High resolution spectral analysis and nonnegative decompositions applied to music signal processing

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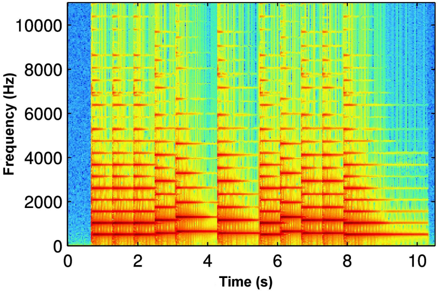
Music representations

Musical score

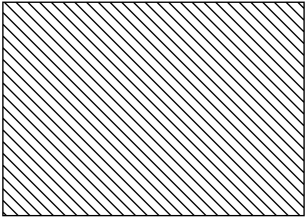
Music representations



Spectrogram of "Au clair de la lune"

Musical score

Low-rank approximations



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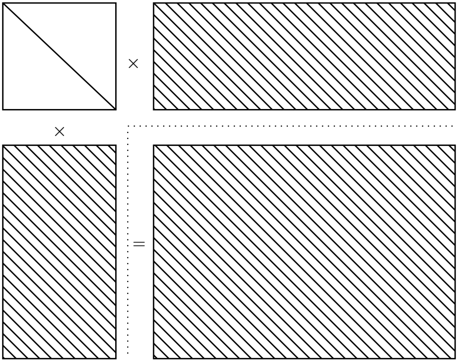
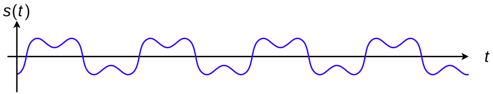


High Resolution (HR) spectral analysis



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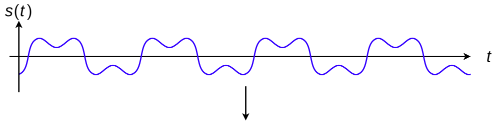

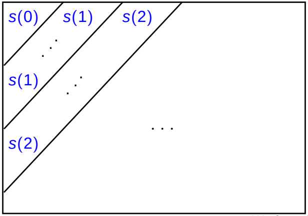

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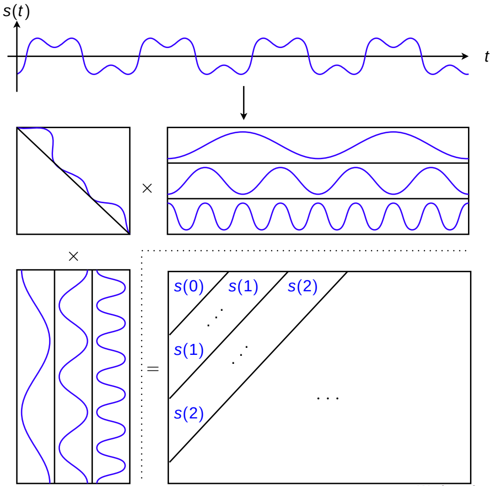
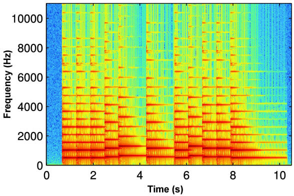


Nonnegative Matrix Factorization (NMF)

Musical score



Spectrogram

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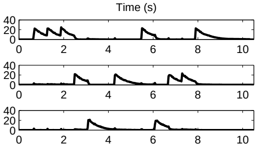
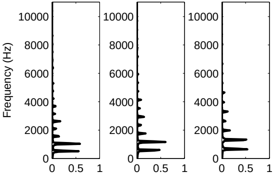
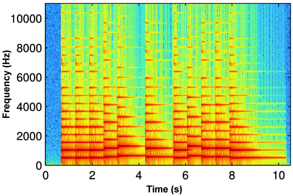


Temporal activations



Spectral atoms



Spectrogram

Applications

- Applications of High Resolution methods
  - Spectral analysis (modal analysis, spectroscopy)
  - Array processing (beamforming, direction of arrival (DOA) estimation)
  - Digital communications (channel identification)
- Applications of NMF
  - Image analysis (face recognition)
  - Text mining, spectroscopy, finance, etc.
- Applications to audio signal processing
  - Source separation, audio coding
  - Pitch and tempo estimation, automatic transcription

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Part I

High Resolution spectral analysis

Exponential Sinusoidal Model (ESM)

- Real-valued model: $s(t) = \sum_{k=1}^{r} a_k\, e^{-\delta_k t} \cos(2\pi\nu_k t + \phi_k)$
  - $a_k \in \mathbb{R}_+^*$ and $\phi_k \in]-\pi, \pi]$ are the **amplitude** and **phase**
  - $\delta_k \in \mathbb{R}$ and $\nu_k \in]-\frac{1}{2}, \frac{1}{2}]$ are the **damping factor** and **frequency**
- Complex-valued model: $s(t) = \sum_{k=1}^{r} \alpha_k\, {z_k}^t$
  - $\alpha_k = a_k\, e^{i\phi_k} \in \mathbb{C}^*$ is a **complex amplitude**
  - $z_k = e^{-\delta_k + i2\pi\nu_k} \in \mathbb{C}^*$ is a **complex pole**
- Noisy model: $x(t) = s(t) + b(t)$ ($b(t)$ is a white Gaussian noise)
- Model estimation
  - Data vector: $\mathbf{s}(t) = [s(t), \ldots, s(t+n-1)]^T$ with $n > r$
  - Fourier analysis: spectral resolution of the order of $\frac{1}{n}$
  - Subspace analysis: **high spectral resolution**

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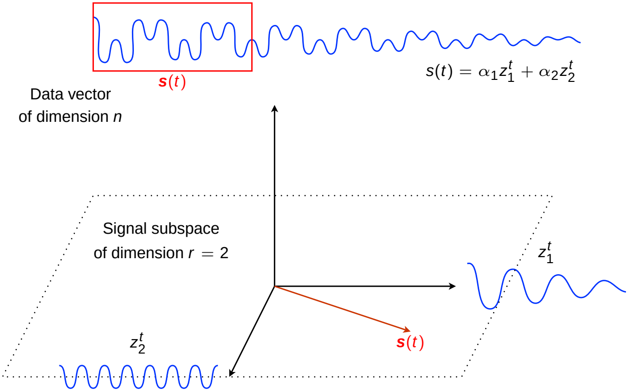

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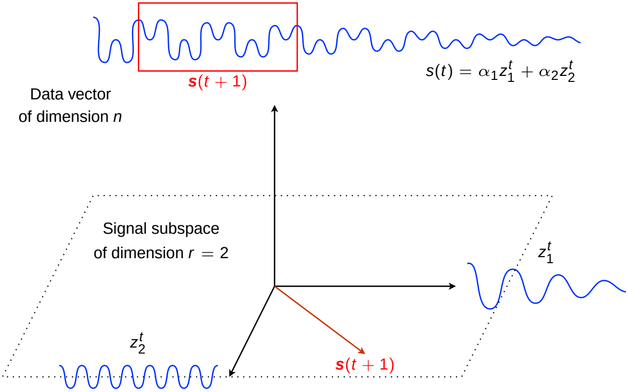

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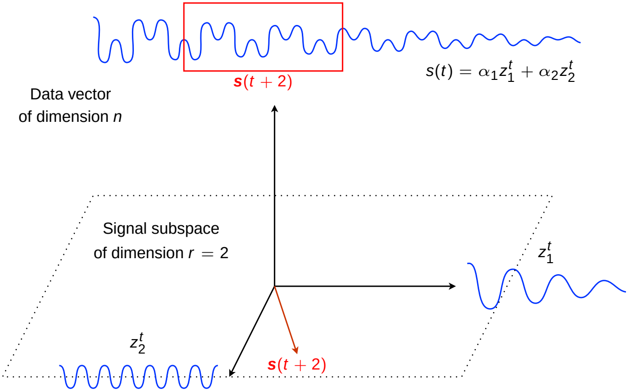

Model estimation

- Choose a window $(\gamma_\tau)_{\tau\in\mathbb{N}}$ (exponential, rectangular, hybrid)
- Compute a "correlation" matrix

$$\boldsymbol{C}_{xx}(t) = \sum_{\tau=0}^{t} \gamma_\tau \, \boldsymbol{x}(t-\tau)\,\boldsymbol{x}(t-\tau)^H$$

- Estimate the model parameters

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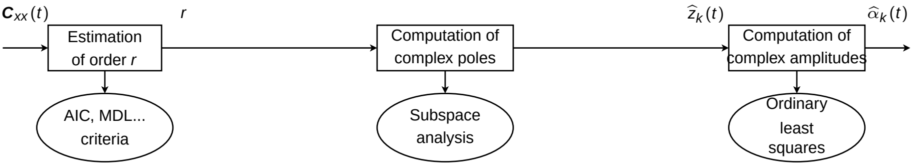

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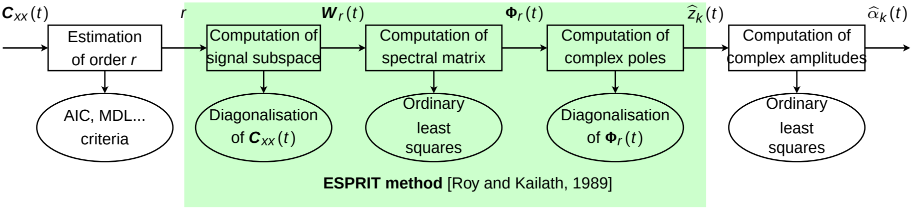

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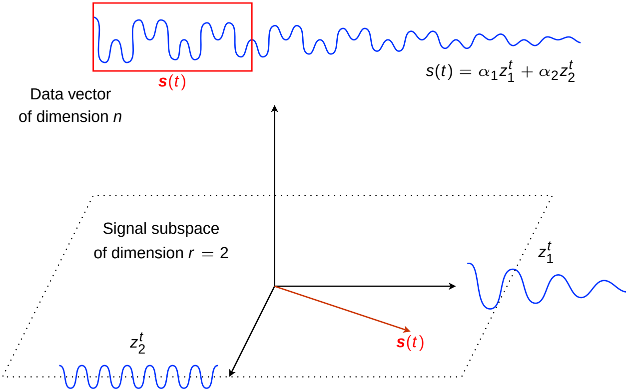

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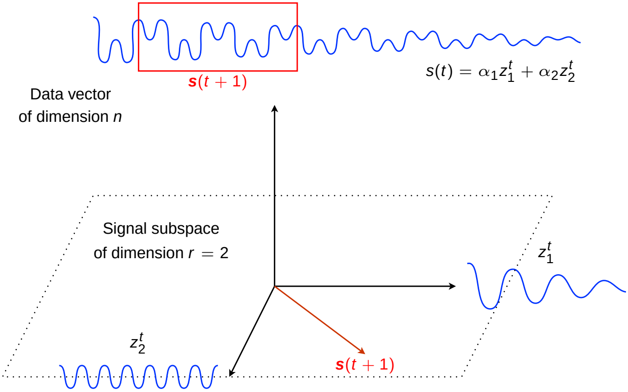

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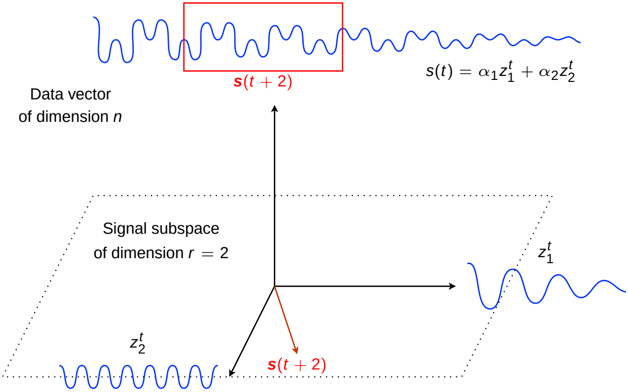

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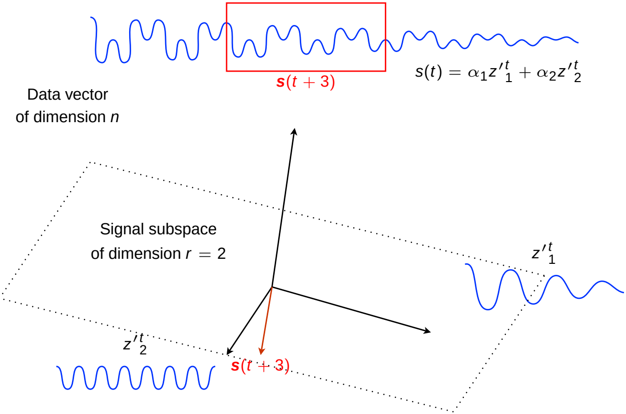

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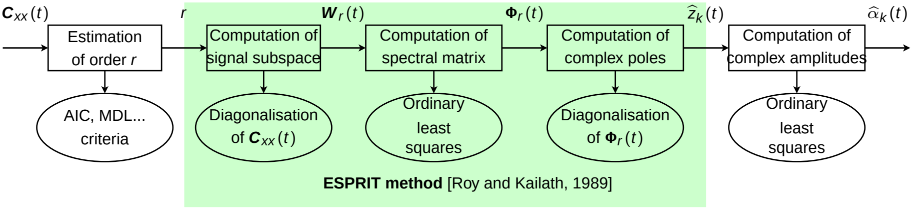
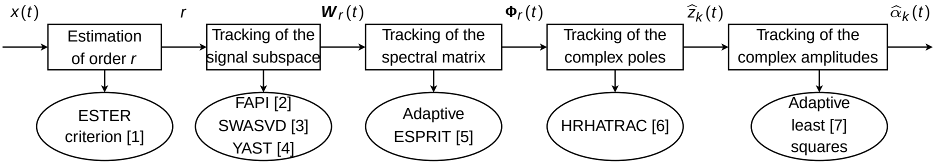


Time-frequency analysis



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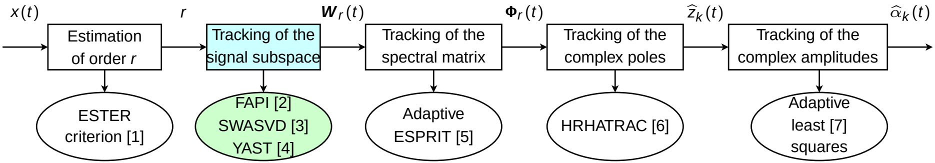

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Power iteration method

- Power iteration method (recursive computation of $\boldsymbol{W}_r(t)$)
  1) $\boldsymbol{C}_{xy}(t) = \boldsymbol{C}_{xx}\boldsymbol{W}_r(t-1)$ (compression of $\boldsymbol{C}_{xx}$)
  2) $\boldsymbol{W}_r(t)\boldsymbol{R}(t) = \boldsymbol{C}_{xy}(t)$ (orthonormalisation of $\boldsymbol{C}_{xy}(t)$)
     - $\mathrm{Span}(\boldsymbol{W}_r(t))$ exponentially converges to the signal subspace
     - If 2) is an orthogonal-triangular (QR) decomposition, $\boldsymbol{W}_r(t)$ converges to the r principal eigenvectors of $\boldsymbol{C}_{xx}$
- Signal subspace tracking if $\boldsymbol{C}_{xx}(t)$ is time-varying
- Fast algorithm [Strobach, 1996] (complexity of nr^2 instead of n^2r)

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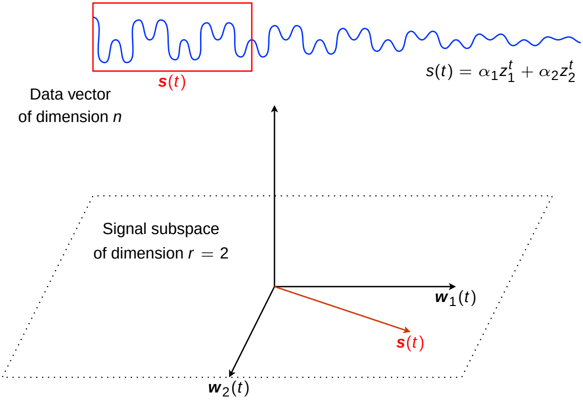

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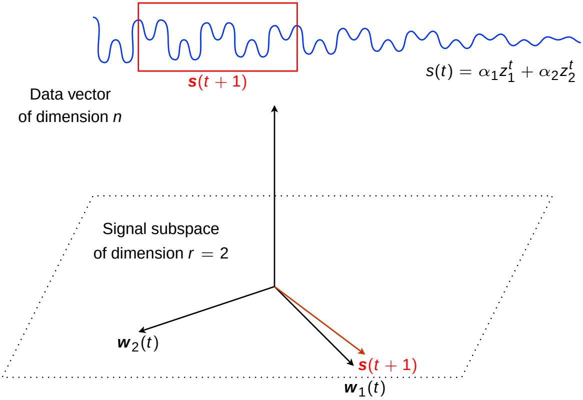

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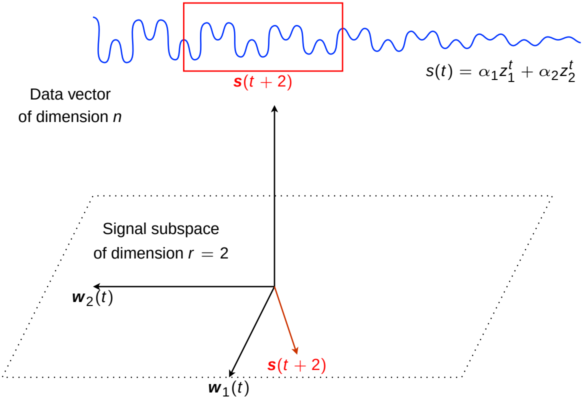

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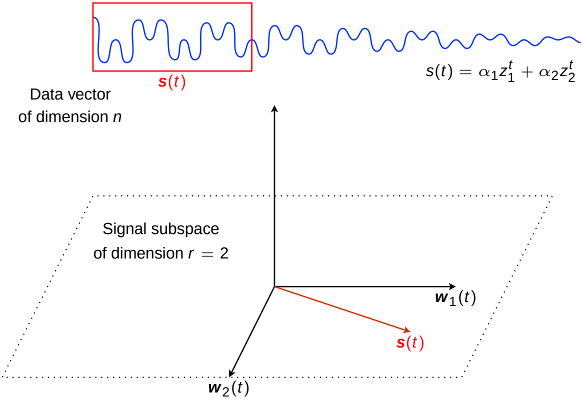

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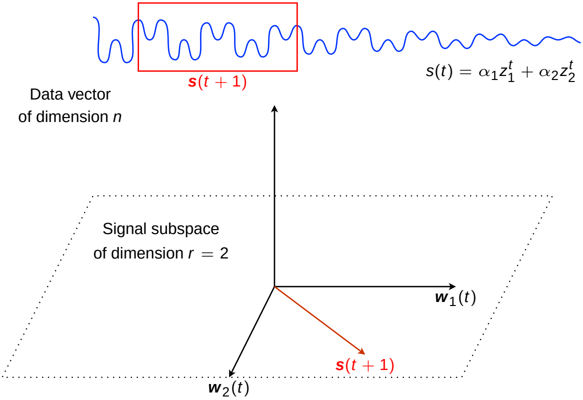

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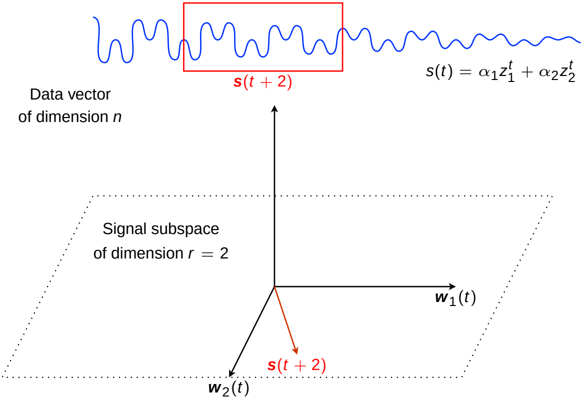

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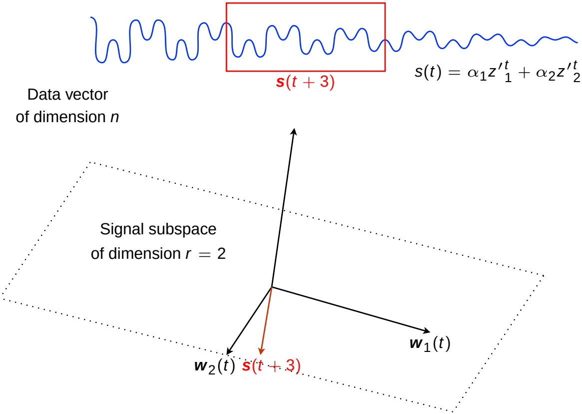

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- FAPI algorithm [1] (complexity of $3nr$ instead of nr^2)
  - reaches the complexity lower bound ($3nr$)
  - converges faster than PAST and its variants
  - guarantees the orthonormality of $\mathbf{W}_r(t)$ and the numerical stability

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  1) $\mathbf{C}_{xy}(t) = \mathbf{C}_{xx}(t)\mathbf{W}_r(t-1)$ (compression of $\mathbf{C}_{xx}$)
  2) $\mathbf{W}_r(t) = \mathbf{C}_{xy}(t)\left(\mathbf{C}_{xy}(t)^H\mathbf{C}_{xy}(t)\right)^{-\frac{1}{2}}$ (orthonormalisation of $\mathbf{C}_{xy}(t)$)
- FAPI algorithm [1] (complexity of $3nr$ instead of nr^2)
  - reaches the complexity lower bound ($3nr$)
  - converges faster than PAST and its variants
  - guarantees the orthonormality of $\mathbf{W}_r(t)$ and the numerical stability

[1] Roland Badeau, Bertrand David, and Gaël Richard. "Fast Approximated Power Iteration Subspace Tracking". *IEEE Transactions on Signal Processing*, 53(8): 2931-2941, August 2005.

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Applications of High Resolution analysis

- Analysis / Synthesis
  - High resolution time-frequency representation
  - Analysis of the sympathetic string modes in a concert harp
  - Audio coding
- Automatic transcription
  - Pitch estimation of piano notes
  - Musical tempo estimation
- Other applications
  - Channel estimation in digital communications
  - Adaptive multilinear SVD for structured tensors

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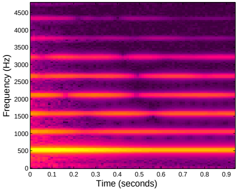
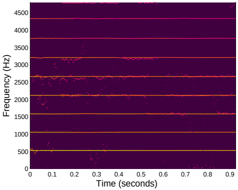
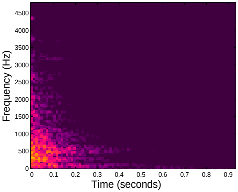
Decomposition of a piano sound



Spectrogram



HR-ogram



Residual

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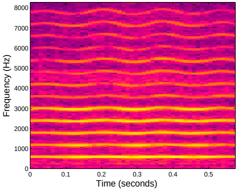
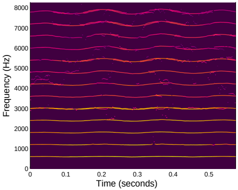
[3] Roland Badeau, Rémy Boyer, and Bertrand David. "EDS parametric modeling and tracking of audio signals". In Proc. of the 5th International Conference on Digital Audio Effects (DAFx), pages 139-144, Hamburg, Germany, September 2002.

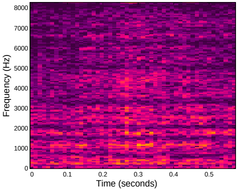
Decomposition of a violin sound



Spectrogram



HR-ogram



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Sinusoids and noise separation

- Principle: projection onto the signal or the noise subspace [1,2]

Instrument	Original	Sinusoids	Noise
Piano			
Guitar			
Violin			
Flute			
Saxophone			
Bell			

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Drum separation and beat estimation

- Drum source separation [1]
  - Original (Aerosmith):
  - Separated drums:
  - Remix - more drums:
  - Remix - less drums:
- Beat tracking [2]
  - Pink Floyd:
  - Brad Mehldau:

[1] Olivier Gillet and Gaël Richard. Transcription and separation of drum signals from polyphonic music. *IEEE Transactions on Audio, Speech, and Language Processing*, 16(3): 529-540, March 2008.
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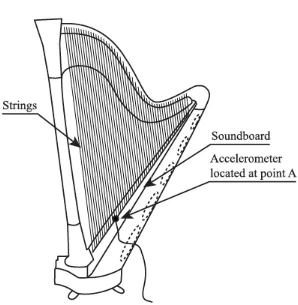
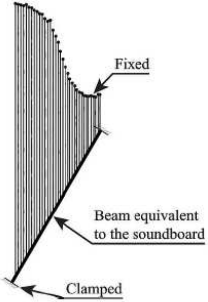
Sympathetic string modes in a concert harp

Modelling sympathetic string modes in a concert harp [1]



Experimental protocol

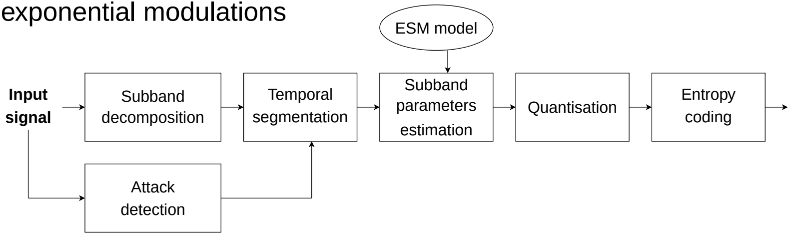


Physical model

[1] Jean-Loïc Le Carrou, François Gautier, and Roland Badeau. "Sympathetic string modes in the concert harp". *Acta Acustica united with Acustica*, 95(4): 744-752, July/August 2009.

Audio coding

- Parametric coder based on the ESM model [1]
  → exponential modulations



- Joint scalar quantisation with entropy constraint [2,3]

[1] Olivier Derrien, Gaël Richard, and Roland Badeau. "Damped sinusoids and subspace based approach for lossy audio coding". In *Acoustics'08*, Paris, France, July 2008.
[2] Olivier Derrien, Roland Badeau, and Gaël Richard. "Entropy-constrained quantization of exponentially damped sinusoids parameters". In *Proc. of IEEE ICASSP*, Prague, Czech Republic, May 2011.
[3] Olivier Derrien, Roland Badeau, and Gaël Richard. "Calculation of an entropy-constrained quantizer for exponentially damped sinudoids parameters". Technical report, Laboratoire de Mécanique et d'Acoustique, Marseille, France, June 2010.

Audio coding

Original sound:

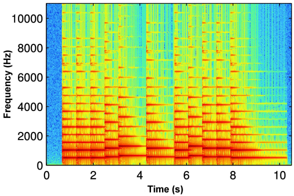
MDCT		ESM	
9 bits/spl		8.9 bits/spl	
8 bits/spl			
7 bits/spl		6.8 bits/spl	
6 bits/spl		6.4 bits/spl	
5 bits/spl		4.7 bits/spl	
4 bits/spl		4.4 bits/spl	
3 bits/spl		3.2 bits/spl	
2 bits/spl		2.1 bits/spl	

Part II

Nonnegative decompositions

Nonnegative Matrix Factorization (NMF)

Musical score



Spectrogram ***V***

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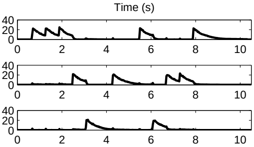
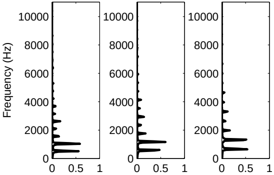
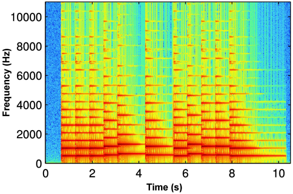


Temporal activations ***H***



Spectral atoms ***W***



Spectrogram ***V***

β-divergence and multiplicative rules

- Minimisation of the criterion $D(\boldsymbol{V}|\boldsymbol{WH}) = \sum_{n=1}^{N}\sum_{f=1}^{F} d\left(v_{fn}\,|\hat{v}_{fn}\right)$
- β-**divergence** [Eguchi and Kano, 2001]:

$$d_\beta(a|b) = \frac{1}{\beta(\beta-1)}\left(a^\beta + (\beta-1)b^\beta - \beta a b^{\beta-1}\right)$$

  - $\beta = 2$ corresponds to **Euclidean distance** (EUC),
  - $\beta = 1$ corresponds to **Kullback-Leibler divergence** (KL),
  - $\beta = 0$ corresponds to **Itakura-Saito divergence** (IS),
  - $d_\beta(a|b)$ is convex w.r.t b if and only if $\beta \in [1,2]$,
- Multiplicative update rules [Kompass, 2007]:
  - $$\begin{cases} \boldsymbol{W} & \leftarrow & \boldsymbol{W} \otimes \frac{(\boldsymbol{V} \otimes (\boldsymbol{WH})^{\beta-2})\boldsymbol{H}^T}{(\boldsymbol{WH})^{\beta-1}\boldsymbol{H}^T} \\ \boldsymbol{H} & \leftarrow & \boldsymbol{H} \otimes \frac{\boldsymbol{W}^T(\boldsymbol{V} \otimes (\boldsymbol{WH})^{\beta-2})}{\boldsymbol{W}^T(\boldsymbol{WH})^{\beta-1}} \end{cases}$$
  - $D(\boldsymbol{V}|\boldsymbol{WH})$ is non-increasing if and only if $\beta \in [1,2]$.

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Stability of multiplicative update rules

- Introduction of an exponentiation step η into NMF multiplicative update rules designed for minimizing the β-divergence [1]:

$$\boldsymbol{W} \leftarrow \boldsymbol{W} \otimes \left(\frac{(\boldsymbol{V} \otimes (\boldsymbol{WH})^{\beta-2}) \boldsymbol{H}^T}{(\boldsymbol{WH})^{\beta-1} \boldsymbol{H}^T} \right)^{\eta} \quad (1)$$

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- Monotonic decrease of the criterion if $\beta \in [1, 2]$ and $\eta \in]0, 1]$
- Exponential or asymptotic stability of both rules (1) and (2) if $\eta \in]0, \eta^*[$, where $\forall \beta \in \mathbb{R}$, $\eta^\star \in]0, 2]$ and if $\beta \in [1, 2]$, $\eta^\star = 2$
- Instability if $\eta \notin [0, 2]$ $\forall \beta \in \mathbb{R}$
- Step η permits to control the convergence rate

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Avoiding local minima

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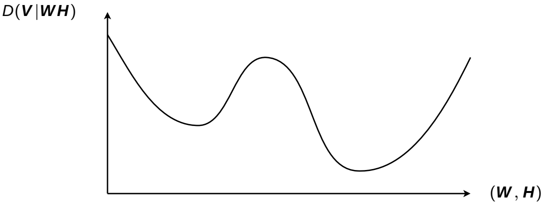

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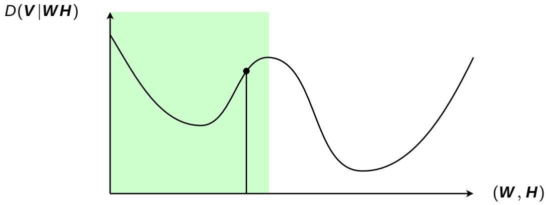

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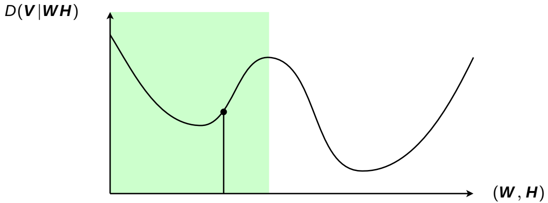

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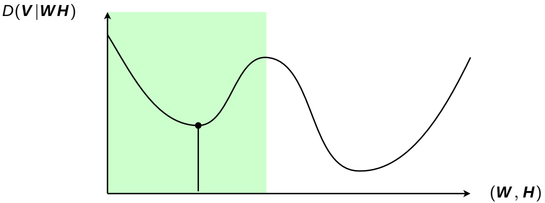

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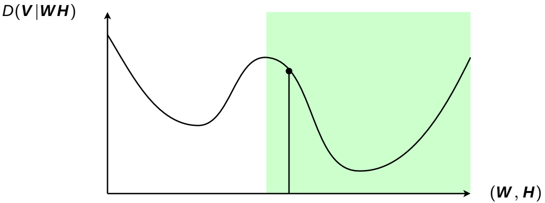

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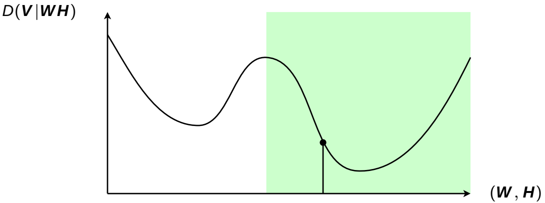

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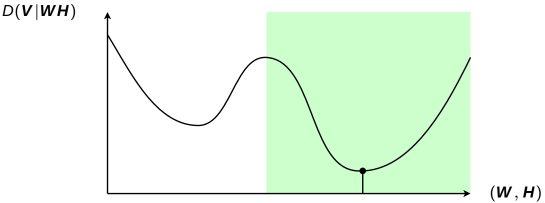

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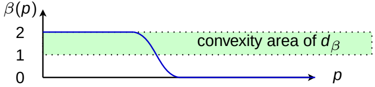

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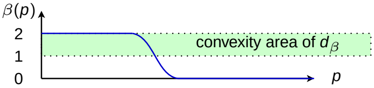

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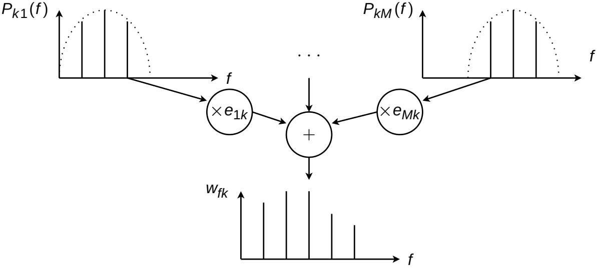
Harmonicity and spectral smoothness

- Model [1]: $\hat{v}_{fn} = \sum_{k=1}^{K} w_{fk}(\theta) h_{kn}$ where $w_{fk}(\theta) = \sum_{m=1}^{M} e_{mk} P_{km}(f)$



- $P_{km}(f)$ is a predefined harmonic spectral pattern
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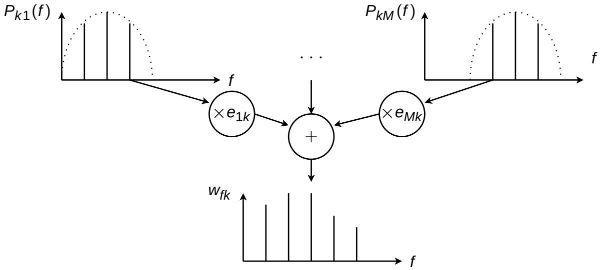

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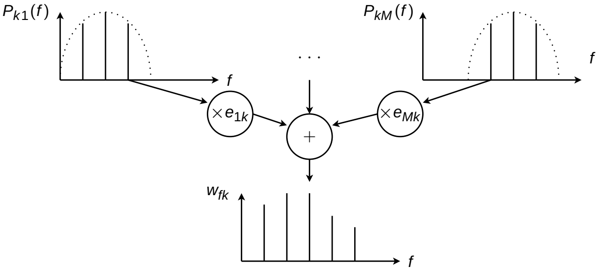

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Temporal smoothness

- MAP estimator: $C(\boldsymbol{\Theta}) = L(\boldsymbol{\Theta}) + \log(p(\boldsymbol{\Theta}))$ où $\boldsymbol{\Theta} = \{e_{mk}, h_{kn}\}$
- Markov chain structured a priori distribution:

$$p(\boldsymbol{H}) = \prod_{k=1}^{K} p(h_{k1}) \prod_{n=2}^{N} p(h_{kn}|h_{k(n-1)})$$

  where $p(h_{kn}|h_{k(n-1)})$ follows an inverse-Gamma distribution
- SAGE algorithm [1] and multiplicative update rules [2] with $\eta = 0.5$

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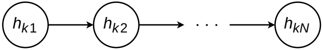

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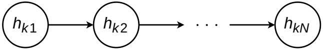

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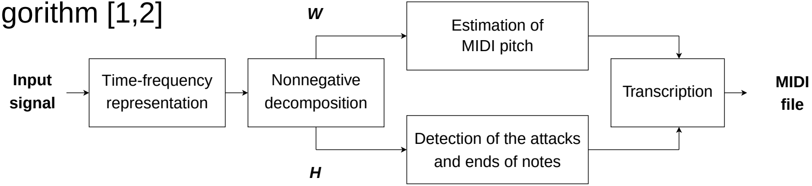
NMF-based automatic transcription

- Algorithm [1,2]



- Demo
  - Original signal (Liszt):
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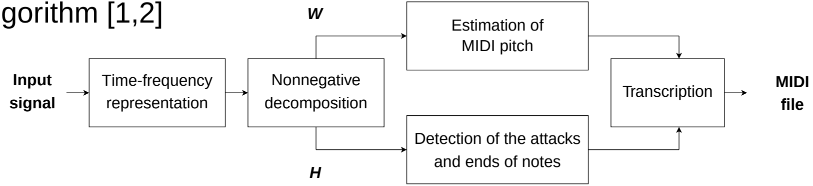

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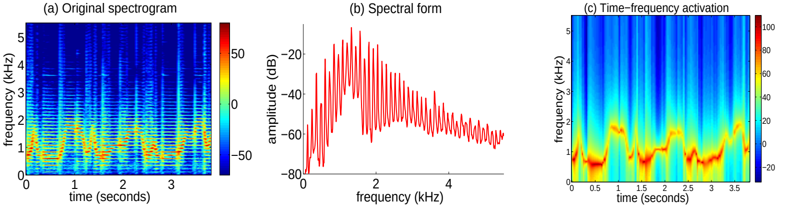
Time-frequency activations

Model [1]: $\hat{v}_{fn} = \sum_{k=1}^{K} w_{fk} h_{kn}(f)$ where $h_{kn}(f) = \sigma_{kn}^2 \frac{\left|1+\sum_{q=1}^{Q} b_{kn}^{(q)} e^{-i2\pi\nu_f q}\right|^2}{\left|1+\sum_{p=1}^{P} a_{kn}^{(p)} e^{-i2\pi\nu_f p}\right|^2}$

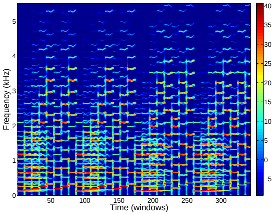


Jew's harp signal decomposed with an ARMA of order (1,1)

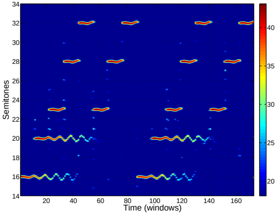
[1] Romain Hennequin, Roland Badeau, and Bertrand David. "NMF with time-frequency activations to model non-stationary audio events". *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 19, no. 4, pp. 744-753, May 2011.

Fundamental frequencies variations

Model [1]: $\hat{v}_{fn} = \sum_{k=1}^{K} w_{fk}(\nu_{kn}^0)\, h_{kn}$ where $w_{fk}(\nu_{kn}^0) = \sum_{h=1}^{n_h(\nu_{kn}^0)} a_h\, g(\nu_f - h\,\nu_{kn}^0)$



Original spectrogram



Temporal activations

Decomposition of an excerpt of J.S. Bach's first prelude

[1] Romain Hennequin, Roland Badeau, and Bertrand David. "Time-dependent parametric and harmonic templates in nonnegative matrix factorization". In *Proc. of DAFx*, Graz, Austria, September 2010.

Score-based informed source separation

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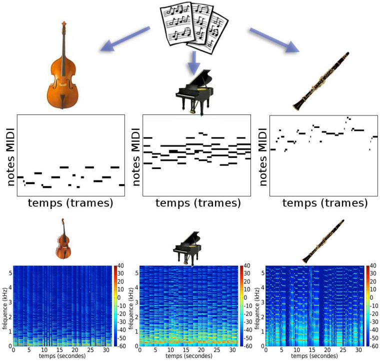

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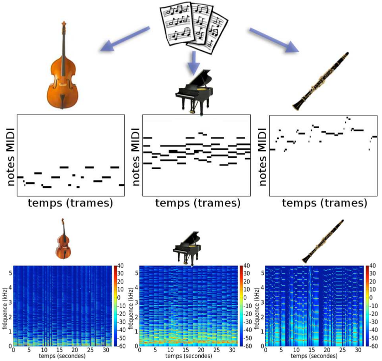

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