LIESSE Course : 5G in all its forms

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Outline

1. Intersymbol interference (ISI) and OFDM
   - Detection theory
   - Diversity and MIMO
   - Information-theoretic point of view

2. Multi-user interference (MUI) and resource allocation
   - Reminder on non-convex optimization
   - Problem 1 : Waterfilling
   - Problem 2 : Power minimization with nonlinear interference
   - Problem 3 : Energy efficiency optimization

3. Energy consumption
   - Modeling issue
   - Some figures
   - Systemic point of view

Section 1 : Intersymbol interference (ISI) and OFDM

System model

- s_n : transmit (complex-valued) symbol related to information bits
- h_n : impulse response of the channel (due to multipath)
- w_n : white Gaussian noise
- y_n : receive samples (to be decoded)

Goal : decode the data from $n = 0, \cdots, N-1$

$$y_n = \sum_{\ell=0}^{L} h_\ell s_{n-\ell} + w_n$$

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}$$

with $\mathbf{y} = [y_{N-1}, \cdots, y_0]^{\mathrm{T}}$
and $\mathbf{s} = [s_{N-1}, \cdots, s_0, \underbrace{s_{-1}, \cdots, s_{-L}}_{known}]^{\mathrm{T}}$

Matrix model also valid for wireless MIMO, PolMux, multimode, multiple access (but with different matrix structures)

Refresher on detection theory

Main result

If data are equilikely, then the optimal receiver (minimizing the error probability) is the Maximum Likelihood (ML)

$$\hat{\mathbf{s}}_{\mathrm{ML}} = \arg\max_{\mathbf{s}} p_{Y|S}(\mathbf{y}|\mathbf{s})$$

Application to linear model

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}$$

with

- matrix of size $N_r \times N_t$
- **w** white Gaussian noise

ML = Least Square (LS)

$$\hat{\mathbf{s}}_{\mathrm{ML}} = \arg\min_{\mathbf{s}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2$$

Optimal detector

$$\hat{\mathbf{s}}_{\mathrm{ML}} = \arg\min_{\mathbf{s}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2$$

Simple optimization without constraint on $\mathbf{s}$

Optimization problem consists in finding out the minimum of a (positive) quadratic form under (nonconvex) conditions

- Exhaustive search : complexity in $\mathcal{O}(M^{N_t})$ with M-QAM
  - ex. : 4×4 MIMO with 64-QAM leads to 16M $\mathbf{s}$
- Special simple cases : separable fcts ($N_t = 1$ or unitary matrix)
- Using the structure of $\mathbf{H}$
  - ISI due to $\mathbf{H}$ band-Toeplitz $\Rightarrow$ Viterbi algo. (complexity in $\mathcal{O}(N.M^L)$)
- Suboptimal detectors
  - Remove constraint and threshold ($\hat{\mathbf{s}} = \mathrm{threshold}(\mathbf{H}^{\#}\mathbf{y})$) $\Rightarrow$ ZF
  - MMSE, DFE
  - Neural networks as a classifier

Optimal detector

$$\hat{\mathbf{s}}_{\mathrm{ML}} = \arg \min_{\mathbf{s} \in \mathcal{C}^{N_H}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2$$

Hard optimization due to constraint on $\mathbf{s}$

Optimization problem consists in finding out the minimum of a (positive) quadratic form under (nonconvex) conditions

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  - Neural networks as a classifier

Special case : $N_r \neq 1, N_t = 1$ (SIMO)

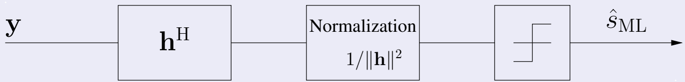
We consider a multivariate signal (typically L antennas)

$$y_\ell = h_\ell s + w_\ell \text{ with } \ell = 1, \cdots, N_r \iff \mathbf{y} = \mathbf{h}s + \mathbf{w}$$

with $\mathbf{h}$ a $N_r \times 1$ column-vector

We have

$$\begin{aligned} \hat{s}_{\mathrm{ML}} &= \arg\min_{s \in \mathcal{C}} \|\mathbf{y} - \mathbf{h}s\|^2 \\ &= \arg\min_{s \in \mathcal{C}} \|\mathbf{y}\|^2 + \|\mathbf{h}\|^2 |s|^2 - \mathbf{y}^{\mathrm{H}}\mathbf{h}s - \bar{s}\mathbf{h}^{\mathrm{H}}\mathbf{y} \end{aligned}$$



- ML=ZF (with pseudo-inverse $\mathbf{h}^{\#} = (\mathbf{h}^{\mathrm{H}}\mathbf{h})^{-1}\mathbf{h}^{\mathrm{H}} = \mathbf{h}^{\mathrm{H}}/\|\mathbf{h}\|^2$)
- Easy to implement

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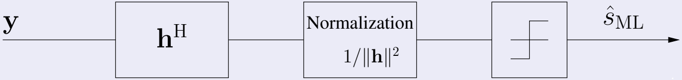
We have

$$\begin{aligned}
\hat{s}_{\mathrm{ML}} &= \arg\min_{s \in \mathcal{C}} \|\mathbf{y} - \mathbf{h}s\|^2 \\
&= \arg\min_{s \in \mathcal{C}} \|\mathbf{h}\|^2 \left\| \frac{\mathbf{h}^{\mathrm{H}}\mathbf{y}}{\|\mathbf{h}\|^2} \right\|^2 + \|\mathbf{h}\|^2 \left(|s|^2 - \frac{\mathbf{y}^{\mathrm{H}}\mathbf{h}}{\|\mathbf{h}\|^2} s - \bar{s} \frac{\mathbf{h}^{\mathrm{H}}\mathbf{y}}{\|\mathbf{h}\|^2} \right)
\end{aligned}$$



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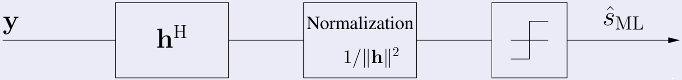


- ML=ZF (with pseudo-inverse $\mathbf{h}^{\#} = (\mathbf{h}^{\mathrm{H}}\mathbf{h})^{-1}\mathbf{h}^{\mathrm{H}} = \mathbf{h}^{\mathrm{H}}/\|\mathbf{h}\|^2$)
- Easy to implement

Go back to ISI model

- $\mathbf{x} = [x_{N-1}, \cdots, x_0]^{\mathrm{T}}$ be the transmit signal (may be different from s_n)
- $\tilde{\mathbf{x}} = [x_{-1}, \cdots, x_{-L}]^{\mathrm{T}}$

$$\mathbf{y} = \mathbf{T}_1\mathbf{x} + \mathbf{T}_2\tilde{\mathbf{x}}$$

- $\mathbf{T}_1$: $N \times N$ Toeplitz matrix whose the k-th row is given by
- $\mathbf{T}_2$: $N \times L$ Toeplitz matrix whose the k-th row is given by

In noisy case, we find again

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

- when null guard interval between blocks : $\mathbf{H} = \mathbf{T}_1$ and $\tilde{\mathbf{x}} = 0$
- when guard interval $\tilde{\mathbf{x}}$ is not empty and linearly depends on current bloc, i.e., $\tilde{\mathbf{x}} = \mathbf{T}_3\mathbf{x}$: $\mathbf{H} = \mathbf{T}_1 + \mathbf{T}_2\mathbf{T}_3$

Question : how obtaining an interference-free system at the receiver side (by modifying the transmitter) ?

Lemma 1

Assumption :

- Perfect CSIT ($\mathbf{H}$ known at the transmitter side)

Let $\mathbf{H} = \mathbf{U}^{\mathrm{H}}\mathbf{\Lambda}\mathbf{V}$ be the singular value decomposition (svd) of $\mathbf{H}$ with $\mathbf{U}$, $\mathbf{V}$ two unitary matrices, and $\mathbf{\Lambda} = \mathrm{diag}(\lambda_0, \cdots, \lambda_{N-1})$

- Instead of sending $\mathbf{x} = \mathbf{s}$, we send $\mathbf{x} = \mathbf{V}^{\mathrm{H}}\mathbf{s}$ (no energy purpose)
- Instead of detecting on $\mathbf{y}$, we detect on $\mathbf{z} = \mathbf{U}\mathbf{y}$

$$\Rightarrow \mathbf{z} = \mathbf{\Lambda}\mathbf{s} + \mathbf{w}'$$

with $\mathbf{w}' = \mathbf{U}\mathbf{w}$ still white Gaussian noise

Remarks

- Information located in eigenvectors (not interfere in-between !)
- Eigenvectors usually depend on $\mathbf{H}$
- Issue : CSIT assumption unrealistic in wireless and even in optic

Lemma 2

Let $\mathbf{C}$ be a $N \times N$ circulant matrix associated with $\{h_\ell\}_{\ell=0,\cdots,L}$

$$\mathbf{C} = \begin{bmatrix} h_0 & h_1 & \cdots & h_L & 0 & \cdots & 0 \\ 0 & h_0 & h_1 & \cdots & h_L & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ h_1 & \cdots & h_L & 0 & \cdots & 0 & h_0 \end{bmatrix}$$

Properties of C

- Eigenvectors : Fourier vectors (independent of the channel !)
- Eigenvalues : filter responses at Fourier frequencies

$$\mathbf{C} = \mathbf{F}^{\mathrm{H}} \mathbf{\Lambda} \mathbf{F}$$

with

- $\mathbf{F}$ FFT matrix (so, $\mathbf{F}^{\mathrm{H}} = \mathbf{F}^{-1}$)
- $\lambda_n = H(e^{2i\pi n/N}) = \sum_{\ell=0}^{L} h_\ell e^{-2i\pi \frac{\ell n}{N}}$

OFDM principle

If $\tilde{\mathbf{x}} = [x_{N-1}, \cdots, x_{N-L}]^{\mathrm{T}}$, then

$$\mathbf{y} = \mathbf{T}_1\mathbf{x} + \mathbf{T}_2\tilde{\mathbf{x}} \Leftrightarrow \mathbf{y} = \mathbf{C}\mathbf{x}$$

Thus we have

$$\mathbf{z} = \mathbf{\Lambda}\mathbf{s}$$

with $\mathbf{z} = \mathbf{F}\mathbf{y}$ and $\mathbf{x} = \mathbf{F}^{-1}\mathbf{s}$

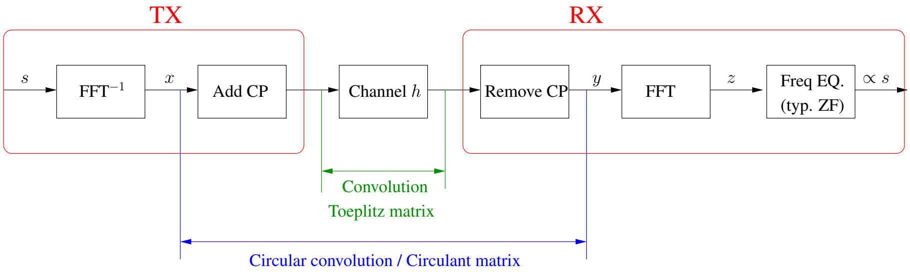
Finally, for the *k*-th block and the *n*-th subcarrier, we get

$$z_n^{(k)} = H(e^{2i\pi n/N})s_n^{(k)} \quad \forall n, k$$

Remarks

- cyclic prefix transforms Toeplitz into Circulant (who diagonalizes within a basis independent of the channel, so no required CSIT !)
- OFDM : Orthogonal Frequency Division Multiplexing

TX/RX Scheme



- mid-80 : European project "Eurêka" for DAB
  - cyclic prefix
  - coding and OFDM relationship in wireless context
- beginning-90 : first standard based on OFDM (DAB)
- end-90 : very popular standards (ADSL, DVBT, Wifi, 4G, 5G, $\cdots$)

Detection : OFDM-SISO

Formally, we have

$$\mathbf{z} = \mathbf{H}\mathbf{s} + \mathbf{w}$$

with $\mathbf{H} = \mathrm{diag}(H(1), \cdots, \underbrace{H(e^{2i\pi n/N})}_{H_n}, \cdots, H(e^{2i\pi(N-1)/N}))$

As $\mathbf{H}$ is diagonal, one can work subcarrier per subcarrier, i.e.,

$$z_n = H_n s_n + w_n$$

Optimal detector

Threshold detector on $H_n^{-1} z_n$

Extension to OFDM-MIMO

$$\mathbf{z}_n = \mathbf{H}_n \mathbf{s}_n + \mathbf{w}_n$$

with $\mathbf{H}_n$ a $N_r \times N_t$ filter response at the n-th subcarrier

Error probability point of view

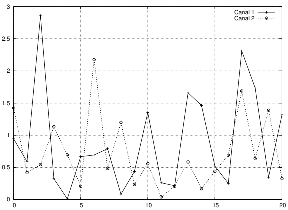
Gaussian channel : $z_n = s_n + w_n$

$$P_e \approx e^{-E_s/N_0}$$

Rayleigh channel : $z_n = h_n s_n + w_n$ with h_n iid Gaussian

$$P_e = \mathbb{E}[P_e(h)] \propto \frac{1}{E_s/N_0}$$

Problem : when fading h_n is weak, transmission quality is bad



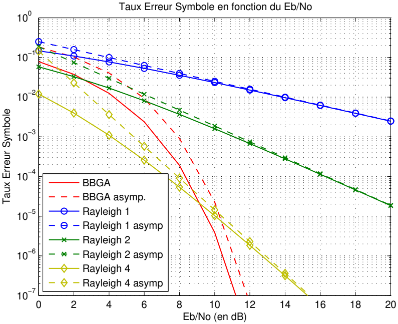
Error probability and diversity

Error probability with N_r independent channels

$$P_e \propto \frac{1}{(E_s/N_0)^{N_r}}$$

at high Signal-to-Noise Ratio (SNR).

$$N_r = \text{diversity order}$$



Examples

- Repetition coding with rate $1/n$. It leads to diversity order of n but weak spectral efficiency
- Let $y_1 = h_1 x_1 + w_1$ and $y_2 = h_2 x_2 + w_2$, with x_1 and x_2 independent BPSK

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

  with $\mathbf{y} = [y_1, y_2]^{\mathrm{T}}$, $\mathbf{H} = \mathrm{diag}(h_1, h_2)$.
  - Diversity of 1
  - Instead of sending $\mathbf{x}$, send $\mathbf{R}\mathbf{x}$ with a rotation $\mathbf{R}$. Diversity of 2 but rate of 1

MIMO code construction

$$\mathbf{Y}_{N_r \times T} = \mathbf{H}_{N_r \times n_t} \mathbf{X}_{N_t \times T} + \mathbf{w}_{N_r \times T}$$

Pairwise error probability

At high SNR, maximum likelihood receiver provides

$$P_e(\mathbf{X} \to \mathbf{X}') \approx \left(\prod_{k=1}^{r} \lambda_k \right)^{-N_r} \left(\frac{1}{E_s/N_0} \right)^{rN_r}$$

with λ_k eigenvalues and r rank of matrix $(\mathbf{X} - \mathbf{X}').(\mathbf{X} - \mathbf{X}')^{\mathrm{H}}$

Diversity and coding gain (pseudo-distance)

$$d = rN_r, \quad \gamma = \left(\prod_{k=1}^{r} \lambda_k \right)^{N_r}$$

Information theory point of view

For memoryless discrete channel, we get

- Entropy : $H(Z) = -\sum_k \Pr(Z = z_k) \log_2(\Pr(Z = z_k))$
- Mutual Information : $I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$

Channel capcity C

$$C = \max_{p(X)} I(X,Y)$$

Capacity theorem

It exists a coding scheme at information rate T and length N s.t. $T < C$ for which

$$\lim_{N \to \infty} P_e = 0$$

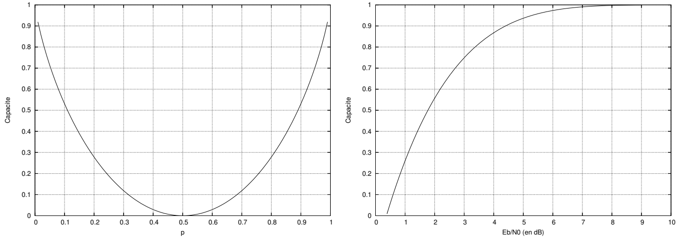
- The fundamental limit is the rate, not the error probability

Example : Binary Symmetric Channel (BSC)

$$C = 1 - H_b(p)$$

- Canal related to binary transmission with hard decision, and for Gaussian channel, we get

$$p = Q\left(\sqrt{\frac{2E_s}{N_0}}\right) = Q\left(\sqrt{\frac{2E_bC}{N_0}}\right)$$



- Capacity vanishes for $p = 1/2$.
- Capacity vanishes if $E_b/N_0 \leq 0.4$ dB

Capacity for Gaussian channel (continuous output/input)

$$y = x + w$$

with $\mathbb{E}[|x|^2] \leq P_x$ and w Gaussian noise with zero-mean and variance N_0

Mutual Information

$$\begin{aligned} I(X;Y) &= \int p(X,Y) \log_2 \left(\frac{p(X,Y)}{p(X)p(Y)} \right) dXdY \\ &= D_{KL}(p(X,Y)||p(X)p(Y)) = H_{diff}(X) - H_{diff}(X|Y) \end{aligned}$$

Capacity closed-form expression

$$C = \log_2 \left(1 + \frac{E_s}{N_0} \right)$$

As $E_s = TE_b$ and $T < C$, $E_b/N_0 \geq (2^T - 1)/(T) \overset{T \to 0}{\geq} \ln(2) \quad (-1.6 \text{ dB})$

MIMO channel

Mutual Information

$$I(X;Y) = \log_2 \det \left(\mathbf{Id} + \frac{1}{N_0} \mathbf{H} \mathbf{Q} \mathbf{H}^{\mathrm{H}} \right)$$

with $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^{\mathrm{H}}]$

If $\mathbf{Q} = \frac{P_{\mathbf{x}}}{N_t} \mathbf{Id}_{N_t}$, then

$$\begin{aligned} I(X;Y) &= \log_2 \det \left(\mathbf{Id} + \frac{P_{\mathbf{x}}}{N_t N_0} \mathbf{H} \mathbf{H}^{\mathrm{H}} \right) \\ &= \sum_k \log_2 \left(1 + \frac{P_{\mathbf{x}}}{N_t N_0} \lambda_k \right) \end{aligned}$$

with λ_k the k-th eigenvalue of $\mathbf{H}\mathbf{H}^{\mathrm{H}}$

MIMO block fading channel

Ergodic capacity (a codeword encounters all the channel realizations)

$$C = \mathbb{E}_{\mathbf{H}}\left[\log_2 \det\left(\mathbf{Id} + \mathrm{SNR}.\mathbf{H}\mathbf{H}^{\mathrm{H}}\right)\right]$$

with $\mathrm{SNR} = P_{\mathbf{x}}/(N_t N_0)$

Marcenko-Pastur distribution : the eigenvalues of $\frac{1}{N_t}\mathbf{H}\mathbf{H}^{\mathrm{H}}$ follow a determined pdf f when $N_r/N_t \to$ constant and $N_r, N_t \to \infty$

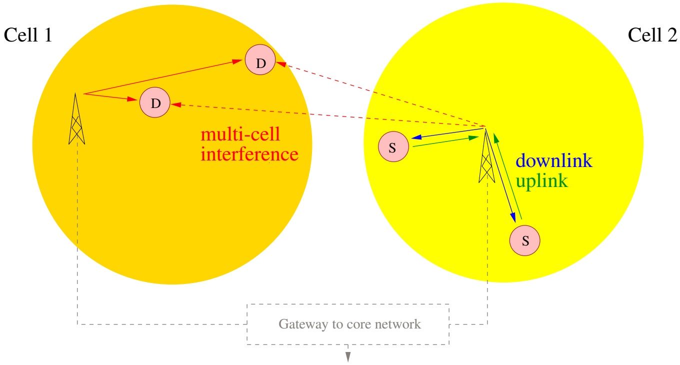
$$\begin{aligned} C &\approx \min(N_t, N_r) \times \frac{1}{\min(N_t, N_r)} \sum_{k=1}^{\min(N_t,N_r)} \log_2(1 + \mathrm{SNR}\lambda_k) \\ &\approx \min(N_t, N_r) \int \log_2(1 + \mathrm{SNR}\lambda) f(\lambda) d\lambda \\ &\approx \min(N_t, N_r) \log_2(\mathrm{SNR}) + \mathcal{O}(1) \end{aligned}$$

- Degrees of freedom (DoF) : $\min(n_t, n_r)$
- $N_t N_r$ independent channels but the rate only increased by $\min(N_t, N_r)$

Section 2 : Multi-user interference and resource allocation

Multi-user Interference issues : where do they come from ?

Cellular network (3G/4G/5G)



- Several information flows to manage
- Different kinds of interference : multi-cell (red), uplink (green), downlink (blue)

⇒ **Multi-User Interference (MUI)**

Example

- Consider MISO downlink with two users.
- Apply beamforming $\mathbf{v}_1$ and $\mathbf{v}_2$

We get

$$\begin{cases} y_1 &= \mathbf{h}_1^{\mathrm{T}}\mathbf{v}_1 s_1 + \mathbf{h}_1^{\mathrm{T}}\mathbf{v}_2 s_2 \\ y_2 &= \mathbf{h}_2^{\mathrm{T}}\mathbf{v}_1 s_1 + \mathbf{h}_2^{\mathrm{T}}\mathbf{v}_2 s_2 \end{cases}$$

Maximizing SINR is ideally equivalent to get

- $\mathbf{v}_1 \in \mathrm{span}(\mathbf{h}_1)$, $\mathbf{v}_2 \in \mathrm{span}(\mathbf{h}_2)$,
- and $\mathbf{v}_1 \in \ker(\mathbf{h}_2)$, $\mathbf{v}_2 \in \ker(\mathbf{h}_1)$

It happens iff $\mathbf{h}_1 \perp \mathbf{h}_2$. If not, by keeping the signal power maximization, we get

$$\begin{cases} y_1 &= \|\mathbf{h}_1\|^2 s_1 + \gamma s_2 \\ y_2 &= \gamma s_1 + \|\mathbf{h}_2\|^2 s_2 \end{cases}$$

with

$$\gamma = \mathbf{h}_1^{\mathrm{T}}\mathbf{h}_2 = <\mathbf{h}_1|\mathbf{h}_2>$$

MU-MIMO with beamforming leads to non-orthogonality

Naive solution 1 : do nothing

- One user of interest but $N-1$ interferers (with same power) :

$$y = x + \sum_{k=1}^{N-1} \gamma x_k + w$$

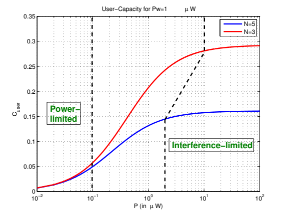
- **Assumption :** interference seen as an extra (Gaussian) noise :

$$C_{\text{user}} = \log_2\left(1 + \frac{P}{(N-1)\gamma^2 P + P_w}\right)$$

with user power P and noise power P_w

Result

- $C_{\text{user}} \to \log_2\left(1+\frac{1}{(N-1)\gamma^2}\right)$ when $P \to \infty$
- C_{target} achievable iff $N \leq 1 + \frac{1}{(2^{C_{\text{target}}}-1)\gamma^2}$
- no DoF issue but SINR issue



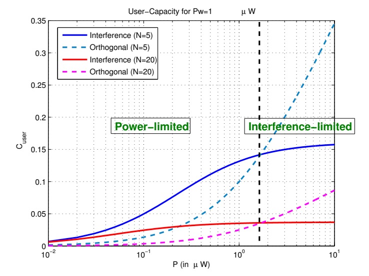
Naive solution 2 : take margin

- TDMA with time margin of $\gamma^2\%$
- FDMA with frequency margin of $\gamma^2\%$

Result

$$C^{\perp}_{\text{user}} = \frac{1}{N(1+\gamma^2)} \log_2\left(1 + \frac{P}{P_w}\right)$$

- $C^{\perp}_{\text{user}} \to \infty$ when $P \to \infty$, no upper bound
- For low and medium P (depending on N) $C_{\text{user}} > C^{\perp}_{\text{user}}$.
- no SINR issue but DoF issue



Comments

Two regimes :

- Interference-limited : if SNR large enough
- Power-limited : if SNR low enough

⇒ **Orthogonality can not be used for any flow (N too large)**
  - in practice in downlink and uplink only, ...

⇒ **Even if orthogonality used, partially broken at the receiver**
  - in practice multi-path, Doppler effect, ...

Degrees of freedom :

- Multiple access techniques
- Receivers
- Resource allocation (scheduling, power)

Reminder on convex optimization

Optimization problem

$$\min_{\mathbf{x}} f(\mathbf{x})$$

s.t.

$$\forall \ell, \ g_\ell(\mathbf{x}) \leq 0$$

$$\forall \ell', \ h_{\ell'}(\mathbf{x}) = 0$$

with f and g_ℓ $(\forall \ell)$ convex, and $h_{\ell'}$ $(\forall \ell')$ affine

Resolution tools :

- Mathematically : KKT conditions (seldom feasible)
- Numerically : algorithms such as gradient-descent, newton, interior-point method, etc

Reminder on non-convex optimization

Successive Convex Approximation (SCA)

At each iteration i, solve

$$\mathbf{x}_{i+1}^* = \arg\min_{\mathbf{x}\in\mathcal{D}} \overline{f}_i(\mathbf{x}, \mathbf{x}_i^*)$$

with $\overline{f}_i$ an upper-bound approximating convex function of f

- $\overline{f}_i(\mathbf{x}_i^*, \mathbf{x}_i^*) = f(\mathbf{x}_i^*)$, $\nabla_{\mathbf{x}}\overline{f}_i(\mathbf{x}, \mathbf{x}_i^*)_{|\mathbf{x}=\mathbf{x}_i^*} = \nabla_{\mathbf{x}} f(\mathbf{x})_{|\mathbf{x}=\mathbf{x}_i^*}$,
- $\forall \mathbf{x} \in \mathcal{D}$, $f(\mathbf{x}) \leq \overline{f}_i(\mathbf{x}, \mathbf{x}_i^*)$.

Then SCA converges to a stationary point of f

Problem : how finding $\overline{f}_i$?

Special case : Difference of Convex (DoC) $\Rightarrow$ easy to exhibit $\overline{f}_i$

- $f(\mathbf{x}) = f_1(\mathbf{x}) - f_2(\mathbf{x})$
- $\overline{f}_i(\mathbf{x}, \mathbf{x}_i^*) = f_1(\mathbf{x}) - f_2(\mathbf{x}_i^*) - \nabla_{\mathbf{x}} f_2(\mathbf{x})_{|\mathbf{x}=\mathbf{x}_i^*}(\mathbf{x} - \mathbf{x}_i^*)$

Extension to other non-convex optimization

Nevertheless, there are some other special cases for non-convex optimization

Geometric Programming (GP) : f and g_ℓ are posynomial, and $x_n \geq 0,\ \forall n$.

$$f(\mathbf{x}) = \sum_m \beta_m \prod_{n=1}^{N} (x_n)^{\alpha_{m,n}}$$

with $\alpha_{m,n} \in \mathbb{R}$ and $\beta_m \in \mathbb{R}_+$

- $g_\ell(\mathbf{x}) \leq 1$
- Change of variables $y_n = \log(x_n)$
- Work on $\log(f)$ and $\log(g_\ell)$
- New problem is convex

Example

$$f(\mathbf{x}) = x_1 x_2$$

- Not jointly convex :

$$\text{Hessian :} \quad \nabla^2 f = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

It is not a positive matrix! $([1,-1].(\nabla^2 f).[1,-1]^{\mathrm{T}} = -2)$

- but $j : \mathbf{y} \mapsto \log(f(e^{\mathbf{y}}))$ is **convex** since

$$\begin{aligned} j(\mathbf{y}) &= \log(e^{y_1} e^{y_2}) \\ &= y_1 + y_2 \end{aligned}$$

Extension to other non-convex optimization (cont'd)

Complementary Geometric Programming (CGP)

$$\min_{\mathbf{P}} \frac{p_0(\mathbf{P})}{q_0(\mathbf{P})} \quad \text{s.t.} \quad \frac{p_i(\mathbf{P})}{q_i(\mathbf{P})} \leq 1 \quad \forall i = 1, \cdots, K$$

where p_i and q_i are posynomial functions $\forall i = 0, \cdots, K$.

- CGP are nonconvex and become GP when q_i are monomials.
- SCA by replacing posynomial denominator with approximate monomial

Signomial Programming (SP)

$$\min_{\mathbf{P}} \frac{a_0(\mathbf{P}) - b_0(\mathbf{P})}{c_0(\mathbf{P}) - d_0(\mathbf{P})} \quad \text{s.t.} \quad \frac{a_i(\mathbf{P}) - b_i(\mathbf{P})}{c_i(\mathbf{P}) - d_i(\mathbf{P})} \leq 1 \quad \forall i = 1, \cdots, K$$

where a_i, b_i, c_i and d_i are posynomial functions $\forall i = 0, \cdots, K$.

- SP problems are nonconvex and can be converted into CGP

$$\min_{\mathbf{P}, t} t \quad \text{s.t.} \quad \frac{a_i(\mathbf{x}) - b_i(\mathbf{x})}{c_i(\mathbf{x}) - d_i(\mathbf{x})} \leq t^{\delta_{0,i}} \rightarrow \frac{a_i(\mathbf{x}) + t^{\delta_{0,i}} d_i(\mathbf{x})}{b_i(\mathbf{x}) + t^{\delta_{0,i}} c_i(\mathbf{x})} \leq 1.$$

Some functions

- R_k rate for user k
- P_k power for user k

Functions to be optimized

- (weighted) Sum rate : $\sum_k w_k R_k$
- Proportional fairness : $\sum_k \log(R_k)$
- Maxmin fairness : $\max \min_k R_k$
- Sum Energy Efficiency : $\sum_k \frac{R_k}{P_k + P_{\text{circuitry}}}$
- Power minimization : $\sum_k P_k$

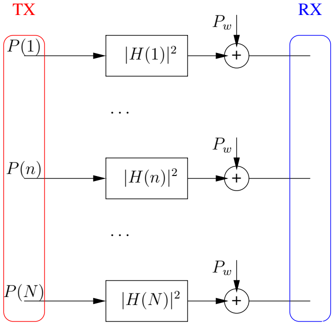
with typically

$$R_k = \log_2(1 + \text{SINR}_k).$$

and

$$\text{SINR}_k = \frac{P_k}{\sum_{m, m \neq k} \gamma_m P_m + P_w}$$

Problem 1 : Waterfilling [1948]



- Sum rate maximization
- Power constraint :

$$\sum_{n=1}^{N} P_n = P_{\max}$$

with maximum power $P_{\max}$

- $P_w = 1$

Problem : maximum capacity ?

$$[P_1^*, \cdots, P_N^*] = \arg \max_{P_1, \cdots, P_N} \sum_{n=1}^{N} \log_2(1 + |H_n|^2 P_n)$$

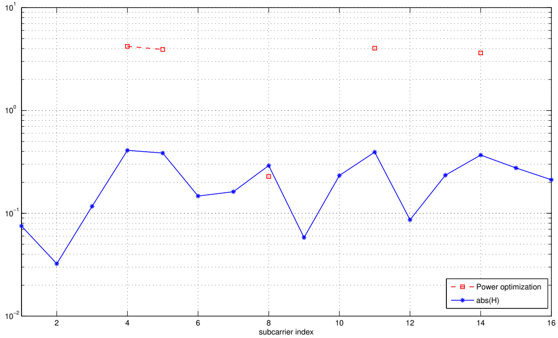
s.t. $P_n \geq 0$, and $\sum_{n=1}^{N} P_n \leq P_{\max}$.
Convex optimization problem ($\Rightarrow$ KKT conditions)

Problem 1 : Result

$$P_n^* = \left(\nu - \frac{1}{|H_n|^2}\right)^+$$

with

- ν chosen s.t. $\sum_{n=1}^{N} P_n^* = P_{\max}$.
- $(\bullet)^+ = \max(0, \bullet)$.



Problem 1 : *sketch of proof*

Lagrangian function

$$\mathcal{L}(\mathbf{P}, \lambda, \mu_n) = -\sum_n \log_2(1 + |H_n|^2 P_n) + \lambda(\sum_n P_n - P_{\max}) - \sum_n \mu_n P_n$$

KKT conditions

$$\begin{cases} -\frac{|H_n|^2}{1+|H_n|^2 P_n} + \lambda - \mu_n = 0 \Leftrightarrow P_n = \frac{1}{\lambda - \mu_n} - \frac{1}{|H_n|^2}, \forall n \\ \lambda(\sum_n P_n - P_{\max}) = 0 \\ \mu_n P_n = 0, \ \forall n \end{cases}$$

- If $\mu_n \neq 0$, then $P_n = 0$
- If $\mu_n = 0$, then $P_n = \frac{1}{\lambda} - \frac{1}{|H_n|^2}$ if this term is positive.

So

$$P_n = \left(\frac{1}{\lambda} - \frac{1}{|H_n|^2}\right)^+$$

Problem 2 : nonlinear interference [2021]

- OFDMA-based return link between terrestrial distributed antennas and satellite (gain G_k for user k)
- Then gateway between satellite and basestation
- Assumption : nonlinear amplifier on satellite board.

$$y_c(t) = \gamma_1 x_c(t) + \gamma_3 x_c(t) x_c(t) \overline{x_c(t)} + w_c(t)$$

For sample n of user/band k, we get

$$z_k(n) = z_k^{\mathrm{L}}(n) + z_k^{\mathrm{NL}}(n) + w_k(n)$$

Let

- $\mathcal{P}_{\mathrm{L}}(k) = \mathbb{E}[|z_k^{\mathrm{L}}|^2]$ be the auto-correlation of the linear part,
- $\mathcal{P}_{\mathrm{NL}}(k) = \mathbb{E}[|z_k^{\mathrm{NL}}|^2]$ be the auto-correlation of the nonlinear part,
- $\mathcal{P}_{\mathrm{LNL}}(k) = \mathbb{E}[z_k^{\mathrm{L}} \overline{z_k^{\mathrm{NL}}}]$ be the cross-correlation between the linear and nonlinear parts.

Problem 2 : capacity expressions - 1

Assuming optimal decoder and Gaussian codebooks

$$C(k) = \log_2 (1 + Q(k))$$

with

$$Q(k) = \frac{\mathcal{P}_{\mathrm{L}}^2(k) + 2\mathcal{P}_{\mathrm{L}}(k)\Re\{\mathcal{P}_{\mathrm{LNL}}(k)\} + |\mathcal{P}_{\mathrm{LNL}}(k)|^2}{\mathcal{P}_{\mathrm{L}}(k)\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{L}}(k)\mathcal{P}_{\mathrm{W}} - |\mathcal{P}_{\mathrm{LNL}}(k)|^2}$$

Assuming nonlinear part as noise and Gaussian codebooks

$$\underline{C}(k) = \log_2 \left(1 + \underline{Q}(k)\right)$$

with

$$\underline{Q}(k) = \frac{\mathcal{P}_{\mathrm{L}}(k)}{\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}}}$$

Remark : if $z_k^{\mathrm{NL}}(n) = 0$, then

$$Q(k) = \underline{Q}(k) = \frac{\mathcal{P}_{\mathrm{L}}(k)}{\mathcal{P}_{\mathrm{W}}}$$

Problem 2 : capacity expressions - 2

$$
\begin{aligned}
\mathcal{P}_{\mathrm{L}}(k) &= \gamma_1^2 G_k P_k, \\
\mathcal{P}_{\mathrm{NL}}(k) &= 4\gamma_3^2 \alpha^{(1)} G_k P_k \sum_{k',k''} G_{k'} G_{k''} P_{k'} P_{k''} \\
&+ 2\gamma_3^2 \alpha^{(2)} \sum_{k_1,k_2,k_3 | k=k_1+k_2-k_3} G_{k_1} G_{k_2} G_{k_3} P_{k_1} P_{k_2} P_{k_3} \\
&+ 4\gamma_3^2 \beta^{(1)} (\delta_{k,1}^c G_{k-1} P_{k-1} + \delta_{k,K}^c G_{k+1} P_{k+1}) \sum_{k',k''=1}^{K} G_{k'} G_{k''} P_{k'} P_{k''} \\
&+ 2\gamma_3^2 \beta^{(2)} \sum_{k_1,k_2,k_3 | k=k_1+k_2-k_3 \pm 1} G_{k_1} G_{k_2} G_{k_3} P_{k_1} P_{k_2} P_{k_3} \\
\mathcal{P}_{\mathrm{LNL}}(k) &= 2\gamma_1 \gamma_3 \lambda G_k P_k \sum_{k'} G_{k'} P_{k'}
\end{aligned}
$$

- All $\alpha^{(1)}$, $\alpha^{(2)}$, $\beta^{(1)}$, $\beta^{(2)}$, λ are positive.
- All terms are posynomials

Problem 2 : power minimization (with $\underline{C}$)

$$\min_{\mathbf{P}} \sum_{k=1}^{K} P_k \quad \text{s.t.} \quad \log_2\left(1 + \frac{\mathcal{P}_{\mathrm{L}}(k)}{\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}}}\right) \geq R_k \quad \forall k$$

which is equivalent to

$$\min_{\mathbf{P}} \sum_{k=1}^{K} P_k$$

s.t.

$$\mathcal{P}_{\mathrm{L}}(k)^{-1}\left(\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}}\right) \leq \frac{1}{2^{R_k} - 1} \quad \forall k = 1, \ldots, K$$

Last problem is GP

Problem 2 : maxmin data rate (with $\underline{C}$)

$$\max_{\mathbf{P}} \min_{k} \frac{\mathcal{P}_{\mathrm{L}}(k)}{\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}}}$$

which is equivalent to

$$\max_{\mathbf{P},t} t$$

s.t.

$$\frac{\mathcal{P}_{\mathrm{L}}(k)}{\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}}} \geq t \quad \forall k$$

$$\min_{\mathbf{P},t} t^{-1}$$

s.t.

$$t\mathcal{P}_{\mathrm{L}}(k)^{-1}\left(\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}}\right) \leq 1 \quad \forall k$$

Last problem is GP

Problem 2 : sum-rate (with $\underline{C}$)

$$\max_{\mathbf{P}} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\mathcal{P}_{\mathrm{L}}(k)}{\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}}}\right) = \max_{\mathbf{P}} \prod_{k=1}^{K} \frac{\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}} + \mathcal{P}_{\mathrm{L}}(k)}{\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}}}$$

$$\Rightarrow \min_{\mathbf{P}} \prod_{k=1}^{K} \frac{\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}}}{\mathcal{P}_{\mathrm{NL}}(k) + \mathcal{P}_{\mathrm{W}} + \mathcal{P}_{\mathrm{L}}(k)} = \min \frac{\text{posynomial}}{\text{posynomial}}$$

Easy to solve if denominator was a monomial !

Result (transforming a sum of monomials into a monomial)

Let $Q_m(\mathbf{P}) = \beta_m \prod_{n=1}^{N} P_n^{\alpha_{m,n}}$ be a monomial

$$Q(\mathbf{P}) := \sum_m Q_m(\mathbf{P}) \geq \tilde{Q}(\mathbf{P}) := \prod_m \left(\frac{Q_m(\mathbf{P})}{\delta_m}\right)^{\delta_m}.$$

If $\delta_m = Q_m(\mathbf{P}_0)/Q(\mathbf{P}_0)$, then $Q(\mathbf{P}_0) = \tilde{Q}(\mathbf{P}_0)$ and $\frac{\partial Q}{\partial \mathbf{P}}_{|\mathbf{P}=\mathbf{P_0}} = \frac{\partial \tilde{Q}}{\partial \mathbf{P}}_{|\mathbf{P}=\mathbf{P_0}}$

Problem 2 : *sketch of proof*

- Comparison between arithmetic mean and geometric mean :

$$\sum_m \delta_m x_m \geq \prod_m x_m^{\delta_m}$$

  with $\delta_m \geq 0$ and $\sum_m \delta_m = 1$.
- Consider $x_m = Q_m(\mathbf{P})/\delta_m$ and $\delta_m = Q_m(\mathbf{P}_0)/Q(\mathbf{P}_0)$

- $\tilde{Q}(\mathbf{P}_0) = \prod_m (Q(\mathbf{P}_0))^{Q_m(\mathbf{P}_0)/Q(\mathbf{P}_0)} = Q(\mathbf{P}_0)^{\sum_m Q_m(\mathbf{P}_0)/Q(\mathbf{P}_0)} = Q(\mathbf{P}_0)$
- $\frac{\partial \log Q}{\partial \mathbf{P}}_{|\mathbf{P}=\mathbf{P_0}} = \frac{\sum_m \partial Q_m/\partial \mathbf{P}_{|\mathbf{P}=\mathbf{P_0}}}{\sum_m Q_m(\mathbf{P}_0)}$
- $\log \tilde{Q}(\mathbf{P}) = \sum_m \delta_m \log_2(Q_m(\mathbf{P})/\delta_m)$
  $\frac{\partial \log \tilde{Q}}{\partial \mathbf{P}}_{|\mathbf{P}=\mathbf{P_0}} = \sum_m \delta_m \frac{\partial Q_m/\partial \mathbf{P}_{|\mathbf{P}=\mathbf{P_0}}}{Q_m(\mathbf{P}_0)} = \sum_m \frac{\partial Q_m/\partial \mathbf{P}_{|\mathbf{P}=\mathbf{P_0}}}{\sum_m Q_m(\mathbf{P}_0)}$

Problem 2 : sum-rate (with C)

Due to sign $-$ in $Q(k)$, we have

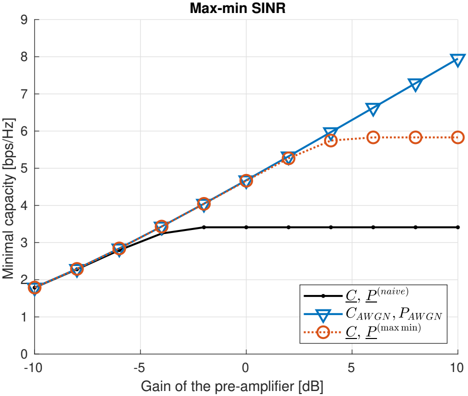
$$\min \frac{\text{signomial}}{\text{signomial}}$$

under ratio of signomials.

- Solution : Signomial Programming

Problem 2 : Numerical illustrations

- $K = 6$ users
- Rainy weather (G_k strongly different between users)
- $P_{\max} = 50$W (47dBm)
- $\gamma_3 = 0.05$



Problem 3 : Energy efficiency [2015]

- K users
- Downlink communications
- FDMA (with equal bandwidth for each user)

$$\max_{\{P_k\}} \frac{\sum_{k=1}^{K} \overbrace{\log_2(1 + G_k P_k)}^{R_k}}{\sum_{k=1}^{K} P_k + P_{\mathrm{circuitry}}}$$

s.t.

$$\begin{aligned} \sum_{k=1}^{K} P_k &\leq P_{\max} \\ P_k &\geq 0, \ \forall k \end{aligned}$$

Remarks :

- Warning : the power constraint is not necessary saturated.
- Ratio between concave function and convex function
- Linear constraints

$\Rightarrow$ Resorting to **Fractional programming (FP)**

Problem 3 : Review on Fractional Programming - 1

$$\max_{\mathbf{x}\in\mathcal{C}} \frac{f(\mathbf{x})}{g(\mathbf{x})}$$

with concave function f, and convex function g with a positive minimum.

- Let q^* be the maximum value (assumed non-negative).
- Let $\mathbf{x}^*$ be the argmax value.

Lemma 1

$\mathbf{x}^*$ is achieved iff

$$\max_{\mathbf{x}\in\mathcal{C}} (f(\mathbf{x}) - q^*.g(\mathbf{x})) = 0$$

Consequence :

- If q^* is known in advance, just solve

$$\max_{\mathbf{x}\in\mathcal{C}} (f(\mathbf{x}) - q^*.g(\mathbf{x}))$$

- As f concave and g convex, $f - q^*.g$ is concave

Convex optimization

Problem 3 : *Sketch of proof*

- Let $\mathbf{x}^*$ be the optimal solution of LHS of Lemma 1. It means

$$f(\mathbf{x}^*) - q^* g(\mathbf{x}^*) = 0 \Rightarrow \frac{f(\mathbf{x}^*)}{g(\mathbf{x}^*)} = q^*$$

  Moreover $\forall \mathbf{x} \neq \mathbf{x}^*$,

$$f(\mathbf{x}) - q^* g(\mathbf{x}) \leq 0 \Rightarrow \frac{f(\mathbf{x})}{g(\mathbf{x})} \leq q^*$$

  which proves that $\mathbf{x}^*$ is the optimal solution of FP.

- Let $\mathbf{x}^*$ be the optimal solution of FP. As $q^* = f(\mathbf{x}^*)/g(\mathbf{x}^*)$, we get

$$\frac{f(\mathbf{x})}{g(\mathbf{x})} < q^* \Rightarrow f(\mathbf{x}) - q^* g(\mathbf{x}) \leq 0$$

  for any $\mathbf{x} \neq \mathbf{x}^*$.

Problem 3 : Review on Fractional Programming - 2

Lemma 2

Let

$$F(q) := \max_{\mathbf{x}\in\mathcal{C}} \left(f(\mathbf{x}) - q.g(\mathbf{x})\right)$$

is a strictly decreasing function in $q \in \mathbb{R}_+$

Consequence :

- $q \mapsto F(q)$ is continuous
- $\lim_{q\to 0} F(q) = \max_{\mathbf{x}\in\mathcal{C}} f(\mathbf{x}) > 0$ (as q^* is non-negative)
- $\lim_{q\to\infty} F(q) = -\infty$ (as g is non-negative)
- q^* is the unique root of F
- Any root-finding algorithm works !
- but each computation of F requires a convex optimization

Problem 3 : *Sketch of proof*

- Assume $q_1 > q_2$
- x_1^* the argmax with q_1, and x_2^* the argmax with q_2

$$
\begin{aligned}
F(q_1) = f(\mathsf{x}_1^*) - q_1 g(\mathsf{x}_1^*) &\overset{(a)}{<} f(\mathsf{x}_1^*) - q_2 g(\mathsf{x}_1^*) \\
&\overset{(b)}{<} f(\mathsf{x}_2^*) - q_2 g(\mathsf{x}_2^*) = F(q_2)
\end{aligned}
$$

(a) $q_1 > q_2$ and g is a positive function. Strict inequality.

(b) x_2^* is the argmax for q_2

Problem 3 : Practical algorithm

Dinkelbach algorithm [1967]

Start with $q_0 = 0$, select an arbitrary small ε.
Iterate over n

1. Given q_n, find
$$\mathbf{x}_n^* = \arg\max_{\mathbf{x}\in\mathcal{C}} f(\mathbf{x}) - q_n.g(\mathbf{x})$$
2. Then
$$q_{n+1} = \frac{f(\mathbf{x}_n^*)}{g(\mathbf{x}_n^*)}$$
3. Stop when $F(q_{n+1}) < \varepsilon$

Result : this algorithm converges to $(q^*, \mathbf{x}^*)$ up to ε.

Problem 3 : *Sketch of proof*

Step 1 : sequence $\{q_n\}_n$ is strictly increasing.

- Assuming $F(q_n) = f(\mathbf{x}_n^*) - q_n g(\mathbf{x}_n^*) > 0$ (True for $F(q_0)$)
- $f(\mathbf{x}_n^*) - q_{n+1} g(\mathbf{x}_n^*) = 0$

$$q_{n+1} - q_n = \frac{F(q_n)}{g(\mathbf{x}_n^*)} > \frac{F(q_n)}{g_{\max}} > 0$$

with $g_{\max} = \max_{\mathbf{x} \in \mathcal{C}} g(\mathbf{x})$ (it exists if $\mathcal{C}$ compact)

Step 2 : convergence to q^*

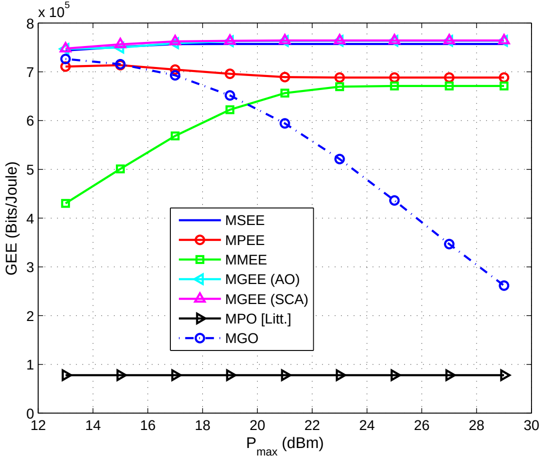
- Due to stopping criterion, bounded increasing sequence, and so $\lim_{n\to\infty} q_n = \overline{q}$
- Assuming that $\lim_{n\to\infty} F(q_n) = F(\overline{q}) > \varepsilon$, i.e, $\overline{q} < q^* - \delta$ with $F(q^* - \delta) = \varepsilon$.
- but as q_n converges, $(q_{n+1} - q_n)$ converges to 0 which implies

$$F(q_n) \to 0 \Rightarrow q_n \to q^*$$

which leads to a contradiction.

Problem 3 : Numerical illustrations

- Here R_k : throughput with HARQ and practical modulation and coding scheme [2018]
- Global Energy Efficiency (GEE) versus P_{max}



Section 3 : Energy consumption issue

Energies ?

- P_{tx} : *power transmission* energy
- $P_{\text{circuitery}}$: *circuitery* energy
  - P_{hardware} : *hardware* circuitery energy (power amplifier, ADC/DAC)
  - $P_{\text{processing}}$: *processing* energy (coding/decoding)
- $P_{\text{manufacturing}}$: *manufacturing* energy related to Life Cycle Analysis (mining, transportation, factory).

Shannon capacity analysis

- Let W be the used bandwidth (so send a sample every $1/W$ seconds)
- Then $P_{\rm tx} = E_s W$ (in Watts)

So

$$C = \underbrace{W}_{\substack{\text{Degree of freedom}\\ \text{(DoF/pre-log term)}}} \log_2\left(1 + \underbrace{\frac{P_{\rm tx}}{WN_0}}_{\text{Signal-to-Noise Ratio (SNR)}}\right) \text{(in bits/s)}$$

Analysis :

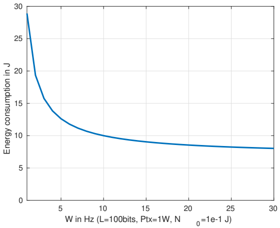
- E_s constant : linear increasing function wrt W.
- $P_{\rm tx}$ constant : increasing function wrt W but asymptotic limit. But asymptotically in $P_{\rm tx}$ (i.e., for $P_{\rm tx}$ large enough), we get

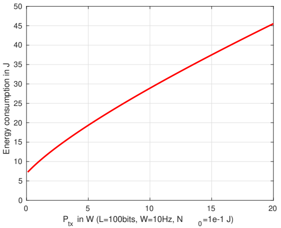
$$C \propto W \log_2(P_{\rm tx}).$$

- Extension to square $n_{\rm tx}$-MIMO : additional DoF. Now $n_{\rm tx} W$

Example : L-bits file transmission

$$E_{\mathrm{tx,file}} = \frac{LP_{\mathrm{tx}}}{W \log_2\left(1 + \frac{P_{\mathrm{tx}}}{WN_0}\right)}$$





$E_{\mathrm{tx,file}}$ vs W (left) and P_{tx} (right)

- DoF relevant to decrease transmission energy consumption
  - Done for 5G (compared to 4G) : $n_{\mathrm{tx}} \nearrow$ and $W \nearrow$
  - Data rate increase and <u>also</u> J/bit decrease
  - Warning : only transmission power consumption !

Numerical illustrations

	4G	5G sMIMO	5G mMIMO
Bandwidth (W)	20MHz	100MHz	100MHz
Antennes (n_{tx})	4	8	100
Energy per bit (E_{tx}/L)	2.08nJ	0.22 nJ	0.027 nJ

with $P_{\mathrm{tx}} = 10\mathrm{W}$ and $N_0 = -170\mathrm{dBm/Hz}$

Result

According to the transmit power, 5G may be 100 times more efficient than 4G for each transmit bit (as commonly heard or advocated by ETSI)

Energy efficiency criterion

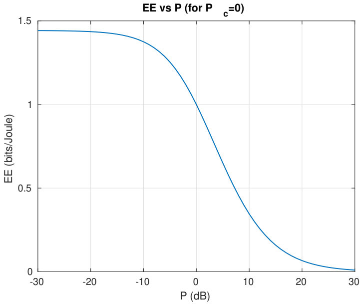
$$\begin{aligned}\mathcal{E}_k &= \frac{\#\text{ total amount of data correctly delivered by user } k}{\#\text{ total consumed energy by user } k} \\ &= \frac{TC}{TP_{\rm tx} + TP_{\rm circuitery}} \\ &= \frac{W\log_2(1 + P_{\rm tx}/WN_0)}{P_{\rm tx} + P_{\rm circuitery}}\end{aligned}$$

where

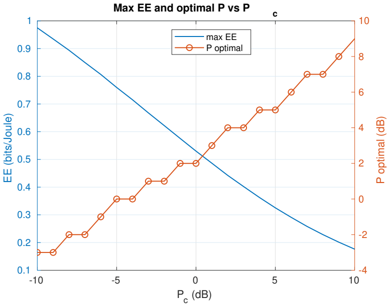
- T duration of transmission
- $P_{\rm circuitery}$ consumed power due to circuitery
  - here : amplifier, ADC/DAC, processing but no manufacturing
  - usually assumed constant
  - sometimes

$$P_{\rm circuitery} = \sum_{i \in components} \prod_j \left(\frac{\text{parameters}_j}{\text{parameters}_{j,ref}} \right)^{\alpha_{i,j}}$$

Energy Efficiency vs P_{tx} or $P_{\text{circuitery}}$



EE vs P_{tx} (for $P_{\text{circuitery}} = 0$)



Max EE and best P_{tx} vs $P_{\text{circuitery}}$

- EE makes sense iff $P_{\text{circuitery}} \neq 0$
- EE operating point strongly depends on $P_{\text{circuitery}}$
  - If $P_{\text{circuitery}}$ is large (not efficient), then EE leads to high P_{tx}
  - If $P_{\text{circuitery}}$ is low (very efficient), then EE leads to low P_{tx}

Massive MIMO

Main breakthrough for 5G (Enhanced Mobile BroadBand-eMMB)

Goal

- After how many years, is it useful to replace 4G with 5G (by taking into account all kinds of energies) ?
- Tradeoff between P_{tx} and $(P_{\mathrm{hardware}}, P_{\mathrm{processing}})$
- Impact of manufacturing power and rebound effect

Metrics :

- utilization (per year and per consumption unit) : $u_{4G} \rightarrow u_{5G}$
- manufacturing : f_{5G} with A depreciation years and f_{4G} for 10-year extra years

$$\text{Energy consumption per year} = \begin{cases} u_{4G} + \frac{f_{4G}}{10+A} \\ u_{5G} + \frac{f_{5G}}{A} \end{cases}$$

Massive MIMO : power model

- K users
- Required Data Rate per user : R
- N antennas on BS
- K_u active users with rate R_u at each timeslot

$$P_{\text{tot}} = P_{\text{tx}} + P_{\text{circuitery}} + P_{\text{manufacturing}}$$

where

- $$P_{\text{tx}} = \frac{WN_0}{\eta}(2^{R_u/W} - 1)\mathcal{D}K_u$$

  with η amplifier efficiency and $\mathcal{D}$ depends on propagation model

- $$P_{\text{circuitery}} = \underbrace{P_{\text{fix}} + P_{\text{tc}}}_{\text{hardware}} + \underbrace{P_{\text{ce}} + P_{\text{cd}} + P_{\text{lp}}}_{\text{processing}} + P_{\text{bh}}$$

- $P_{\text{manufacturing}} = NP_{\text{m,bs}} + KP_{\text{m,ue}}$ with A the depreciation years, and

$$P_{\text{m,d}} = E_{\text{m,d}}/(365 \times 24 \times 3600 \times A)$$

Massive MIMO : power model (cont'd)

- P_{fix} sleep power (cooling system for instance)
- P_{tc} hardware power (amplifier, oscillator, ...)

$$P_{\text{tc}} = NP_{\text{bs}} + P_{\text{lo}} + KP_{\text{ue}}$$

- P_{ce} channel estimation power
- P_{cd} coding and decoding power

$$P_{\text{cd}} = K_u R_u (P_{\text{cod}} + P_{\text{dec}})$$

  with P_{cod} and P_{dec} unitary coding and decoding powers
- P_{lp} signal processing power (linear precoding, for instance)

$$P_{\text{lp}} = P_{\text{mc}} + P_{\text{mo}}$$

  with P_{mc} precoding application and P_{mo} precoding computation
- P_{bh} core network power dedicated to this trafic (P_{bt} unitary power)

$$P_{\text{bh}} = K_u R_u P_{\text{bt}}.$$

Massive MIMO : some values

W	20 MHz	$E_{\mathrm{m,bs}}$	60 GJ
N_0	-140 dBm/Hz	$E_{\mathrm{m,ue}}$	0.175 GJ
η	0.39	P_{fix}	18 W
U	1800	P_{bs}	1 W
τ	1	P_{lo}	2 W
L_{ue}	5 Gflops/W	P_{ue}	0.1 W
L_{bs}	12.8 Gflops/W	P_{cod}	0.1 W/(Gb/s)
d_0	$10^{-3.53}$	P_{dec}	0.8 W/(Gb/s)
κ	3.76	P_{bt}	0.25 W/(Gb/s)

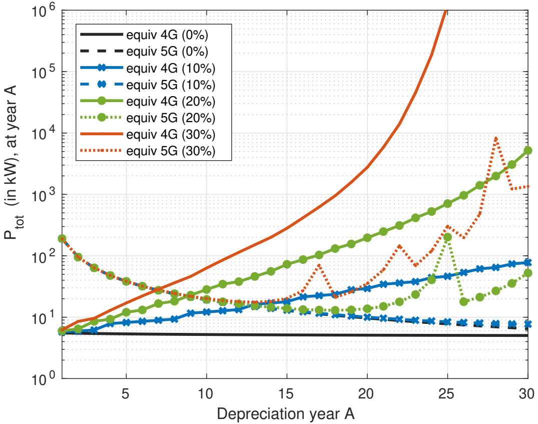
Table – Elementary parameters values

	P_{tx}	P_{fix}	P_{tc}	P_{ce}	P_{cd}	P_{lp}	P_{bh}
$K = 5, R = 10\mathrm{k}$	**13**	**18**	**22.5**	10^{-4}	10^{-5}	0.3	10^{-5}
$K = 50, R = 10\mathrm{M}$	**1660**	18	**1491**	0.02	0.5	**235**	0.15

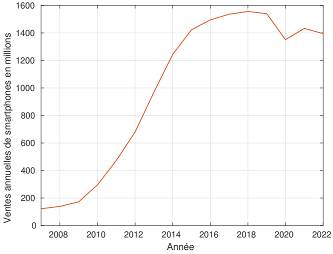
Table – Involved Power values (in Watts)

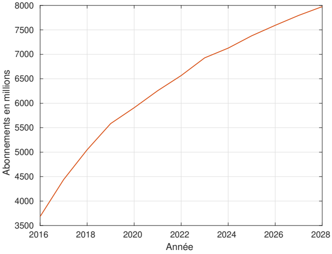
Numerical illustrations

- 4G-like : 4 antennas carried out for 10 years
- 5G-like : 100 antennas
- Percentage corresponds to traffic increase per year



On the real life : terminals sales

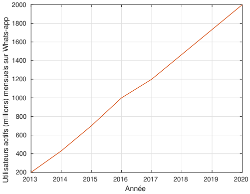


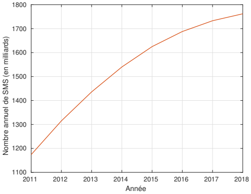


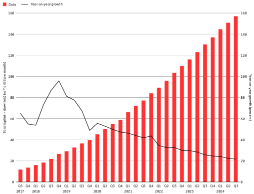
Stagnation of new smartphones annual sales
but subscribers still increase

source : Statista2022, WeAreSocial

Traffic figures



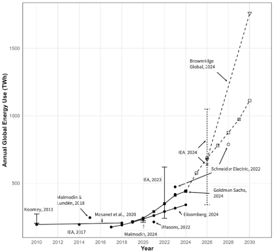


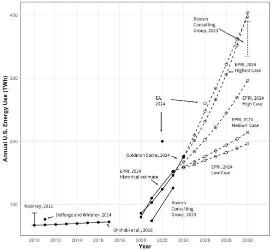


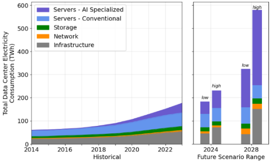
Tools/apps accumulation and no substitution

source : Statista2022, Ericsson Mobility Report

Data (computation) centers figures







- 460 TWh/yr in 2024
  - more than French nuclear plants
  - 2 % of worldwide electricity production
- Increase mainly due to AI

source : US data center energy usage report 2024 (Berkeley Lab)

Digital technologies impact and possible solutions

$\sim$ 4 % of worldwide GHG emissions (CO_2e)

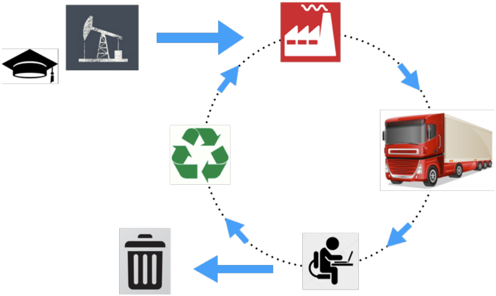
- Solution 1 : *GreenIT*

$$\textbf{Energy Efficiency} = \frac{\textbf{performances metric}}{\textbf{consumed energy}}$$

  - relative objective (less GHG per unit)
  - rebound effect (number of units increases)
  - technological answers not sufficient for fixing the issue
- Solution 2 : *IT for Green*
  - deported objective (less GHG elsewhere)
  - enablement effect
  - transfer of energy efficiency, thus same issue
- Solution 3 : *Sufficiency*
  - pre-defined consumed power
  - avoid rebound effect and ensure enablement effect

Efficiency related to optimization while sufficiency to way of life

How evaluating ? Life Cycle Assessment

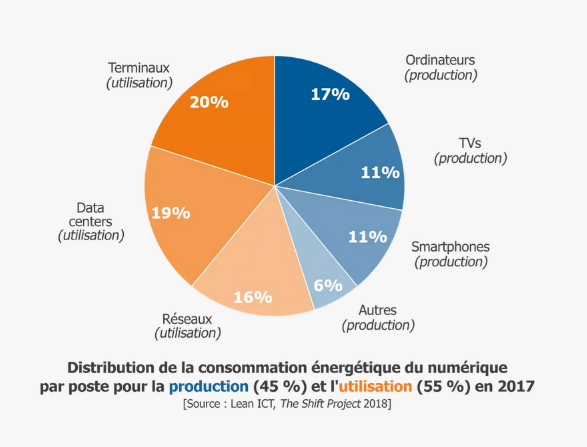


- Design
- Materials
- Manufacturing
- Transportation/logistic
- Use
- Waste/recycling

Fundamental limits

- Complex models
- Few data available
  - If data, often too aggregated
  - If data, often too optimistic
  - Fast modification of data

Use vs manufacturing



Use not necessary the main source of consumption ! depends on the device

Examples

- Laptop MacBook 16 inches storage 521 Go, frequency 2.6 GHz
  - Use phase duration : 4 years
  - Carbon footprint 394 kgCO_2e
    - ↪ manufacturing : 75 %
    - ↪ transportation : 5 %
    - ↪ use : 19 %
    - ↪ end of life : 1 %

  Use this laptop **4 times as long** for having use part equal to manufacturing part

- Email
  - around 4 mgCO_2e for 1 Mo email (without storage and LCA)
  - 20 gCO_2e with storage and LCA

Rebound effect or Jevons paradox

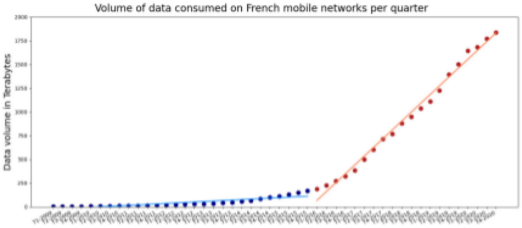
- A technogly is improved
- Its use is increased

and finally the initial consumption is exceeded (sometimes, it is intentional)

	Effect	Examples 5G
First order	manufacturing	new devices
	waste	device's end of life
Second order	direct	more transmit data
	induction	new applications
	optimization	more efficient data emissions
	substitution	less meetings and more visio
Third order	indirect	fingerprint during the savetime
	systemic	structural change in consumption

Examples coming from digitalization

- Improvement of machine learning
  - more apps using them
  - more managed data
- Improvement of electronic devices and batteries
  - more phone use
- Improvement of communication standards
  - more transmit data



source : P. Ciblat, J. Combaz, M. Coupechoux, K. Marquet, and A.-C. Orgerie, "Environmental impacts of 5G", 1024 newsletter, April 2024

Enablement effect

Thanks to digitalization, other areas drastically decrease their energy consumption

1 gCO_2e consumed by ICT avoids 10 gCO_2e elsewhere : fictitious figure

- self-driving car (public transportation alternative)
- logistic
- remote working
- smart farming

↪ Multiple and constrained solutions except if limited area

↪ Introduction of a new technology related to expected increase of productivity (French report on AI2024) : "transparence environnementale (*evaluation*), recherche dans modèles à faible impact (*energy efficiency*) et utilisation pour transitions environnementales (*enablement effect*)"

source : GSMA, "The State of mobile Internet connectivity", 2019 ; G. Roussilhe, "Explications sur l'empreinte environnementale du secteur numérique", 2021

Virtual conference

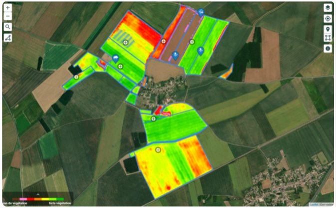
- real conference : 10.000 km per flight and capita
  - 150 capita at 0 km and 150 at 20.000 km return
  - Paris-NY per capita = 1 tCO_2e for 12.000 km so 1 km/passager = 83 gCO_2e. Then $150 \times 20000 \times 83$ (gCO_2e)= 249 tCO_2e
- remote conference : 80 videos per capita
  - 1 video duration is 20 mn, 20 videos/d, 4 day conference
  - 24.000 files of 45 Mo, so 1 To and thus 6 $kgCO_2e$ (with storage and LCA)

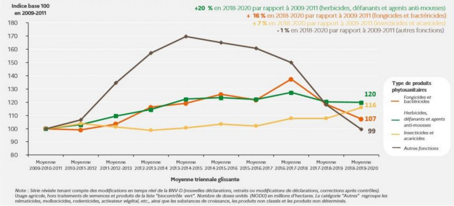
Remarks

- 40.000 less for remote confernece (but 4 if LCA but then LCA for plane is required)
- still a conference (no networking) ?
- continental real conference more relevant ?
- and air traffic still increases

Smart farming

- Data lead to Decision-helping tools (OAD)
  - geolocation, satellite, sensors, data networks, decision-making algorithms → big techno-structure
- Goals
  1. yield increase
  2. less fertilizer and/or water <u>but</u> Ecophyto plan failure

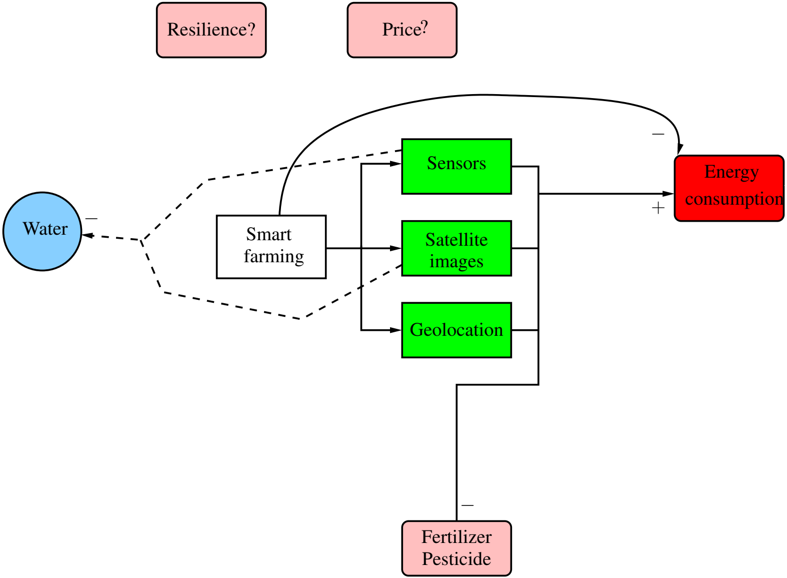




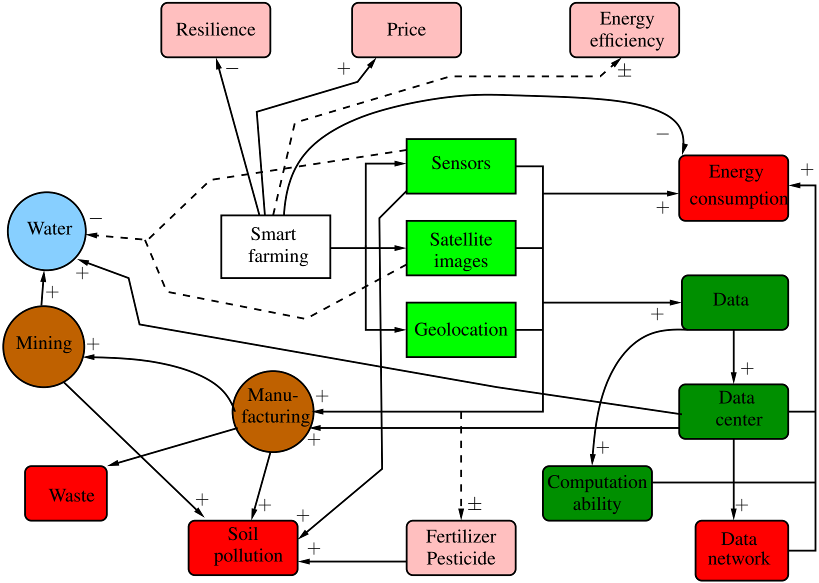
- Digital Twin for biological systems

source : https ://blog.spotifarm.fr ; http ://www.ofb.gouv.fr ; J. Oui, "Produire une faute -conforme-. Outils numériques et normes environnementales en agriculture", Sociologies Pratiques, 2024

Systemic point of view on smart farming



Systemic point of view on smart farming



Conclusion

- Technological solutions exist for use phase
- Measurements are possible but hard to be precise
- Economical point of view leads to rebound effect