

Speed-Accuracy Tradeoff:
**A Formal Information-Theoretic
Transmission Scheme (FITTS)**

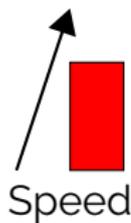
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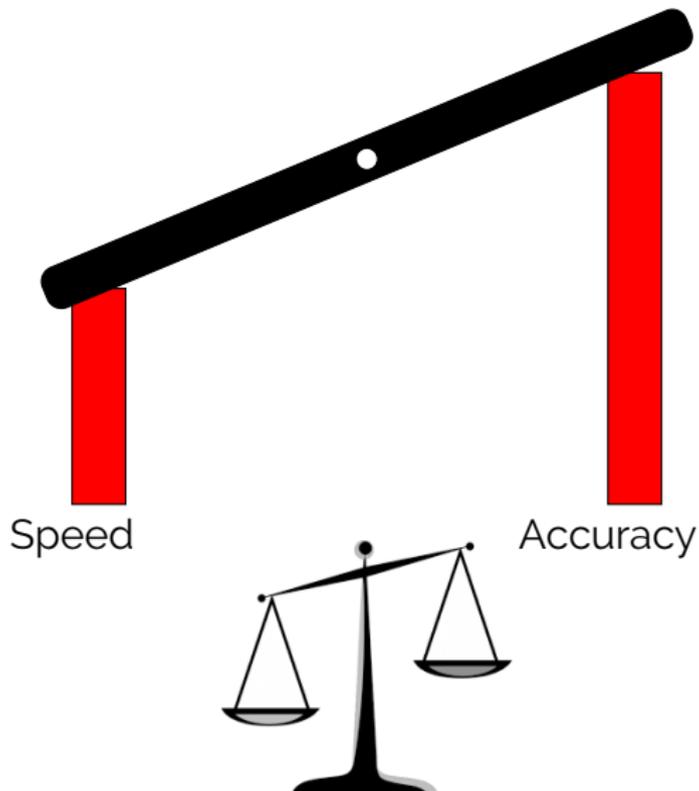
THE PROBLEM OF THE SPEED-ACCURACY TRADEOFF



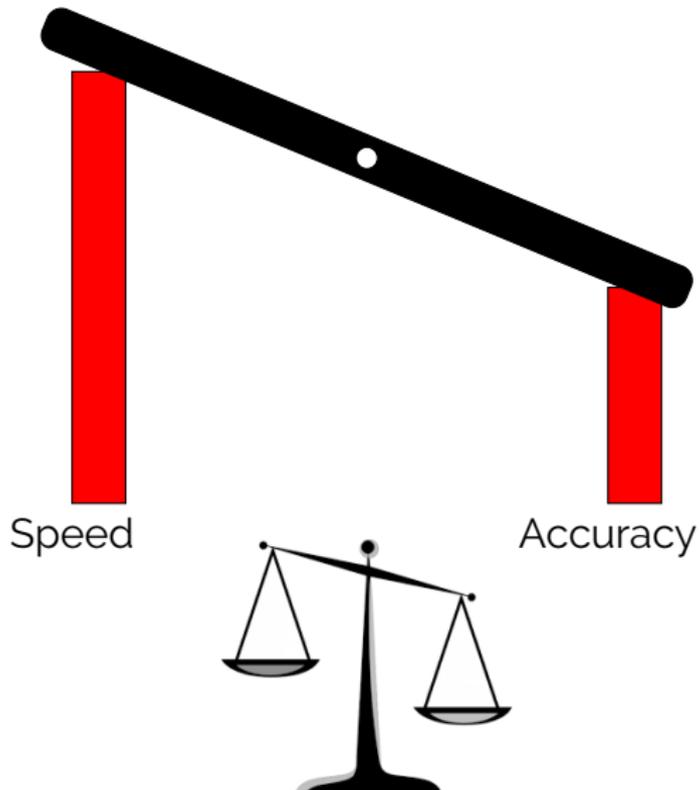
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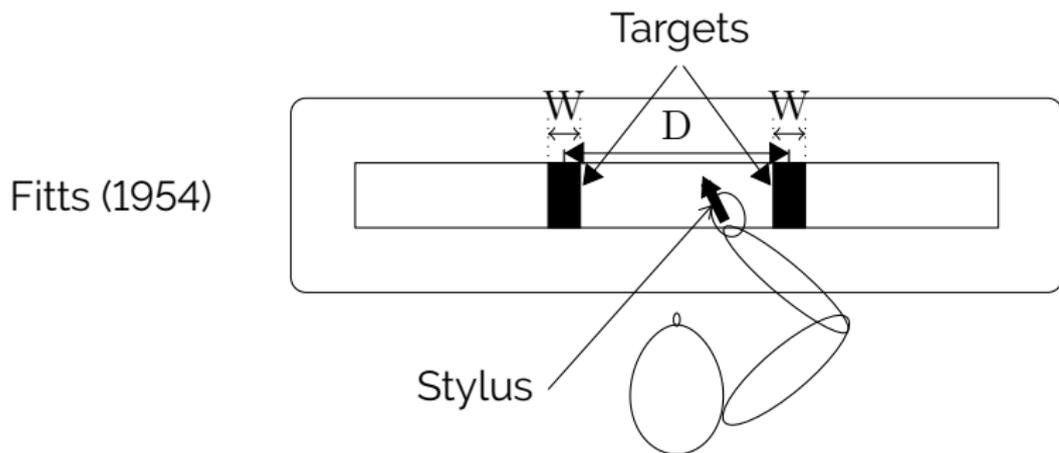
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THE PROBLEM OF THE SPEED-ACCURACY TRADEOFF



QUANTIFYING THE SPEED-ACCURACY TRADEOFF



Index of Difficulty: $ID = \log(1 + D/W)$ (bit)

Fitts' law for Movement Time:

$$MT = a + b ID$$

Vary according to
participant, device

FITTS' LAW IN HCI

$$MT = a + b ID$$

- Evaluate (by estimating values a and b):
 1. Device performance
 2. User performance
- Predict (by assuming values for a and b)
 1. Movement time for a pointing task
 2. Complex interaction times (optimize interfaces, keyboards)

Even though Fitts' law is widely used in HCI, it suffers from many issues.

I have selected 3 problems for this presentation

PROBLEM 1: MANY DIFFERENT FORMULATIONS

How many can you think of ?

PROBLEM 1: MANY DIFFERENT FORMULATIONS

$$MT = a + b \log_2(2D/W)$$

$$MT = a + b \log_2(D/W)$$

$$MT = a + b \log_2(1/2 + D/W)$$

$$MT = a + b D + c(1/W - 1)$$

$$MT = a + b\sqrt{D}$$

$$MT = a + (D/W)^b$$

$$MT = a + b \log_2 D + c \log_2 W$$

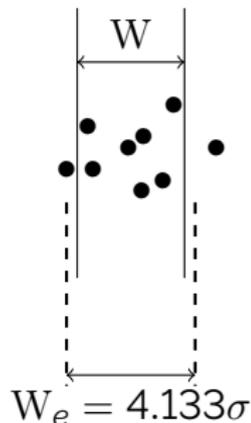
$$MT = \dots$$

More than 15 different formulations !

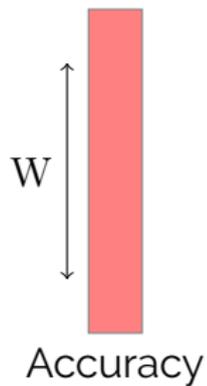
PROBLEM 2: SOMETIMES PARTICIPANTS MISS THE TARGET

Solution : Correct W a-posteriori to account for misses (usually $W_e = 4.133\sigma$)
(Crossman 1957, MacKenzie 1992)

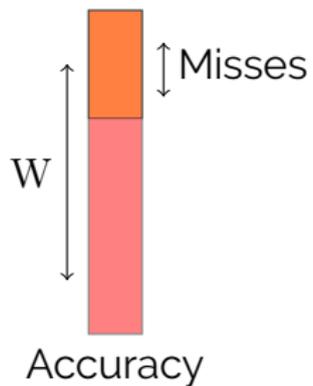
Refer to paper for a critique



PROBLEM 2: NOISY MANIPULATION OF ACCURACY

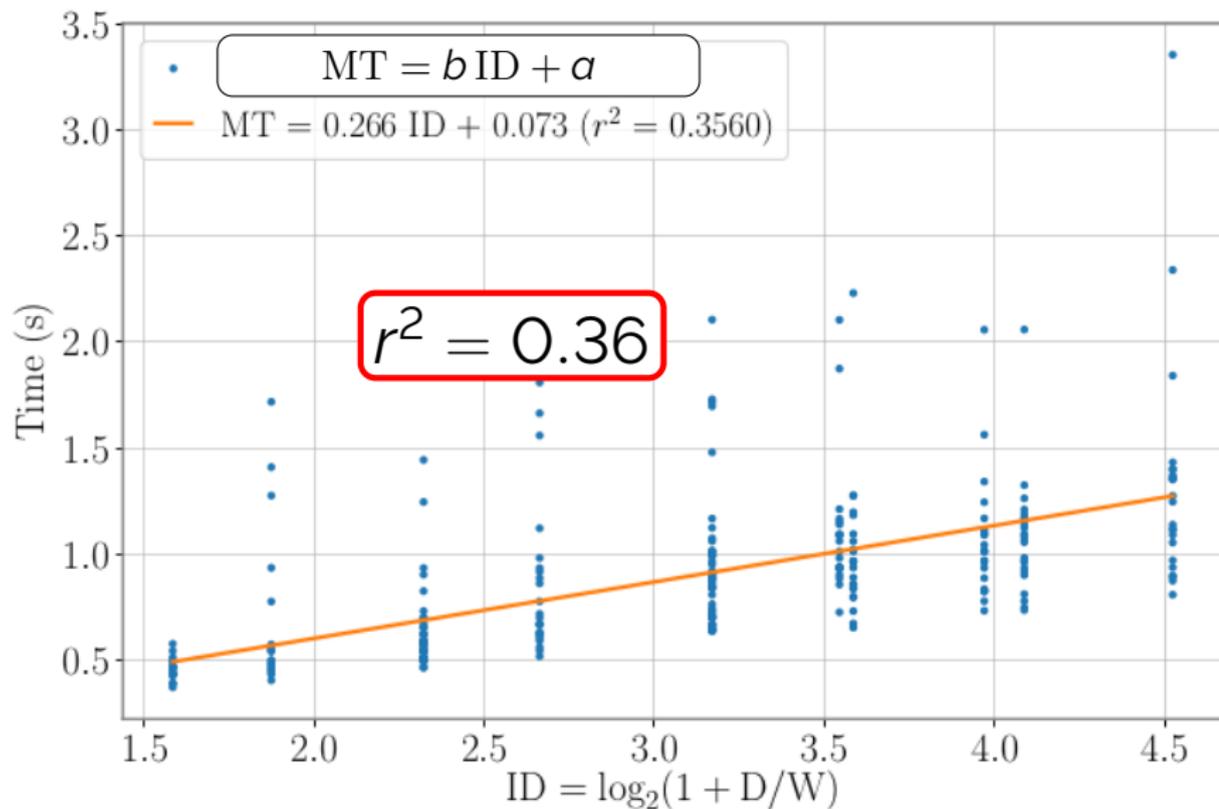


PROBLEM 2: NOISY MANIPULATION OF ACCURACY

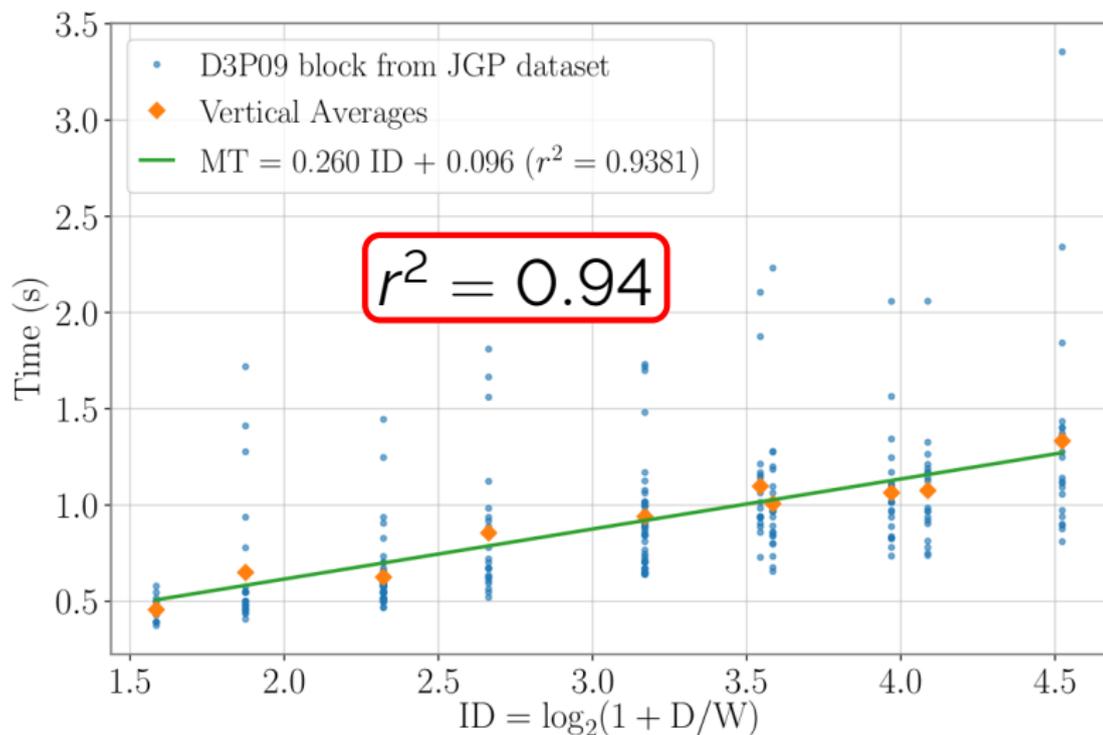


Accuracy's definition in
Fitts' task is not accurate...

PROBLEM 3: REGRESSION

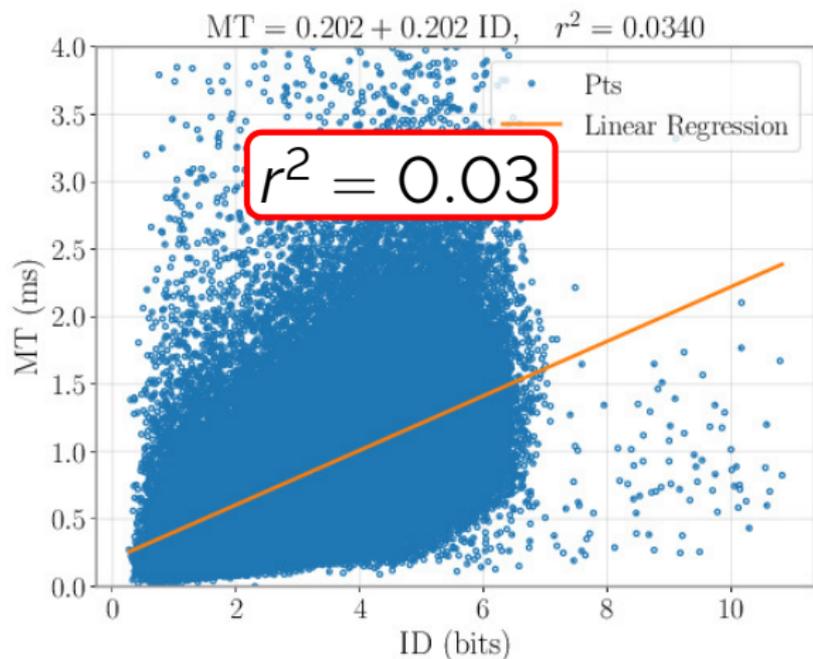


PROBLEM 3: REGRESSION



Fitts' law is a poor model for MT and can only model **average** MT

PROBLEM 3: REGRESSION



How can high variability data be handled (e.g. data from a field experiment) ?

GOAL OF THIS WORK:

A theoretical framework for the speed-accuracy tradeoff:

- simple
- rigorous
- not completely driven by empirical considerations
- address problems 1, 2 and 3

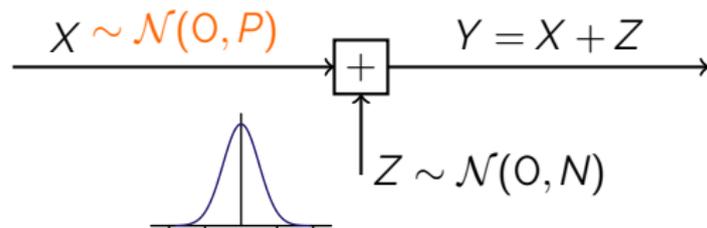
Tool used: Shannon's Information Theory

- Historically, Fitts' law is derived from an analogy with Shannon's capacity formula
- The channel capacity theorem can be interpreted as a speed-accuracy tradeoff

CAPACITY OF THE GAUSSIAN CHANNEL:

Capacity:

$$C = \max_X I(X; Y)$$



- **Shannon's Theorem 17 (1948):**

$$C = B_W \log\left(1 + \frac{P}{N}\right) \text{ bit/sec.}$$

- Tradeoff between time (B_W) and bits $\log\left(1 + \frac{P}{N}\right)$

A CLOSER LOOK AT FITTS' 1954 ANALOGY

Fitts' 1954 **analogy**: Humans have finite rate communication channels

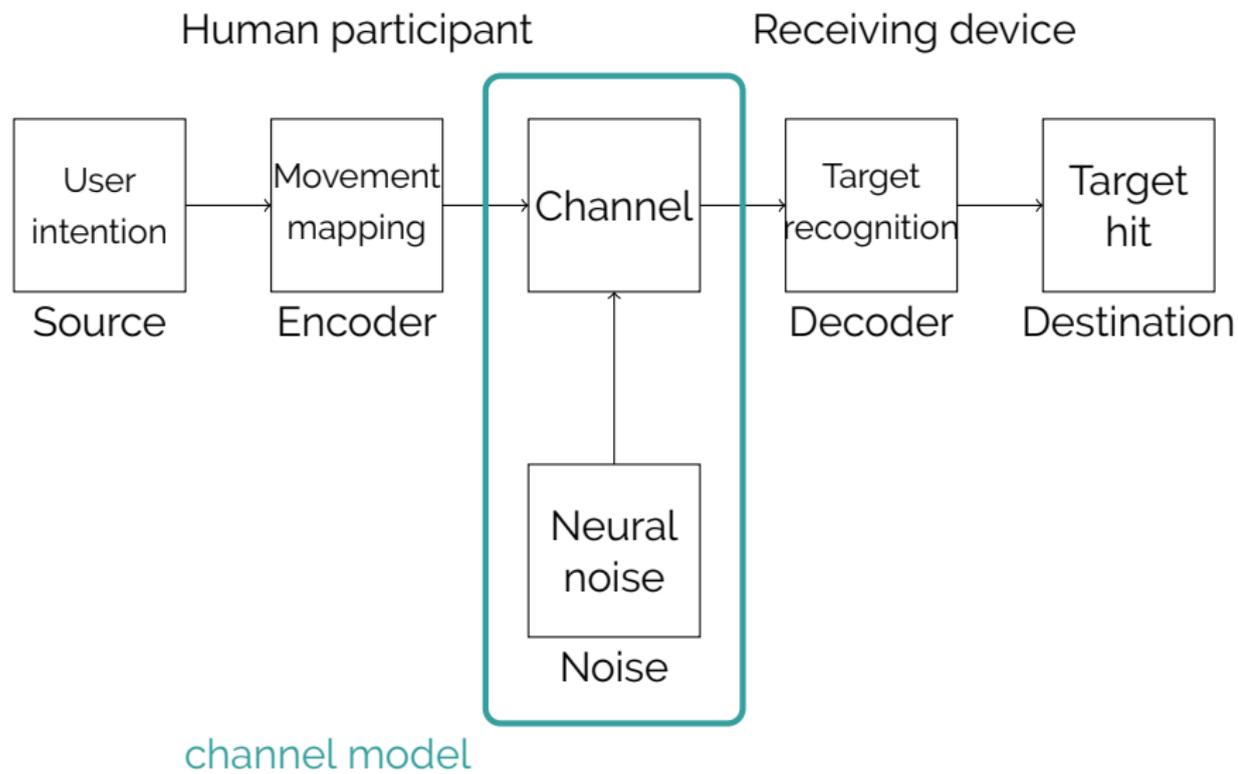
- " The average amplitude of a human movement is equivalent to signal " $\rightarrow P \equiv D$
 - " Movement variability is equivalent to noise " $\rightarrow N \equiv W$
 - B_W is expressed in Hertz $\equiv s^{-1} \simeq MT^{-1}$
 - $MT \propto \frac{1}{C} \log(1 + D/W)$ (1954) $\simeq a + b \log(1 + D/W)$ (1964)
-

- Why equate a **variance** ($P \propto X^2$) to **amplitude** D ?
- What is the channel model of the aiming task?
- Which input? Which output?

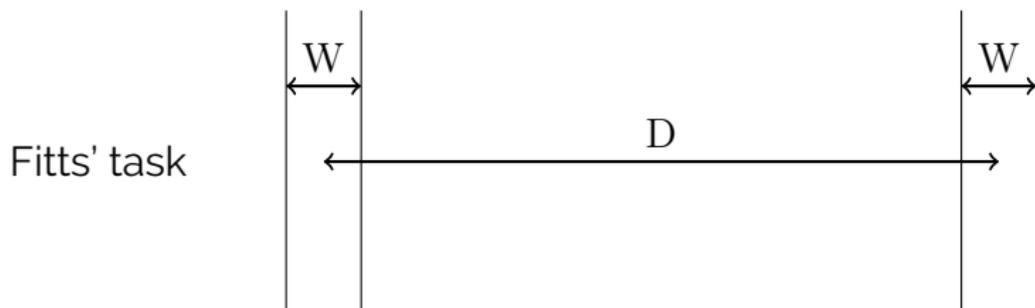
Our model (FITTS) answers these questions.

FITTS: A FORMAL INFORMATION-THEORETIC TRANSMISSION SCHEME

COMMUNICATION MODEL



CONSTRAINTS ON X , Y AND Z



- Hit the target: aim at the center and allow variability less than $\frac{W}{2}$ (*no miss*):

$$-\frac{W}{2} \leq Z \leq \frac{W}{2}$$

- The amplitude range of the input signal is D :

$$-\frac{D}{2} \leq X \leq \frac{D}{2}$$

- Additive noise model: $Y = X + Z$

$$-\frac{D+W}{2} \leq Y \leq \frac{D+W}{2} \quad (\text{triangular inequality})$$

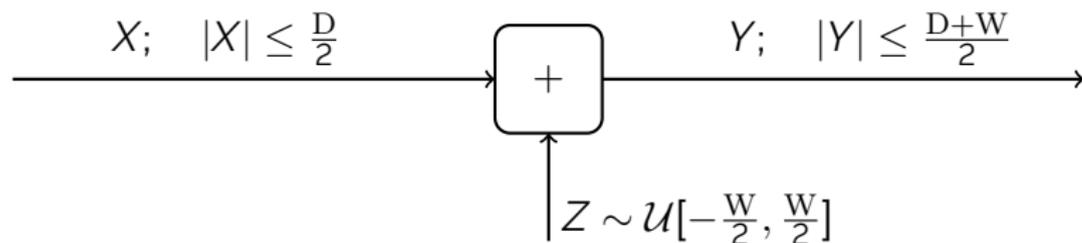
WHICH NOISE Z ?

- We assumed $|Z|$ is bounded by $\frac{W}{2}$, but choosing the noise distribution is not evident
- **Principle of Maximum Entropy** (Jaynes 1957): The probability distribution that best represents the current state of knowledge is the one with *largest* entropy (i.e., *the worst noise!*)

The maximum entropy distribution for $-\frac{W}{2} \leq Z \leq \frac{W}{2}$ is the uniform distribution $\mathcal{U}[-\frac{W}{2}, \frac{W}{2}]$:

$$p_Z(z) = \frac{1}{W}, \quad z \in [-\frac{W}{2}, \frac{W}{2}]$$

UNIFORM CHANNEL MODEL

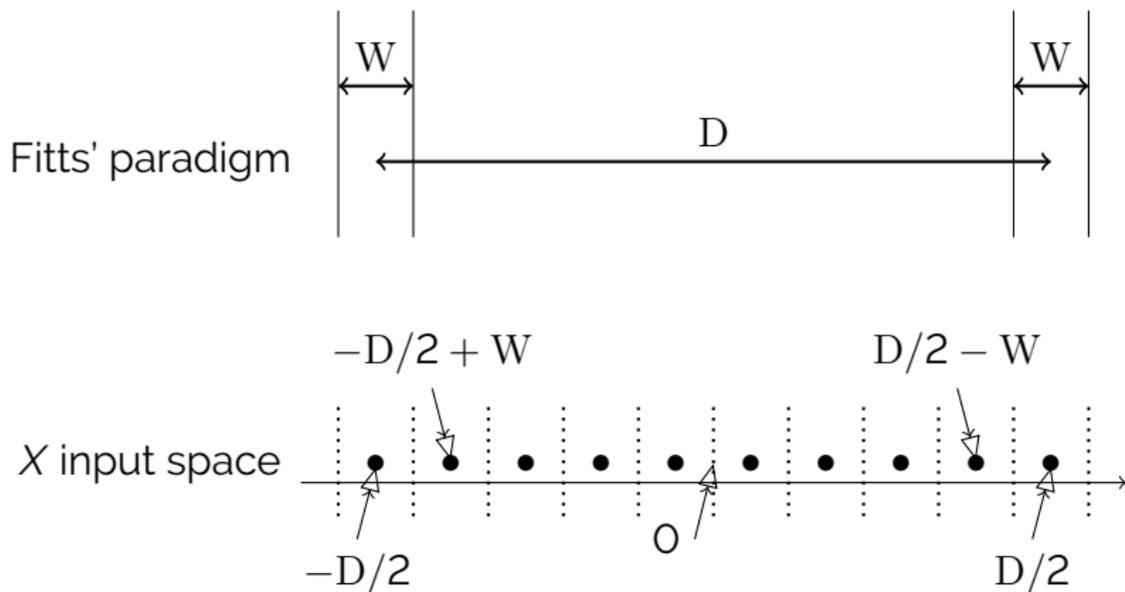


- Capacity of the uniform channel:

$$C = \max_X I(X; Y) = \log(1+D/W) = \text{ID (MacKenzie 1989)}$$

- Discrete uniform input X reaches capacity
- **Addresses Problem 1 (Formulations)**

CAPACITY ACHIEVING DISTRIBUTION



Aiming is choosing

C AS A LOWER BOUND

- Uniform noise = maximum entropy noise = worst noise.

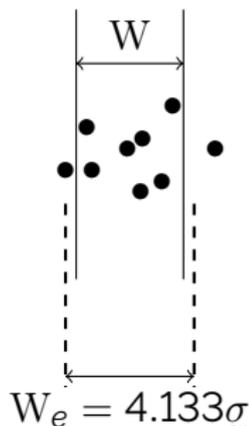
$$C = \log\left(1 + \frac{D}{W}\right)$$

What if the noise is not uniform, but e.g. Gaussian ?

- With a Gaussian noise, limited to $W = 4.133\sigma$, the resulting capacity C' differs by at most 0.2 bit:

$$C \leq C' \leq C + 0.2 \text{ bit}$$

(Crossman-MacKenzie correction)



Model for errorless task, with Uniform Noise



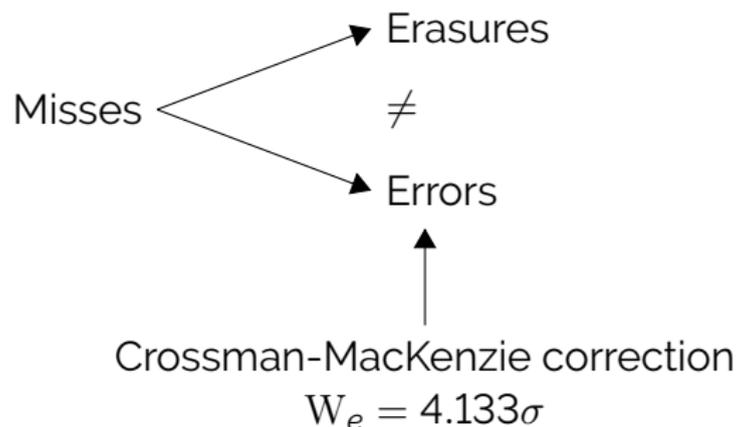
Model for errorless task, with any Arbitrary Noise



Model with misses

DISTINGUISHING ERRORS FROM ERASURES

Participants sometimes miss the target (rate ϵ).



- Using σ = considering the distance from each endpoint to the target's center
- 100% of movements involve metrical errors (of varying amplitude)

ERASURES



In GUIs, what matters is

- **Whether or not** the click falls in the intended area
- Not **where** the click takes place
- Not go **as close** to the center of the target as possible

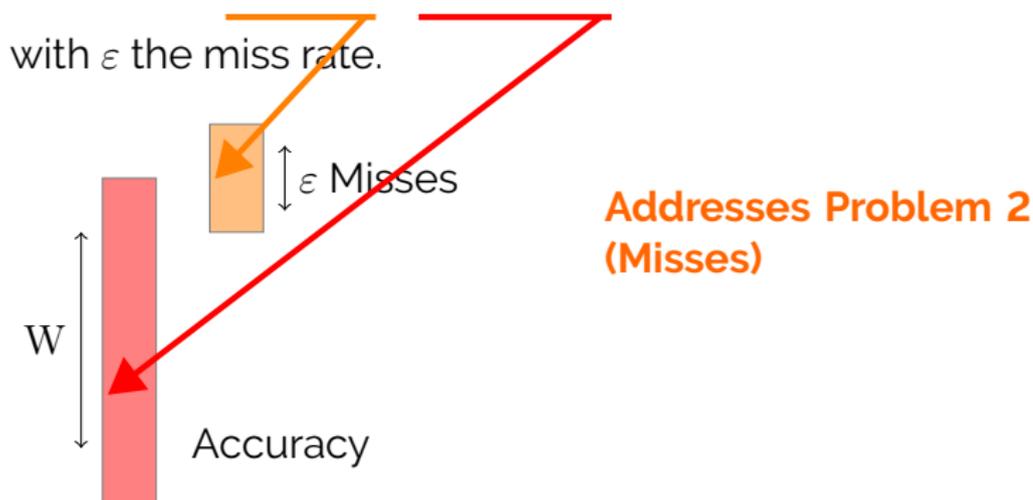
Dichotomous hit or miss corresponds to erasures in information-theory.

CAPACITY FOR THE CHANNEL WITH ERASURES

We show that

$$C = (1 - \varepsilon) \log_2(1 + D/W) = (1 - \varepsilon)ID = ID(\varepsilon),$$

with ε the miss rate.



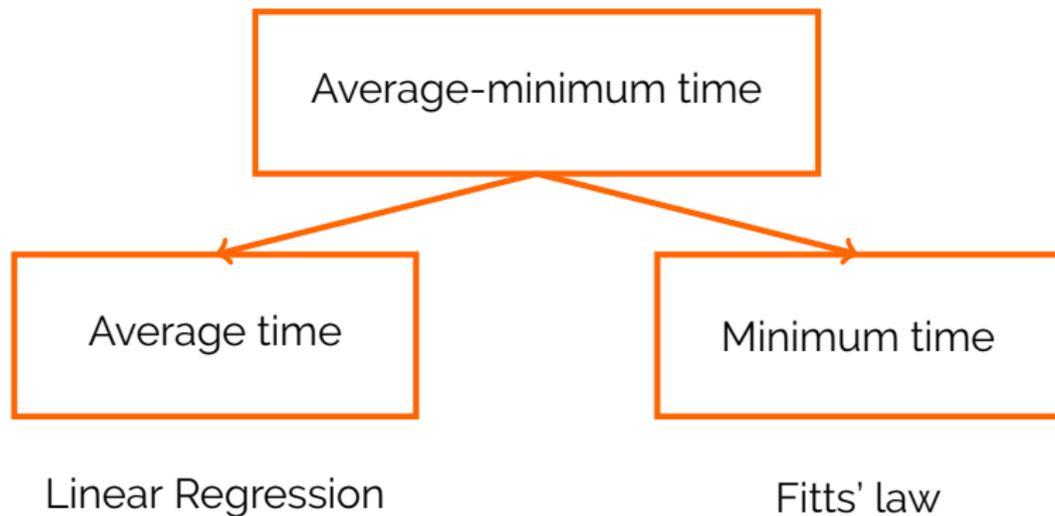
COMPARISON WITH CROSSMAN-MACKENZIE CORRECTION

Crossman-Mackenzie	Erasure Model
$ID_e = \log\left(1 + \frac{D}{4.133\sigma}\right)$	$ID(\varepsilon) = (1 - \varepsilon)ID$
Mistakes = Metrical Errors	Mistakes = Erasures
Continuous	Discrete Binary
Gaussian hypothesis	No hypothesis
$\lim_{\varepsilon \rightarrow 0} ID_e = \infty$	$ID(0) = (1 - 0)ID = ID$

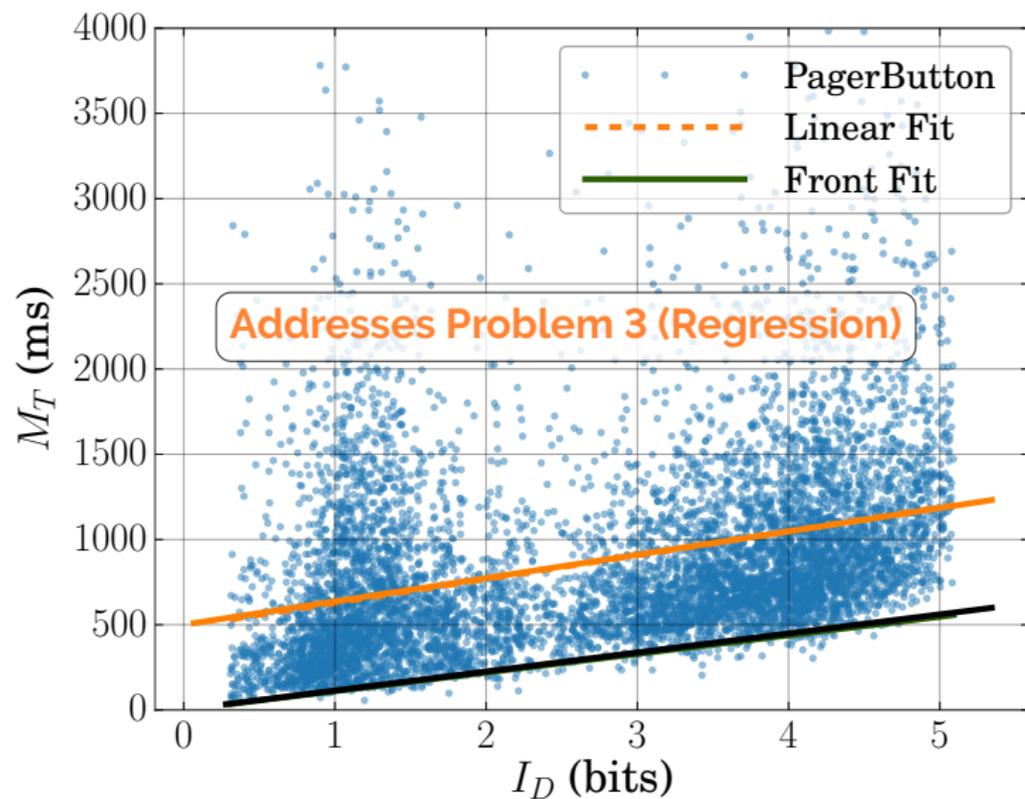
WHICH MOVEMENT TIME ?

A complete model for movement time (MT) in Fitts' task ...

- Channel capacity gives **minimum** MT ($C = \max I(X; Y)$)
- "The operator was instructed to work **as rapidly as possible. [...] Compute average time**"



EXAMPLE — FIELD STUDY



CONCLUSION

Our FITTS model

- unambiguously links ID to a channel capacity,
- in a simple scheme where input, output and noise are defined and interpreted.

FITTS is a “black-box” model for endpoints:

- only concerned with MT (not about the full trajectory)
- Not based on empirical findings (neuroscience, psychology)
- Based on (abstract) information theory
- No feedback taken into account

Additional problem: Compute the *minimum* time regression

Speed-Accuracy Tradeoff: **A Formal Information-Theoretic Transmission Scheme (FITTS)**

J. Gori, O. Rioul, and Y. Guiard, "Speed-accuracy tradeoff of aimed movement: A formal information-theoretic transmission scheme (FITTS)," ACM Transactions on Computer-Human Interaction (TOCHI), Vol. 25, No. 5, pp. 27:1-27:33, Oct. 2018

- FITTS 2: Scheme for full trajectories with feedback control [arXiv]:1804.05021
<http://www.theses.fr/2018SACL022>
- EMG Regression pre-print:
<https://hal.archives-ouvertes.fr/hal-02111178>