Probabilistic modelling of time-frequency representations with application to music signals

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C4DM, Wednesday, March 6, 2013
**Introduction**

- NMF applied to time-frequency distributions:
  - is a powerful tool for modelling music signals
  - has many applications in audio signal processing

- Most probabilistic models for NMF:
  - permit to exploit some a priori knowledge
  - do not take phase into account
  - assume that all time-frequency bins are independent

- The proposed HR-NMF model:
  - takes phases and local correlations into account
  - achieves high spectral resolution
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Advantages and drawbacks of NMF probabilistic models

Choosing an appropriate TF representation

Modelling phases and correlations in the TF domain
  - HR-NMF model
  - Algorithms

Preliminary results
  - Audio source separation
  - Audio inpainting

Conclusions
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Non-negative Matrix Factorization (NMF)

Musical score
Non-negative Matrix Factorization (NMF)

Musical score

Spectrogram V
Non-negative Matrix Factorization (NMF)

Musical score

Temporal activations $H$

Spectral templates $W$

Spectrogram $V$
Non-negative Matrix Factorization (NMF)

- Factorization of a matrix $V \in \mathbb{R}^{F \times T}$ as a product $V \approx WH$
- Rank reduction: $W \in \mathbb{R}^{F \times K}_{+}$ and $H \in \mathbb{R}^{K \times T}_{+}$ where $K < \min(F, T)$
- Usual applications:
  - Image analysis, data mining, spectroscopy, finance, etc.
  - Audio signal processing:
    - Multi-pitch estimation, onset detection
    - Automatic music transcription
    - Musical instrument recognition
    - Source separation
    - Audio inpainting
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NMF probabilistic models

- Mixture models with (hidden) latent variables
  - can exploit a priori knowledge
  - can use well-known statistical inference techniques

- Probabilistic models of time-frequency distributions:
  - Additive Gaussian noise [Schmidt 2008],
  - Probabilistic Latent Component Analysis [Smaragdis 2006],
  - Mixture of Poisson components [Virtanen 2008],
  - Mixture of Gaussian components [Févotte 2009],
  - Only model taking the existence of phase into account, and justifying the use of Wiener filtering for separating the components
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Gaussian model (IS-NMF) [Févotte 2009]

\[ C^{(k)}_{ft} \sim \mathcal{N}(0, w_{fk} h_{kt}) \]

all time-frequency bins are independent

\[ \hat{V}^{(k)}_{ft} = w_{fk} h_{kt} \]
Gaussian model (IS-NMF) [Févotte 2009]

\[ c_{ft}^{(k)} \sim \mathcal{N}(0, w_{fk}) \]

\[ \hat{V}_{ft}^{(k)} = w_{fk} h_{kt} \]

\[ X_{ft} = \sum_{k=1}^{K} c_{ft}^{(k)} \sim \mathcal{N}(0, W) \]

\[ \hat{V} = WH \]

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Gaussian model (IS-NMF) [Févotte 2009]

\[
\begin{align*}
C^{(k)} & \sim \mathcal{N}(0, W_{fk}) \\
C_{ft}^{(k)} & \sim \mathcal{N}(0, W_{fk}) \\
X_{ft} &= \sum_{k=1}^{K} C_{ft}^{(k)} \\
X & \sim \mathcal{N}(0, W_{fk}) \\
\hat{V} &= WH \\
\hat{V}_{ft} &= \sum_{k=1}^{K} \hat{V}_{ft}^{(k)} \\
\hat{V}_{ft}^{(k)} &= w_{fk} h_{kt} \\
\hat{V} & \sim \mathcal{N}(0, 0, 0) \\
\max L(X) & \iff \min D_{IS}(V = |X|^2 |\hat{V})
\end{align*}
\]
A priori knowledge in probabilistic models

- Various kinds of a priori knowledge:
  - Harmonicity [Virtanen 2008, Vincent 2008...]
  - Smoothness of spectral envelopes [Schmidt 2008, Vincent 2008...]
  - Smoothness of temporal activations [Virtanen 2008, Févotte 2009...]
  - Spectral or temporal sparsity [Schmidt 2008, Smaragdis 2009...]

- Standard approaches:
  - Parametrisation of \( W \) and / or \( H \)
  - Use of a predefined dictionary \( W \) (parametric or non-parametric, learned beforehand)
  - Bayesian methods (a priori distribution of the parameters)
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The low-level model raises several issues:

- Phase is not (or insufficiently) taken into account
- Sinusoids are not modelled as such (they cannot be properly separated by Wiener filtering)
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Preservation of whiteness (PW)
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White noise

Filter bank

Frequency

Time

2D white noise
Perfect reconstruction (PR)

Input signal

Filter bank

Analysis

Frequency

Time-frequency representation

Time
Perfect reconstruction (PR)

Input signal → Filter bank → Frequency → Analysis

Time-frequency representation

Synthesis → Time → Filter bank → Input signal
Solution of (PW) + (PR)

- Critically sampled paraunitary filter banks: \( R(z) = \tilde{E}(z) \)

- "Decorrelating" effect onto a stationary process
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Graphical model of IS-NMF ($X \sim \mathcal{N}(0, WH)$)
Autoregressive filtering of the channels

\[ f * a^{(k)}_F \]
\[ f * a^{(k)}_{F-1} \]
\[ \vdots \]
\[ f * a^{(k)}_2 \]
\[ f * a^{(k)}_1 \]
Graphical model of HR-NMF (AR1)
Graphical model of HR-NMF (AR2)
HR-NMF model

- Frequency bands are independent and non-stationary
- Particular cases:
  - IS-NMF model
  - Autoregressive process
  - Exponential Sinusoidal Model (ESM)
HR-NMF model

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Maximum likelihood estimation

- Expectation-Maximization (EM) algorithm:
  - E-step:
    - Kalman filtering with smoothing (forward-backward)
    - Complexity: \( O(FTK^3P^3) \)
  - M-step:
    - Iterative algorithm which switches between \((W, a)\) and \(H\)
    - Complexity: \( O(FTKP^2) \)

- Processing realistic data requires faster algorithms:
  - Improve the convergence speed
  - Reduce the computational complexity

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EM as Minorize-Maximize (MM) method

\[ L(\theta) = \ln(p(x; \theta)) \]
EM as Minorize-Maximize (MM) method

\[ Q(\theta, \theta_0) = \int \ln(p(x, c; \theta))p(c|x; \theta_0)dc \]
\[ M(\theta, \theta_0) = L(\theta_0) + Q(\theta, \theta_0) - Q(\theta_0, \theta_0) \]
EM as Minorize-Maximize (MM) method

\[ \theta_1 = \arg \max_{\theta} M(\theta, \theta_0) \]
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EM as Minorize-Maximize (MM) method

\[
\theta_2 = \operatorname{argmax}_\theta M(\theta, \theta_1)
\]
Computing the gradient of $L$:

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Computing the gradient of $L$

\[ \nabla L(\theta_0) = \nabla Q(\theta_0, \theta_0) \]

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Multiplicative update rules

- **Purpose:** improve the convergence rate of EM
- **Observation:** the E-step permits to efficiently compute the gradient of the log-likelihood function
- **Principle:** replace the M-step by any gradient-based optimizer
- **New update rules parametrized by** $\varepsilon \geq 0$, which generalize both IS-NMF multiplicative updates ($\varepsilon = 0$) and EM ($\varepsilon = 1$)
- **Enhanced convergence speed obtained with a "simulated cooling" strategy (make $\varepsilon$ decrease over iterations)**

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Prior distribution of latent variables in band $f$ ($P = 1$, $K = 2$)
Joint distribution of complete data in band $f$ ($P = 1$, $K = 2$)
Variational Bayesian EM algorithm

Posterior distribution of latent variables in band \( F \) \((P = 1, K = 2)\)
Variational Bayesian EM algorithm

- Structured mean field approximation in band $f$ ($P = 1$, $K = 2$)

\[
\begin{align*}
\cdots & \rightarrow C_{f,t-1}^{(1)} & \rightarrow C_{f,t}^{(1)} & \rightarrow C_{f,t+1}^{(1)} & \rightarrow \cdots \\
\cdots & \rightarrow C_{f,t-1}^{(2)} & \rightarrow C_{f,t}^{(2)} & \rightarrow C_{f,t+1}^{(2)} & \rightarrow \cdots 
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Variational Bayesian EM algorithm

- Mean field approximation in band $f$ ($P = 1$, $K = 2$)

$$
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Variational Bayesian EM algorithm

- **Purpose:** reduce the computational complexity of EM
- **Principle:** the posterior distribution of the latent variables is approximated by a factorized distribution
- **Complexity reduction:**
  - Exact E-step: $O(FTK^3(1 + P)^3)$
  - Structured mean field (no dependency over $k$): $O(FTK(1 + P)^3)$
  - Mean field (no dependency over $k$ and $t$): $O(FTK(1 + P))$
- **Performance loss:**
  - The increase of log-likelihood function is no longer guaranteed
  - In practice, no perceptual difference

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Application to piano tones

Spectrogram of the input piano sound (C4 + C3)
Source separation

Separation of two sinusoidal components

(a) First component (C4)

(b) Second component (C3)

IS-NMF:

HR-NMF:
Audio inpainting

Spectrogram of the input piano sound (C4 + C3)
Audio inpainting

Masked spectrogram of the input piano sound

C4+C3:
C4 alone:
IS-NMF:
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Recovery of the full C4 piano tone

C4+C3:  
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Contributions

- Critically sampled paraunitary filter banks satisfy both PW and PR
- HR-NMF time-frequency model:
  - models phases and local correlations in each frequency band
  - generalizes IS-NMF, mixtures of AR processes, and ESM models
- Algorithms:
  - EM algorithm: too computationally demanding
  - Multiplicative update rules: improved convergence speed
  - Variational Bayesian EM algorithm: lower computational complexity
- Preliminary results:
  - Separation of overlapping sinusoids without perceptible artefacts
  - Restoration of missing observations without perceptible artefacts
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Outlooks

- Design consistent TF representation and TF probabilistic model
- Extensions of HR-NMF:
  - Extension to multichannel signals (e.g. stereo)
  - Correlations between frequency bands (attacks, vibratos, chirps)
  - Correlations between components (sympathetic modes)
  - Replace NMF by other parametric models, or priors enforcing harmonicity, sparsity, smoothness...
- Algorithms:
  - Variational Bayesian methods
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