

# Real BCH Codes as Joint Source Channel Codes for Satellite Images coding

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## ABSTRACT

In this paper, a Bose-Chaudhuri-Hocquenghem (BCH) coder in the field of the real numbers is investigated for simultaneous source coding and impulse noise cancellation of satellite images. Our channel is a binary symmetric channel (BSC). Our approach is to make a carefully designed interpolation of the subband images prior to quantization and transmission.

Compared to a classical tandem source and channel coding (TSC) scheme, in which a binary BCH coder would take place *after* quantization, our approach makes use of BCH coding *prior to* quantization, thus allowing *joint* source and channel decoding.

We also examine the issue of robust transmission of wavelet compressed images through the noisy channel: Simulations show that we obtain a 3.7dB improvement in PSNR over the classical entropy coder for a global rate of 3.25 transmitted bits per pixel and small BSC crossover probability.

## I. INTRODUCTION

Images acquired on board spacecrafts (earth observation, satellites, scientific probes, ...) represent in most cases very large volumes of data. The need to transmit large amounts of data over a band limited channel has led to the development of various data compression schemes. Many of these schemes [1] function by attempting to remove redundancy from the data stream.

An unwanted side-effect of this approach is to

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make the information transfer process more vulnerable to channel noise: loss of synchronization in the entropy coded image data results in the misplacement of decoded coefficients and causes a severe degradation of reconstructed images. A classical approach to solving this problem is to use *channel coding*: one inserts redundancy in the output of the source encoder to make it easier for the receiver to detect and/or correct the erroneously received data. This classical approach does not take full advantage of the redundancy introduced by the channel coder: if no error occurred in the channel this redundancy is wasted whereas it could have been useful to reduce the quantization noise by increasing the quantizer's precision.

In this paper, we follow the approach suggested by Sayood and Borkenhagen [2] which combines both channel and source statistics in the design of the source coder. Our approach inserts a carefully designed redundancy to the subbands prior to quantization and exploits the knowledge of the channel characteristics and quantization to restore the received data.

## II. TRANSMISSION SCHEME

The proposed coding scheme is depicted in fig. 1. The satellite image is decomposed to 3 dyadic scales using a wavelet transform, this yields 10 subbands images. Each source word  $\underline{i}$ , issued from a subband ( $k$  samples) is first encoded using a real BCH coder that produces a codeword  $\underline{c}$  on  $n$  samples, which is then quantized on  $b_i$  bits per sample using a Lloyd-Max quantizer.

Natural index assignment is then applied to

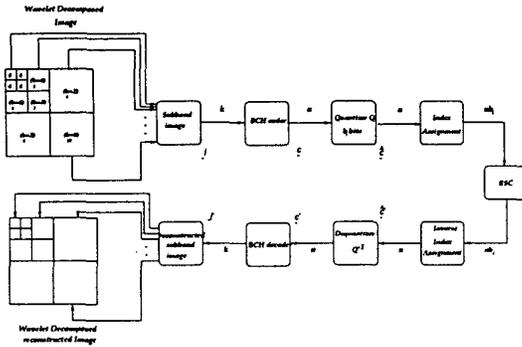


Fig. 1. Principle of the real BCH coding scheme

$\hat{c} = Q(c)$  and the resulting  $n \times b_i$  bits are transmitted over the BSC defined by its crossover (bit error) probability  $\epsilon$ .

### III. BCH ENCODER ON THE REALS NUMBER: DESCRIPTION

We describe these codes in the framework of BCH codes [3], whose definition and properties can be investigated using the Discrete Fourier Transform (DFT).

The BCH coder diagram is shown in fig. 2. A block  $\underline{l}$  of  $k$  spectral components is computed from the original data block  $\underline{i} = (i_0, i_1, \dots, i_{k-1})$  by applying a length- $k$  DFT.

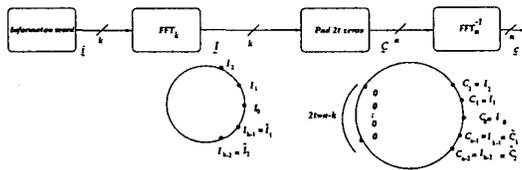


Fig. 2. BCH coding from the information word to the expanded BCH encoded signal

This block is then padded with  $n - k$  consecutive zeros in such a way that Hermitian symmetry is preserved (see fig. 2), and a length- $n$  inverse DFT is applied, resulting in a real encoded signal  $\underline{c} = (c_0, c_1, \dots, c_{n-1})$ .

This codeword is normalized so that energy is preserved:  $\|\underline{c}\|^2 = \|\underline{i}\|^2$ . Compared to a classical systematic BCH coding scheme as described by Blahut [5], this encoding procedure has the nice feature that it roughly preserves the amplitude range of data samples.

### IV. RELATION TO FRAME EXPANSION METHODS

The real BCH encoding procedure is a special case of the expansion-quantization-reconstruction scenario depicted in [4]. In our case the encoding equation can be written as

$$\underbrace{\underline{c}}_{n \times 1} = \underbrace{W_n^{-1}}_{n \times n} \underbrace{\underline{P}}_{n \times k} \underbrace{W_k}_{k \times k} \times \underbrace{\underline{i}}_{k \times 1}$$

where  $\underline{W}_l$  is the length- $l$  DFT matrix,  $\underline{P}$  is the  $n - k$  zero-padding matrix.

The codeword components  $c_j, j = 0, 1, \dots, n-1$  are then quantized as in [4]. The problem is how to use the redundancy present in the quantized codeword to correct errors introduced by the channel. This is solved in the next section using the language of BCH codes.

### V. REAL BCH DECODER: DESCRIPTION

The decoding problem can be rephrased as follows. Given a noisy codeword  $\hat{\underline{c}}$  input to the decoder, estimate the transmitted codeword  $\underline{c}'$  in order to finally reconstruct the initial source word  $\underline{i}'$ .

From fig. 1 and the assumption that the quantizer is scalar and the BSC channel is memoryless, we find that each codeword sample is affected (independently of the others) according to

$$\hat{c}'_j = c_j + n_j + e_j \quad j = 0, 1, \dots, n-1$$

where  $n_j$  is the quantization noise (assumed white) and  $e_j$  is the impulse "error" due to the BSC channel. The impulse error probability that  $e_j \neq 0$  is  $p = 1 - (1 - \epsilon)^{b_i}$  where  $b_i$  is the number of quantized bits per sample and  $\epsilon$  is the BSC bit error probability. Typically when  $e_j \neq 0$ ,  $e_j$  takes larger values than  $n_j$ .

We use the redundancy introduced by BCH coding to simultaneously localize and correct impulse noise samples and reduce quantization noise. The BCH decoding algorithm is based upon the fact that  $n - k$  consecutive DFT components of the codeword  $\underline{c}$  vanish. After quantization and transmission, the corresponding components of  $\hat{\underline{c}}$  will no longer take zero values even when no channel errors have been introduced. These components are first computed in the spectral domain by the

BCH decoder. They constitute the so-called *syndrome* [5] and is used as a “signature” of the impulse noise to be removed in the presence of the background noise.

The decoding algorithm is then in three steps: (1) evaluate the number of impulses considered as “errors,” (2) find the error locations and (3) find and correct the error values to recover  $\underline{c}'$ . Blahut [5] describes several efficient algorithms for doing this, but unfortunately these are very sensitive to quantization noise. Therefore, we have followed the approach taken in [6], which is a modified version of the classical Peterson-Gorenstein-Zierler algorithm adapted to the real number case: (1) The number of “error impulses” is first determined as the rank of a suitable “syndrome matrix,” taking the statistical contribution of the quantization noise into account. (2) Then, we solve a Yule-Walker system to compute the error-locator polynomial, whose roots give the location of the impulses. (3) Finally, from the estimated locations we solve an overdetermined Van der Monde system in the least squares sense to estimate the impulse amplitudes.

At each step of the algorithm, we are able to detect probable malfunction of the decoder. If malfunction is detected, and if we insist on correcting errors there will be a significant increase in distortion due to the decoder because additional errors will be introduced. Therefore, in this case, the algorithm stops and the noisy input data  $\underline{c}$  is directly output as  $\underline{c}'$ .

As a final step, the corrected word is “projected back to the code”: the  $n - k$  spectral components are removed in order to recover the source word  $\underline{z}'$  by inverting the encoding process of section III. Notice that when no “impulse” is detected at the reception, this last step will always reduce the quantization distortion. This, as we have already noticed, cannot occur in a classical “tandem” scheme.

## VI. PRODUCT BCH CODES ON THE REALS

The concept of real BCH product codes [7] is a simple and relatively efficient method to construct powerful codes capable of solving the decoder malfunction by iterating the decoding algorithm.

Given a code  $\mathcal{C} = (n, k)$ , the product code is

obtained by:

- placing  $(k \times k)$  information samples in a matrix,
- coding the  $k$  rows by the code  $\mathcal{C}$ ,
- coding the  $n$  columns using the code  $\mathcal{C}$ .

The resulting product codeword is a  $n \times n$  matrix. On receiving the matrix, the first decoder performs the decoding of the columns (and rows) of the matrix, estimates and correct the errors when no malfunction is detected, and gives as output to the next decoder the resulting decoded matrix [6].

In fact, a very simple loop procedure is used to achieve decoding: A first pass is made on the lines of the matrix, a second pass is then performed on the columns. Next iteration the same procedure is repeated. Even when only a few impulses are suitably corrected in the beginning of the algorithm, such correction greatly reduces the task of the following step, which is performed in the other direction in the subband matrix.

## VII. SIMULATION RESULTS

We compare our method (fig. 1) to the classical TSC scheme depicted in fig. 3.

The source rate, calculated from fig. 1 is  $R_s = \frac{\sum_{i=1}^4 b_i + 4 \sum_{i=5}^7 b_i + 16 \sum_{i=8}^{10} b_i}{64}$  and gives us the number of bits per pixel in the output of the source coder. The tandem scheme consists of uniform quantizing the subband, followed by entropy coding and binary BCH coding with the  $(N, K)$  BCH code.

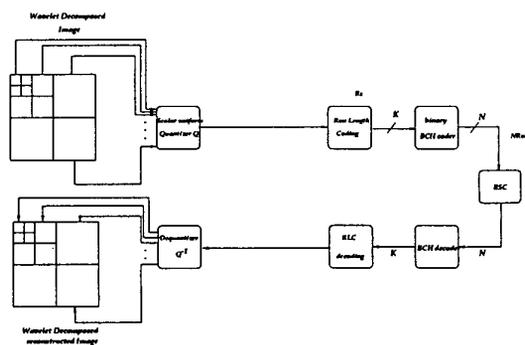


Fig. 3. Principle of the TSC scheme

The binary coded flow is then transmitted over the BSC. When a binary BCH decoder malfunction occurs, it leads to the loss of synchronization. The periodic insertion of a synchronization word allows the reconstruction of the next 16-block length group when the BCH decoder fails.

Both transmission schemes use the same global rate  $\frac{n}{k} \times R_s \approx \frac{N}{K} \times R_s$  transmitted bits per pixel. The comparison is made using natural index assignment.

Simulations were done using a  $1000 \times 1000$  satellite image of Los Angeles, and the (9-7) Daubechies bi-orthogonal filter. Figure 4 gives the end-to-end PSNR in the reconstructed image relative to the initial satellite image, as a function of the BSC bit error probability  $\epsilon$  for both real BCH and TSC schemes.

The numerical values were  $R_s = 2$  bits/pixels,  $(n = 51, k = 31, t = 10)$ , the code used for the product code was the  $(n' = 19, k' = 15, t' = 2)$  and the binary BCH code was  $(N = 63, K = 39, t = 4)$ , giving a global rate around 3.25 transmitted bits per pixel. Also shown in fig. 4 is the PSNR

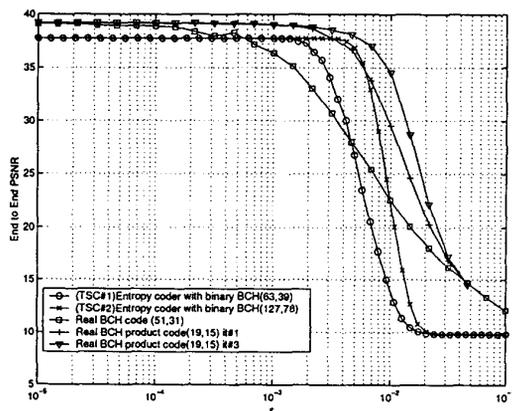


Fig. 4. End-to-end PSNR comparison of real BCH coding and Binary BCH (TSC) coding schemes.

obtained by an entropy coder with the same  $R_s$  and the binary BCH code was  $(N = 127, K = 78)$

We observe that, for small BSC channels ( $\epsilon < 10^{-3}$ ), the real BCH scheme always outperform the case of simple quantization with entropy coding, for the same global rate, as was to be expected. Moreover, the real BCH scheme is more robust to channel noise than the TSC scheme for a large range of  $\epsilon$  values. For example, we obtain a 1.4dB improvement in PSNR over the classical TSC scheme for very small BSC crossover probability.

If we use a binary BCH  $(N, K, t)$  coding after an entropy coder, the error probability after decoding

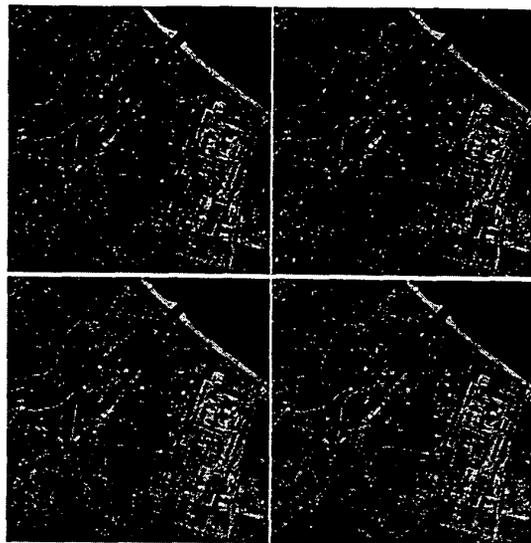


Fig. 5. Iterative denoising of Los Angeles (a), after first (b), second(c) and third(d) iterations with  $\epsilon = 10^{-2}$

will be:

$$P_e = \sum_{i=t+1}^{i=N} C_N^i \epsilon^i (1 - \epsilon)^{N-i}$$

For the binary BCH  $(N = 63, K = 39, t = 4)$  code and  $\epsilon = 2.10^{-3}$ , that means that 14.5% of  $1000 \times 1000$  compressed images with a rate of  $R_s = 2$  bits per pixel will have a severe degradation in the reconstruction due to the loss of synchronization. Figure 5 show simulation results for real BCH product code after three iterations, We observe the great reduction of impulse noise as shows the numerical values in fig. 4.

Figure 6 show simulation results for classical TSC coding scheme of the same image passed through a BSC channel with  $\epsilon = 10^{-2}$  with the same global rate of 3.2 bits per pixel. We observe the loss of synchronization due to the BCH decoder malfunction, in the entropy coded image data resulting in the misplacement of decoded coefficients and therefore causing a severe degradation of reconstructed images.

## VIII. SHOAM-GERSHO OPTIMISATION

For a given crossover probability and a global rate, the presented method of real BCH coding prior to quantization can be optimized using a set of quantizers, and a set of real BCH prod-



Fig. 6. Synchronisation loss effect on the reconstructed image in TSC coding scheme ( $\epsilon = 10^{-2}$ )

uct codes for each subband image. This allows to use the global bit allocation algorithm [8], capable of efficiently allocating a given quota of bits to the set of different real BCH coding followed by scalar quantizers for each subband image. This useful algorithm combined to our real BCH coding scheme gives the good tradeoff between quantizer accuracy and protection for each subband. The figure7 show the performance obtained by such optimisation, compared to the traditionnal TSC scheme. The real BCH coding scheme outperforms the TSC scheme: since “channel coding” occurs before quantization, high amplitude errors issued from quantization, are also considered as impulse noise and can be removed. For a rate of 3.2 bits per pixel, a 3.7 dB improvement over the TSC scheme is observed.

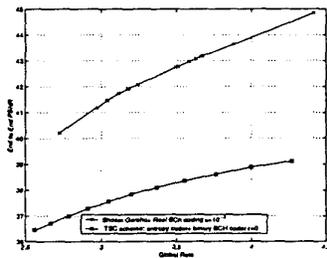


Fig. 7. PSNR comparison of the real BCH coding approach with the TSC scheme

Furthermore, the approach gives us the hierachical protection and the resolution to achieve, for each subband.

## IX. CONCLUSION

The presented method of real BCH coding prior to quantization and the decoding algorithm is able to reduce distortion introduced not only by the transmission channels errors but also by the quantizer. The decoding algorithm is therefore suitable to increase the end-to-end PSNR since it deals simultaneously with the quantization noise and impulse channel noise.

Compared to classical TSC scheme, our real BCH coding approach is more robust to channel noise for a large range of crossover propability, thus allowing efficient *joint* source and channel decoding.

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