Workshop 7

April 3, 2017

Topics: polynomial division, partial fractions, trigonometric substitution, integration review

Practice exercises:

1. **Polynomial Division and Partial Fractions** We use these techniques to integrate ratios of polynomials

$$\frac{f(x)}{g(x)} = \frac{f_n x^n + f_{n-1}x^{n-1} + ... + f_0}{g_m x^m + g_{m-1}x^{m-1} + ... + g_0}$$

Firstly we check if we need to use long division by asking is $\deg(f) \geq \deg(g)$, if it is we must use long division, if not we can skip straight to partial fractions.

Integrate the following using these techniques.

(a) $\frac{x+7}{x^2-x-6}$.

(b) $\frac{x^3+4x^2+3}{x^2+2x+1}$.

(c) $\frac{x-8}{x^2-x-6}$

(d) $\frac{x}{x^2-4x-5}$

(e) $\frac{2x+1}{x^2+2x+1}$

(f) $\frac{1}{x^2-4}$

(g) $\frac{x^4-x^3-x-1}{x^3-x^2}$

(h) $\frac{2x^3-4x-8}{(x^2-x)(x^2+4)}$

2. **Trigonometric substitution** There is no trick here, it just takes lot's of practice! Integrate the following using trigonometric substitution.

(a) $\frac{1}{\sqrt{9-x^2}}$.

(b) $\frac{1}{x^2+25}$.

(c) $\frac{1}{x^2-4}$

(d) $\frac{\sqrt{25x^2-4}}{x}$

(e) $\frac{1}{x^4\sqrt{9-x^2}}$

(f) $\frac{x}{\sqrt{2x^2-4x-7}}$

(g) $e^{4x}\sqrt{1+e^{2x}}$

(h) $\int_0^{1/6} \frac{x^5}{(36x^2+1)^{3/2}}$

3. **General Integration Techniques** Just integrate, using any appropriate technique.

(a) $\int_0^{\pi} \sin^2(\theta)\, d\theta$.

(b) $\int_1^3 \frac{\ln(2x)}{x^2}\, dx$.

(c) $\int e^{-2t}\cos t\, dt$

(d) $\int \frac{3x^3-17x^2+36x-35}{x^2-4x+4}\, dx$

(e) $\int \tan^4(s)\sec^4(s)\, ds$

(f) $\int \arcsin(x-1)\, dx$

(g) $\int \sqrt{9-4x^2}\, dx$

(h) $\int \frac{2x-1}{x^2-2x+10}\, dx$