

ONLINE SUMMER SCHOOL — FREE OF CHARGE
For Ukrainian students — open to the wider mathematical community

Introduction to Stochastic Calculus and Error Theory

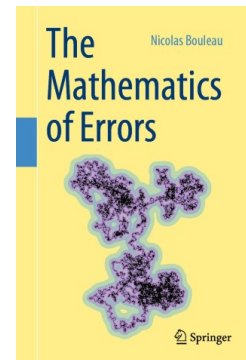
Error propagation with Dirichlet forms: from Gauss to N. Bouleau

Monday 17 – Friday 21 August 2026

Mornings **9:00–10:30** and **10:45–12:00** (Kyiv time): lectures

Afternoons **14:00–15:00** and **15:10–16:00** (Kyiv time): lecture and hands-on session

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What the school is about. How do errors and uncertainties propagate through a computation? Gauss answered this question for finitely many small Gaussian errors; the modern answer, due to Nicolas Bouleau following his work with Francis Hirsch on Dirichlet forms, views Dirichlet forms as tools for differential calculus compatible with probability theory: to each random quantity one attaches an operator (the “carré du champ” operator Γ) that propagates its variance. The theory works directly in infinite dimensions (Wiener, Monte-Carlo and Poisson spaces) and can characterize the errors of stochastic differential equations — something Gauss-type error calculus cannot do. The field is fully alive: a conjecture stated by Bouleau and Hirsch in 1986 (*energy image density*, EID) was settled only in October 2025.

The course follows an inter-semester course taught for three years at Télécom Paris. It is *self-contained*: stochastic calculus is one of the subjects taught, not a prerequisite — a basic undergraduate background in probability suffices (see the detailed programme and prerequisites overleaf).

Practical information.

Format	Online (Zoom); the link will be sent to registered participants shortly before the school begins.
Schedule	Monday 17 to Friday 21 August 2026, Kyiv time (one hour earlier in Paris). Mornings, two lectures: 9:00–10:30 and 10:45–12:00. Afternoons: lecture 14:00–15:00, then applications (exercises, simulations) 15:10–16:00.
Tuition	Free of charge.
Language	Lectures and written materials in English; questions may also be asked in French or Ukrainian (written).
Audience	Ukrainian undergraduate and Master’s students in mathematics, applied mathematics, physics, engineering or related fields; PhD students, researchers, and colleagues are most welcome.
Materials	Lecture notes and exercise sheets (with solutions, including Python simulations) are distributed to participants.

How to register. Send an e-mail to victor.rabet@telecom-paris.fr with subject “**Summer school registration**”, indicating your name, university, and year of study, **by Friday 14 August 2026**. You will receive the Zoom link and the course materials by e-mail.

The detailed day-by-day programme is given overleaf. Any update (schedule, materials) will be posted on this webpage.

Programme

Introduction to Stochastic Calculus and Error Theory

17–21 August 2026 — mornings 9:00–12:00, afternoons 14:00–16:00 (Kyiv time)

Monday 17 — Probabilistic foundations and Gauss-type error calculus. A refresher on measure theory: σ -algebras, simple functions, π - and λ -systems, the Radon–Nikodym theorem; conditional expectation and conditional variance; Gauss’s formulas for small errors and their connection with differential operators; classification of error calculi, examples in finite dimension.

Tuesday 18 — Construction of Brownian motion; from error germ to error structures.

Random processes; the construction of Brownian motion via Schauder’s tent functions; an intuitive introduction to error structures: orders of magnitude of errors, from the error germ γ to the carré du champ operator Γ ; simulation of random variables and of Brownian motion.

Wednesday 19 — Stochastic calculus: Itô integral. Quadratic variation of Brownian motion; filtrations, adaptation and martingales; construction of the Itô integral: progressively measurable processes, integration of step processes, the Itô isometry and its extension by density; Itô processes and Itô’s formula, with first examples of stochastic differential calculus.

Thursday 20 — Stochastic calculus: diffusions; semigroups and Dirichlet forms. The Markov property, diffusions and their simulation by the Euler scheme (the Langevin equation, the Black–Scholes model); strongly continuous semigroups on a Banach space; the Ornstein–Uhlenbeck semigroup; Dirichlet forms; the Dirichlet form associated with the Langevin equation as a worked example of error propagation through an SDE.

Friday 21 — Error structures in the general case; applications. General definition of an error structure $(\Omega, \mathcal{A}, \mathbb{P}, \mathbb{D}, \Gamma)$: dense domain $\mathbb{D}$, functional calculus of class $\mathcal{C}^1 \cap \text{Lip}$, closedness of the form (Dirichlet norm), Markovian case; images and products of error structures; applications and outlook: sensitivity of stochastic models, the EID conjecture (settled in 2025), current research on thermal fluctuations in statistical physics.

Prerequisites and level. The school is accessible to anyone with a basic knowledge of probability, as taught in the first two years of undergraduate studies (Bachelor’s level). Stochastic calculus itself is taught from scratch — no prior knowledge of it is assumed — and the measure-theoretic notions used (conditional expectation, π - and λ -systems, Radon–Nikodym) are recalled on the first day. For a few parts (the construction of Brownian motion, the Itô integral), some familiarity with Hilbert bases and L^2 convergence is helpful, though not indispensable. The last day, devoted to error structures in the general case, is the most advanced one; its emphasis can be adjusted to the audience.

Exercises and simulations. Each afternoon session ends with one hour of applications: exercise sheets (with solutions) and Python notebooks (simulation of Brownian motion, Euler scheme for diffusions), so that participants can experiment on their own.

References.

- [1] N. Bouleau, F. Hirsch, *Dirichlet Forms and Analysis on Wiener Space*, De Gruyter, 1991 (where the EID conjecture is stated).
- [2] N. Bouleau, *Error Calculus for Finance and Physics: The Language of Dirichlet Forms*, De Gruyter, 2003.
- [3] N. Bouleau, *The Mathematics of Errors*, Springer, 2022 (French original: *Théorie des erreurs*, Spartacus-IDH & Cassini, 2019).
- [4] S. Eriksson-Bique, M. Murugan, *On the energy image density conjecture of Bouleau and Hirsch*, arXiv:2510.13659 (preprint), 2025.
- [5] For probability and stochastic calculus, in Ukrainian: М. В. Карташов, *Імовірність, процеси, статистика*, ВПЦ «Київський університет», Kyiv, 2008 — official PDF: https://probability.knu.ua/userfiles/kmv/VPS_Pv.pdf.

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Course

webpage: <https://perso.telecom-paristech.fr/rioul/cours/stochastic-summer-school.html>