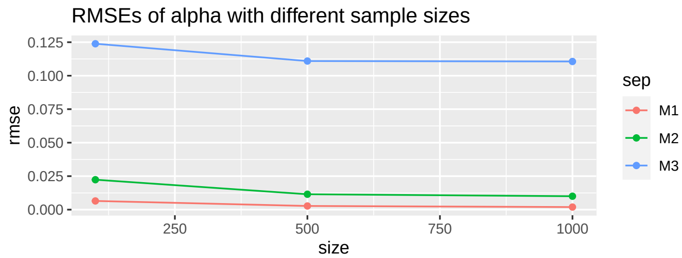


(d)

Figure 16: Comparing biases of parameter estimations for σ (standard deviation for the Gaussian component)

Figure 17



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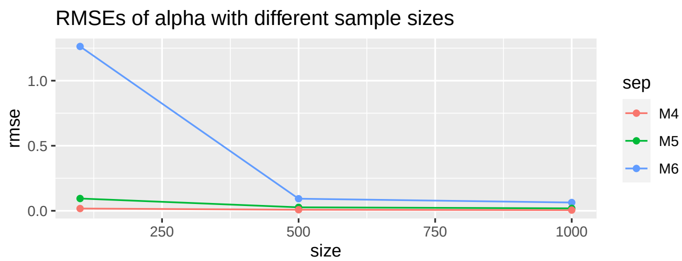

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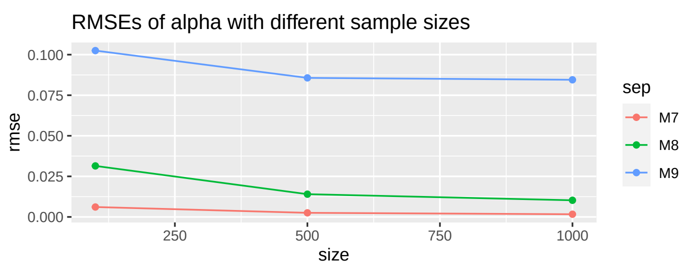

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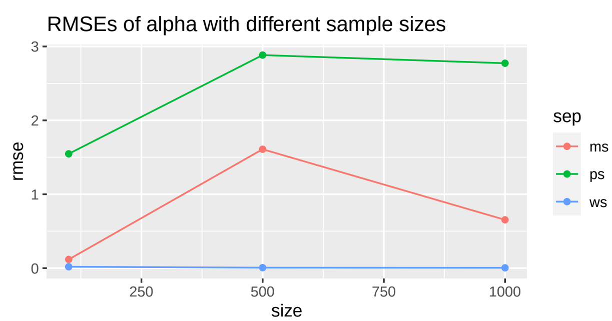
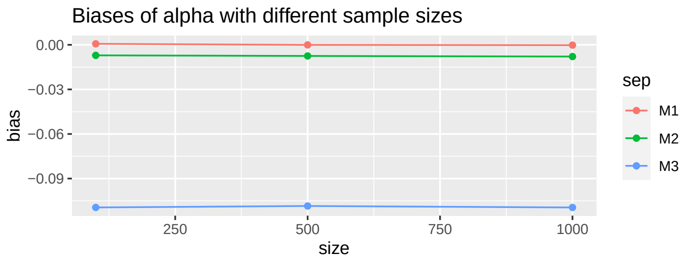


(d)

Figure 17: Comparing RMSEs of parameter estimations for α (rate for the exponential component)

Figure 18



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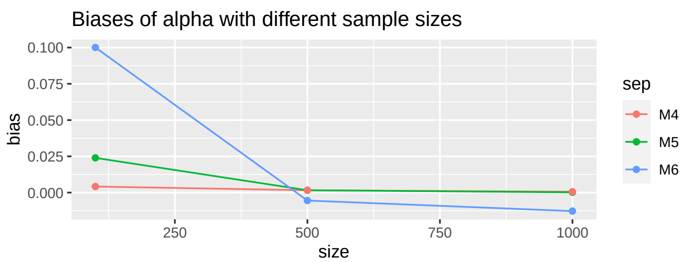

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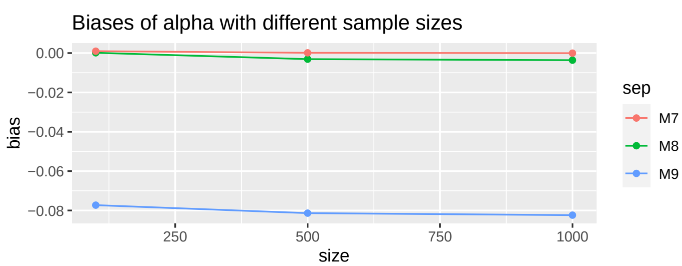

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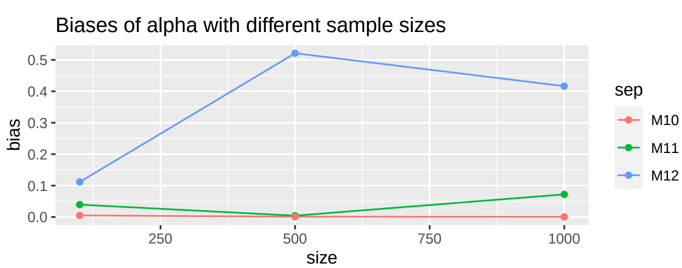

(d)

Figure 18: Comparing biases of parameter estimations for α (rate for the exponential component)

Figure 19

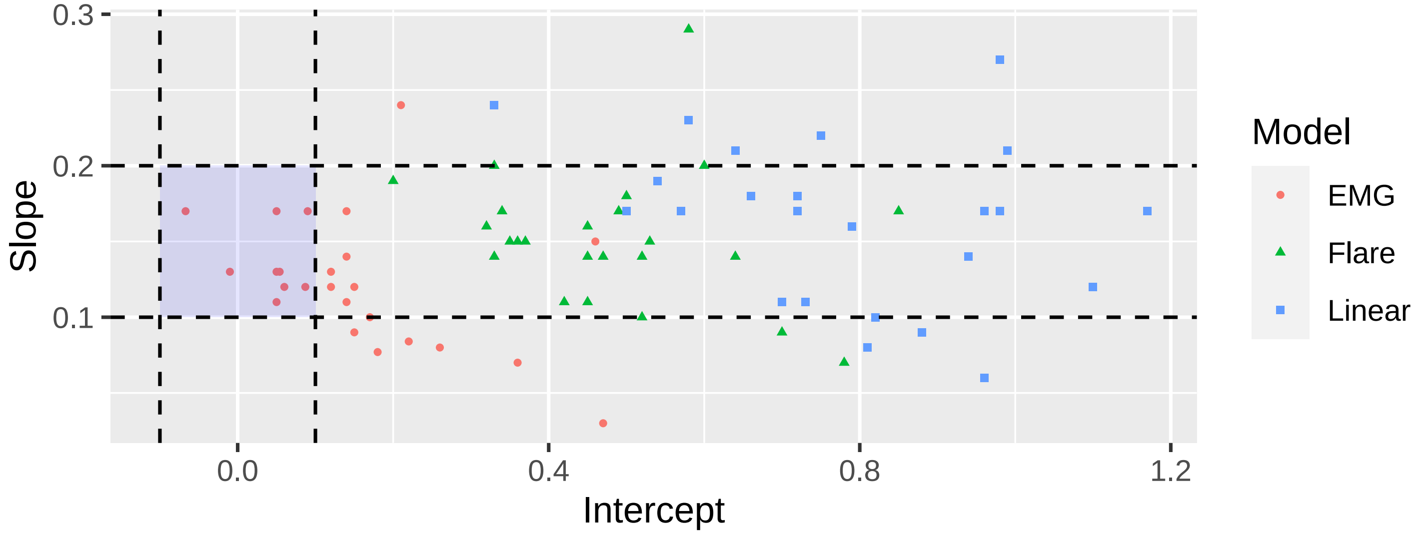

Figure 19: Visualization of intercept versus slope parameters estimated by three regression models, where the interval between two vertical dashed lines indicates the typical interval for the intercept in controlled studies; whereas the interval between two horizontal dashed lines indicates the typical interval for the slope in controlled studies, and the shaded rectangle is their intersection

Reference

[1] T. A. Louis. Finding the Observed Information Matrix when Using the EM Algorithm. *Journal of the Royal Statistical Society Series B (Methodological)*, 44(2):226–233, 1982.