Antiderivatives and Integration

Extra Workshop A

25-2-2014

Topics: Antiderivatives, definite and indefinite integrals, the fundamental theorem of calculus.

Practice exercises:

1. Find the most general antiderivative of each of the following functions. (Check the chart on p.284 of your book if you are stuck.)
   (a) $f(x) = x^2 + 8x - 3$.
   (b) $f(x) = \sqrt{x} + \sqrt[5]{x} + 1$.
   (c) $f(x) = \frac{1}{x^3} - \frac{1}{\sqrt[3]{x}}$.
   (d) $f(x) = \sin x - \cos x$.
   (e) $f(x) = \frac{2}{x} - \frac{2}{x^2}$.
   (f) $f(x) = \sec^2 x$.
   (g) $f(x) = \csc x \cot x$.
   (h) $f(x) = 2e^x + (2e)^x$.
   (i) $f(x) = \frac{1}{x^2+1} + \frac{1}{x^2}$.

2. Compute the following definite integrals. (You are using Part 2 of the Fundamental Theorem of Calculus, p.295.)
   (a) $\int_0^2 (x^2+5)(x-3)\,\mathrm{d}x$.
   (b) $\int_{-1}^1 (1-r^2)^2\,\mathrm{d}r$.
   (c) $\int_{-e^2}^{-1} \frac{1}{x}\,\mathrm{d}x$.
   (d) $\int_0^{\pi/4} \frac{1+\cos^2 t}{\cos^2 t}\,\mathrm{d}t$.
   (e) $\int_1^8 \frac{1+r}{\sqrt[3]{r}}\,\mathrm{d}r$.
   (f) $\int_0^1 \frac{1}{2x^2+2}\,\mathrm{d}x$.
   (g) $\int_0^1 e^t + e^{t+2} + e^{2t}\,\mathrm{d}t$.
   (h) $\int_0^{\sqrt{3}} \frac{s^2-1}{s^4-1}\,\mathrm{d}s$.
   (i) $\int_{-3}^3 |x-1|\,\mathrm{d}x$.

3. Use Part 1 of the Fundamental Theorem of Calculus (p.295) to compute the **derivatives** of the following functions.
   (a) $g(x) = \int_0^x \cos t + 3\,\mathrm{d}t$.
   (b) $g(x) = \int_2^x \ln t + \frac{1}{t^3-t}\,\mathrm{d}t$.
   (c) $g(x) = \int_x^{10} \frac{\sin s}{\sin s+2}\,\mathrm{d}s$.
   (d) $h(x) = \int_0^{\sqrt{x}} t^2 - \tan t\,\mathrm{d}t$. ($0 \le x < \sqrt{\pi/2}$).
   (e) $h(x) = \int_1^{\sin x} 2e^s - 2^s + 2\,\mathrm{d}s$.
   (f) $h(x) = \int_x^{5x} (t^5-1)^5\,\mathrm{d}t$.

Solutions:

1. Find the most general antiderivative of each of the following functions. (Check the chart on p.284 of your book if you are stuck.)
   (a) $F(x) = \frac{1}{3}x^3 + 4x^2 - 3x + C.$
   (b) $F(x) = \frac{2}{3}x^{3/2} + \frac{5}{6}x^{6/5} + x + C.$
   (c) $F(x) = -\frac{1}{2x^2} - \frac{3}{2}x^{2/3} + C.$
   (d) $F(x) = -\cos x - \sin x + C.$
   (e) $F(x) = 2\ln|x| + \frac{2}{x} + C.$
   (f) $F(x) = \tan x + C.$
   (g) $F(x) = -\csc(x) + C.$
   (h) $F(x) = 2e^x + \frac{(2e)^x}{\ln 2+1} + C..$
   (i) $F(x) = \tan^{-1} x - \frac{1}{x} + C.$

2. Compute the following definite integrals. (You are using Part 2 of the Fundamental Theorem of Calculus, p.295.)
   (a) $\int_0^2 x^3 - 3x^2 + 5x - 15\,\mathrm{d}x = -24.$
   (b) $\int_{-1}^1 1 - 2r^2 + r^4\,\mathrm{d}r = \frac{16}{15}.$
   (c) $\int_{-e^2}^{-1} \frac{1}{x}\,\mathrm{d}x = -2.$
   (d) $\int_0^{\pi/4} \sec^2 t + 1\,\mathrm{d}t = 1 + \frac{\pi}{4}.$
   (e) $\int_1^8 r^{-1/3} + r^{2/3}\,\mathrm{d}r = \frac{231}{10}.$
   (f) $\int_0^1 \frac{1}{2}\left(\frac{1}{x^2+1}\right)\,\mathrm{d}x = \frac{\pi}{8}.$
   (g) $\int_0^1 e^t + (e^2)e^t + (e^2)^t\,\mathrm{d}t = e^3 - \frac{1}{2}e^2 + e - \frac{3}{2}.$
   (h) $\int_0^{\sqrt{3}} \frac{1}{s^2+1}\,\mathrm{d}s = \frac{\pi}{3}.$
   (i) $\int_{-3}^1 -(x-1)\,\mathrm{d}x + \int_1^3 (x-1)\,\mathrm{d}x = 10.$

3. Use Part 1 of the Fundamental Theorem of Calculus (p.295) to compute the **derivatives** of the following functions.
   (a) $g'(x) = \cos x + 3.$
   (b) $g'(x) = \ln x + \frac{1}{x^3 - x}.$
   (c) $g'(x) = -\frac{\sin x}{\sin x + 2}.$
   (d) $h'(x) = \frac{1}{2\sqrt{x}}(x - \tan\sqrt{x})$
   (e) $h'(x) = \cos x(2e^{\sin x} - 2^{\sin x} + 2).$
   (f) $h'(x) = 5(3125x^5 - 1)^5 - (x^5 - 1)^5.$