Workshop 9

March 22, 2014

Topics: Arc lengths, ODEs, Sequences

Practice exercises:

1. **Arc Length**
   (a) Calculate the length of $y = 2(x-1)^{\frac{3}{2}}$ for $1 \leq x \leq 5$
   (b) Calculate the length of $y = \frac{2}{3}(x^2+1)^{\frac{3}{2}}$ for $1 \leq x \leq 4$
   (c) Calculate the length of $y = \ln(\cos(x))$ for $0 \leq x \leq \frac{\pi}{4}$
   (d) Calculate the length of $y = \frac{x^3}{6} + \frac{1}{2x}$ for $1 \leq x \leq 3$

   Solution:
   (a) $\int_1^5 (9x-8)^{\frac{1}{2}} = \frac{2}{27}(37\sqrt{37}-1)$
   (b) $\int_1^4 2x^2 + 1dx = 45$
   (c) $\int_0^{\frac{\pi}{4}} \sec x dx = \ln(\sqrt{2}+1)$
   (d) $\frac{1}{2}\int_1^3 x^2 + x^{-2}dx = \frac{14}{3}$

2. **Seperable ODEs**
   (a) $\frac{dy}{dx} = x^2y^2 + x^2$
   (b) $\frac{dy}{dx} = 6y^2x$ with $y(1) = \frac{1}{25}$
   (c) $\frac{dy}{dx} = \frac{3x^2+4x-4}{2y-4}$ with $y(1) = 3$
   (d) $\frac{dy}{dx} = e^{-y}(2x-4)$ with $y(5) = 0$

   Solution:
   (a) $y = \tan(\frac{1}{3}x^3 + C)$
   (b) $y = \frac{1}{28-3x^2}$
   (c) $y = 2 + \sqrt{x^3+2x^2-4x+2}$
   (d) $y = \ln(x^2-4x-4)$

3. **Sequences** Determine if the following sequences diverge or converge as $n \to \infty$. If they converge, give the limit (with proof!). If they diverge, prove that they diverge!

   By proof I mean make sure you know which theorems you are using, or use an epsilon or two!

   (a) $a_n = \frac{3n^2-1}{10n+5n^2}$

   (b) $(-1)^n$

   (c) $\frac{(-1)^n}{n}$

   (d) $\frac{n^n}{n!}$

   (e) $\frac{2^n}{n!}$

   (f) $\frac{n+47}{\sqrt{n^2+3n}}$

   (g) $\sqrt{n+47} - \sqrt{n}$

Solution:

(a) Converges to 3/5

(b) Diverges by choosing $\epsilon = 1$

(c) Converges to 0

(d) Tends to ∞

(e) Converges to 0

(f) Tends to 1 (I believe)

(g) Converges to 0