Antiderivatives and Integration

Extra Workshop A

25-2-2014

Topics: Antiderivatives, definite and indefinite integrals, the fundamental theorem of calculus.

Practice exercises:

1. Find the most general antiderivative of each of the following functions. (Check the chart on p.284 of your book if you are stuck.)
   (a) $f(x) = x^2 + 8x - 3$.
   (b) $f(x) = \sqrt{x} + \sqrt[5]{x} + 1$.
   (c) $f(x) = \frac{1}{x^3} - \frac{1}{\sqrt[3]{x}}$.
   (d) $f(x) = \sin x - \cos x$.
   (e) $f(x) = \frac{2}{x} - \frac{2}{x^2}$.
   (f) $f(x) = \sec^2 x$.
   (g) $f(x) = \csc x \cot x$.
   (h) $f(x) = 2e^x + (2e)^x$.
   (i) $f(x) = \frac{1}{x^2+1} + \frac{1}{x^2}$.

2. Compute the following definite integrals. (You are using Part 2 of the Fundamental Theorem of Calculus, p.295.)
   (a) $\int_0^2 (x^2 + 5)(x - 3)\,\mathrm{d}x$.
   (b) $\int_{-1}^1 (1 - r^2)^2\,\mathrm{d}r$.
   (c) $\int_{-e^2}^{-1} \frac{1}{x}\,\mathrm{d}x$.
   (d) $\int_0^{\pi/4} \frac{1+\cos^2 t}{\cos^2 t}\,\mathrm{d}t$.
   (e) $\int_1^8 \frac{1+r}{\sqrt[3]{r}}\,\mathrm{d}r$.
   (f) $\int_0^1 \frac{1}{2x^2+2}\,\mathrm{d}x$.
   (g) $\int_0^1 e^t + e^{t+2} + e^{2t}\,\mathrm{d}t$.
   (h) $\int_0^{\sqrt{3}} \frac{s^2-1}{s^4-1}\,\mathrm{d}s$.
   (i) $\int_{-3}^3 |x - 1|\,\mathrm{d}x$.

3. Use Part 1 of the Fundamental Theorem of Calculus (p.295) to compute the **derivatives** of the following functions.
   (a) $g(x) = \int_0^x \cos t + 3\,\mathrm{d}t$.
   (b) $g(x) = \int_2^x \ln t + \frac{1}{t^3-t}\,\mathrm{d}t$.
   (c) $g(x) = \int_x^{10} \frac{\sin s}{\sin s+2}\,\mathrm{d}s$.
   (d) $h(x) = \int_0^{\sqrt{x}} t^2 - \tan t\,\mathrm{d}t$. $(0 \leq x < \sqrt{\pi/2})$.
   (e) $h(x) = \int_1^{\sin x} 2e^s - 2^s + 2\,\mathrm{d}s$.
   (f) $h(x) = \int_x^{5x} (t^5 - 1)^5\,\mathrm{d}t$.