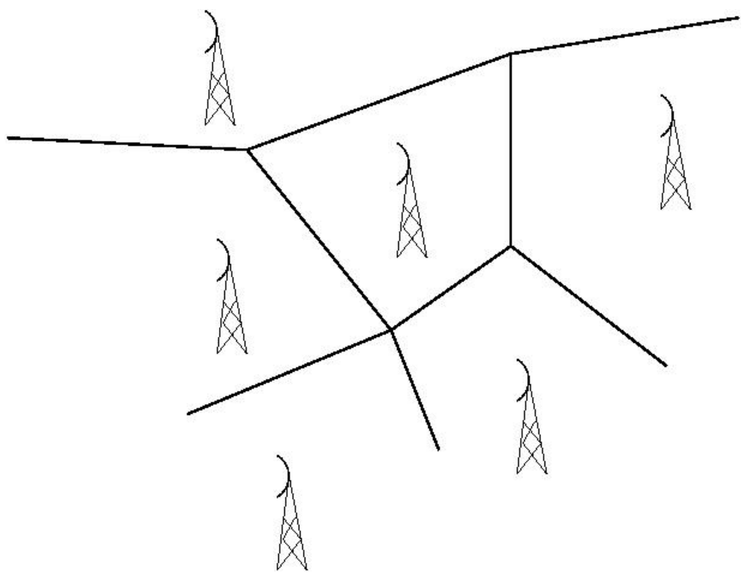
Radio access networks

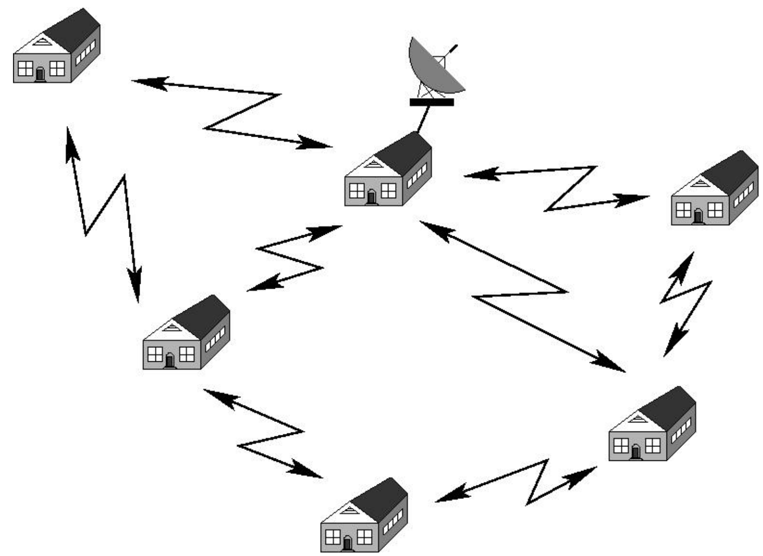
Jérôme Galtier BIZZ/DIAM

November 16, 2006

Two different kinds of networks



cellular



meshed

Main characteristics

Cellular networks:

- Operator-driven
- Planification required to share ressources

Meshed networks:

- "Who's paying what?" problem
- Coverage issues (nice probabilistic models)

Main QoS issues for radio

→ Fairness

→ Fast reconfiguration (meteo, business, week-end time, . . .)

→ Scarce ressources

→ Reliability/accessibility

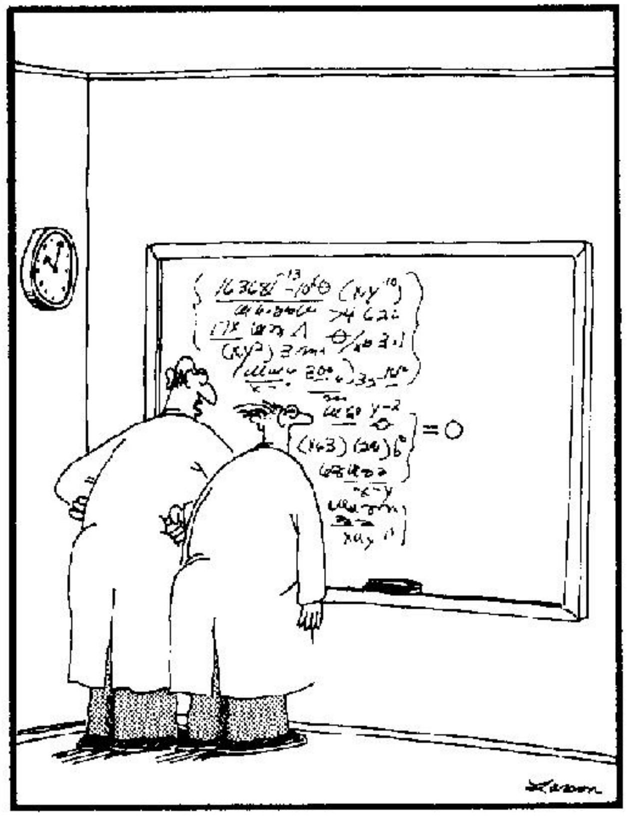
A view on UMTS networks

A natural objective for an operator is to. . .

Maximize the revenues

An natural objective can also be to address fairness issues

How to put this in equations?



"No doubt about it, Ellington—we've mathematically expressed the purpose of the universe. Gad, how I love the thrill of scientific discovery!"

A fairness formulation

There exists a natural formula that generalizes several 'fairness' objective functions (Mo and Walrand 1998):

$$\text{Maximize } \frac{1}{1-\alpha} \sum_{i \in I} r(i)^{1-\alpha}, \quad \alpha \geq 0, \alpha \neq 1.$$

The special case $\alpha = 1$, corresponds to *proportional fairness*, and can be described as:

$$\text{Maximiser } \prod_{i \in I} r(i).$$

Modeling of an uplink problem (1/2)

Datas:

MR_i	Minimum required rate for i
PR_i	Maximum required rate for i
δ_i	Thresshold of C/I for i (depends on the BER)
$\overline{p}_i$	Maximum power for i
ν_{a_i}	Noise factor for the base a_i
$g_{a_i,j}$	Gain on j for base a_i

Variables:

$\delta(i)$	Rate assigned to i
p_i	Power assigned to i

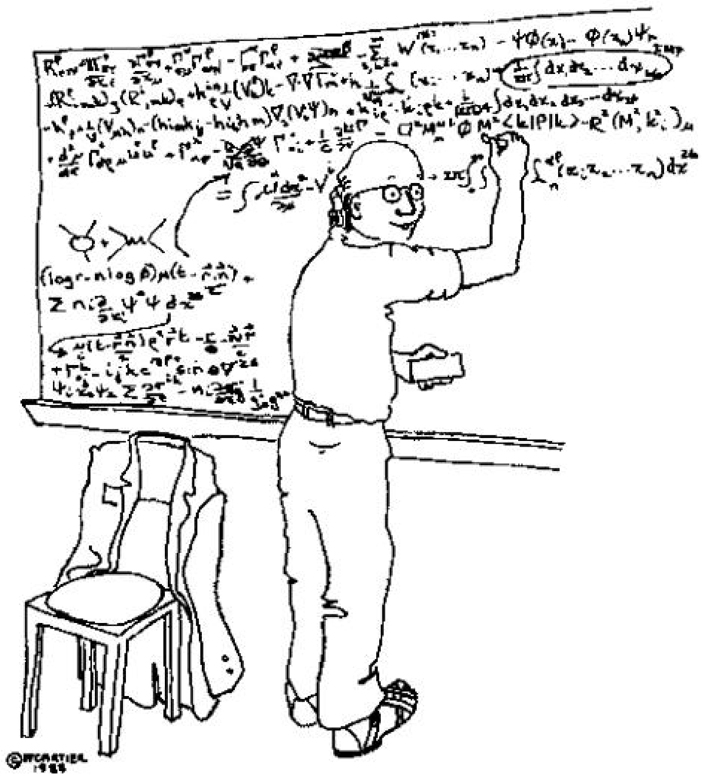
Modeling of an uplink problem (2/2)

$$\left\{\begin{array}{l} MR_i \le r(i) \le PR_i, \\ \\ 0 \le p_i \le \overline{p}_i, \\ \\ \delta_i r(i) \le \dfrac{g_{a_i,i} p_i}{\nu_{a_i} + \sum_{j \ne i} g_{a_i,j} p_j}, \end{array}\right. \qquad 1 \le i \le N.$$

A fair assignment can then be obtained by solving the following problem:

$$\text{Maximize} \sum_{i=1}^{N} \frac{r(i)^{1-\alpha}}{1-\alpha}. \tag{1}$$

It is even not convex!



"At this point we notice that this equation is beautifully simplified if we assume that space–time has 92 dimensions."

Bibliographical notes

1992	Bambos and Pottie	Powers with no white noise in polynomial time
1993	Foschini and Miljanic/Hanly	Powers in polynomial time
1994	Yun and Messerschmitt	Analytical solution
1999	Leelahakriengkrai and Agrawali	Power/rates is non-convex
2001	Andersin and Henrikson	Patent for B&B solution
2002	Won Lee, Mazumbar, and Shroff	Downling using branch-and-price (almost linear)
2004	Altman, Touati and G.	Solution usind SDP
2006	Galtier	Uplink using combined branch-and-price/analytical in l

A first simplification...

Given an adequate change of variables: $\forall i \in [1..N],\ \rho(i) = \frac{r_i}{1+\delta_i r(i)}$,

A first simplification...

Given an adequate change of variables: $\forall i \in [1..N],\ \rho(i) = \dfrac{r_i}{1+\delta_i r(i)}$,and manipulating somehow the equations, the problem turns to:

$$\begin{cases} \dfrac{MR_i}{1+\delta_i MR_i} \leq \rho(i) \leq \dfrac{PR_i}{1+\delta_i PR_i}, \\ 0 \leq \nu_{a_i}\delta_i\rho(i) \leq g_{a_i,i}\overline{p}_i\left(1 - \sum_{j=1}^{N} \delta_j \rho(j)\right) \end{cases}$$

A first simplification...

Given an adequate change of variables: $\forall i \in [1..N]$, $\rho(i) = \frac{r_i}{1+\delta_i r(i)}$,and manipulating somehow the equations, the problem turns to:

$$\begin{cases} \frac{MR_i}{1+\delta_i MR_i} \leq \rho(i) \leq \frac{PR_i}{1+\delta_i PR_i}, \\ 0 \leq \nu_{a_i}\delta_i\rho(i) \leq g_{a_i,i}\overline{p}_i \left(1 - \sum_{j=1}^{N} \delta_j \rho(j)\right) \end{cases}$$

It is linear!

A first simplification...

Given an adequate change of variables: $\forall i \in [1..N]$, $\rho(i) = \dfrac{r_i}{1+\delta_i r(i)}$,and manipulating somehow the equations, the problem turns to:

$$\begin{cases} \dfrac{MR_i}{1+\delta_i MR_i} \leq \rho(i) \leq \dfrac{PR_i}{1+\delta_i PR_i}, \\ 0 \leq \nu_{a_i}\delta_i\rho(i) \leq g_{a_i,i}\bar{p}_i \left(1 - \sum_{j=1}^{N} \delta_j \rho(j)\right) \end{cases}$$

It is linear!

But the objective function becomes: $\displaystyle\sum_{i=1}^{N} \frac{1}{1-\alpha}\left(\frac{\rho(i)}{1-\delta_i\rho(i)}\right)^{1-\alpha}$.

Problem reformulation

$$
\begin{aligned}
&\text{Maximize } \sum_{i=1}^{p} f_i(\lambda_i) \\
&\text{under the constraints} \quad
\begin{array}{ll}
\lambda_i - m_i \geq 0 & \forall i \in \{1, \ldots, p\} \\
M_i - \lambda_i \geq 0 & \forall i \in \{1, \ldots, p\} \\
C - \displaystyle\sum_{i=1}^{p} \lambda_i \geq 0 & \forall i \in \{1, \ldots, p\}.
\end{array}
\end{aligned}
$$

where p is an integer, f_i a concave function, continuously derivable for $i \in \{1, \ldots, p\}$, and C, m_i, M_i be positive real numbers, $(i \in \{1, \ldots, p\})$.

Characteristics of the optimal solution λ^*

Analyzing the Kuhn-Tucker conditions, noting the dual coefficients $\mu_i, \nu_i \; i \in \{1, \ldots, p\}$ and ρ, for each i, $i \in \{1, \ldots, p\}$, three cases can occur:

- $\lambda_i^* = m_i$. Then $\nu_i = 0$ and we have $-f_i'(\lambda_i^*) - \mu_i + \rho = 0$, which gives $f_i'(\lambda_i^*) \leq \rho$.
- $\lambda_i^* = M_i$. In this case $\mu_i = 0$ and then $-f_i'(\lambda_i^*) + \nu_i + \rho = 0$, which gives $f_i'(\lambda_i^*) \geq \rho$.
- $\lambda_i^* \in]m_i, M_i[$. We then have $\nu_i = 0$ and $\mu_i = 0$, so $f_i'(\lambda_i^*) = \rho$.

The key of the problem

It consists in *backtracking* the value of ρ. We can associate, to each ρ, a capacity used, as:

$$\varphi(\rho) = \sum_{i:\rho > f_i'(m_i)} m_i + \sum_{i:\rho < f_i'(M_i)} M_i + \sum_{i:f_i'(M_i) \le \rho \le f_i'(m_i)} (f_i')^{-1}(\rho),$$

and then all the problem consists in solving:

$$\varphi(\rho^*) = C.$$

An algorithm

- By dichotomy find an interval $]y_1; y_2[$ for ρ where $\varphi(\rho^*) = C$ occurs and for all i, $m_i \notin]y_1; y_2[$ and $M_i \notin]y_1; y_2[$.
- Once this interval is found, compute

$$\rho^* = \left[\sum_{\substack{i: f_i'(M_i) \le y_1 \\ y_2 \le f_i'(m_i)}} (f_i')^{-1} \right]^{-1} \left(C - \sum_{i: y_2 > f_i'(m_i)} m_i - \sum_{i: y_1 < f_i'(M_i)} M_i \right).$$

- Backtrack corresponding values for λ^*.

It is almost linear!

Conclusion & further work

$\longrightarrow$ Many UMTS radio problems can get avantadge of linear/convex optimization tools.

$\longrightarrow$ We can test and readjust very quickly the capacity for an individual cell.

$\longrightarrow$ Apply this to all the cells respects the constraints, at the price of (rarely) degrading the capacity.

Perspectives:

- More than one cell.
- More on real implementation.

WiFi networks: typical model

$\longrightarrow$ With the actual protocol, inside a given cell C (i.e. every stations sees all the others) capacity decreases with the number of stations due to **collisions**.

$\longrightarrow$ We would like to have a system with global behavior given by

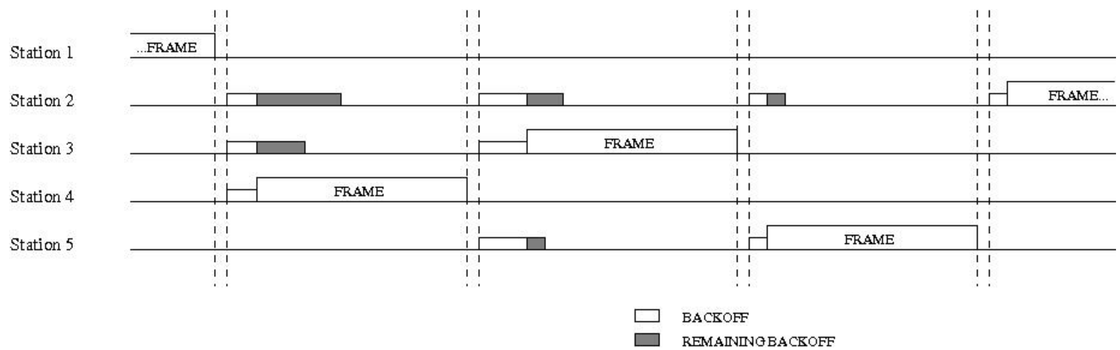
$$\sum_{i \in C} d_i \leq K$$

where user i receives average rate d_i.

The question of the Backoff counter

Each station has a contention window size CW, and a backoff counter k.

When a packet needs to be send, k is chosen randomly between 0 and CW.



→ how is regulated CW ?

Laws for congestion window (CW)

Classical 802.11:

After success (ACK received): $CW := CWmin$
After failure (no ACK received): $CW := min(CWmax, 2 \times CW)$

Slow *multiplicative* congestion window decrease: (Ni et al. 2003)

After success (ACK received): $CW := max(CWmin, CW/\eta)$
After failure (no ACK received): $CW := min(CWmax, \eta \times CW)$

Slow *additive* congestion window decrease:

After success (ACK received): $CW := max(CWmin, CW - \omega)$
After failure (no ACK received): $CW := min(CWmax, CW + \omega)$

Contribution of this paper

$\longrightarrow$ deeper analysis of the Markov chains

$\longrightarrow$ identification of an optimal mode

Modelisation using Markov chains

(see also Kleinrock and Tobagi 1975)

2-stage discrete Markow chain

$$\begin{cases} \text{First component s(t): backoff counter} \\ \text{Second component b(t): CW window size } (W_i) \end{cases}$$

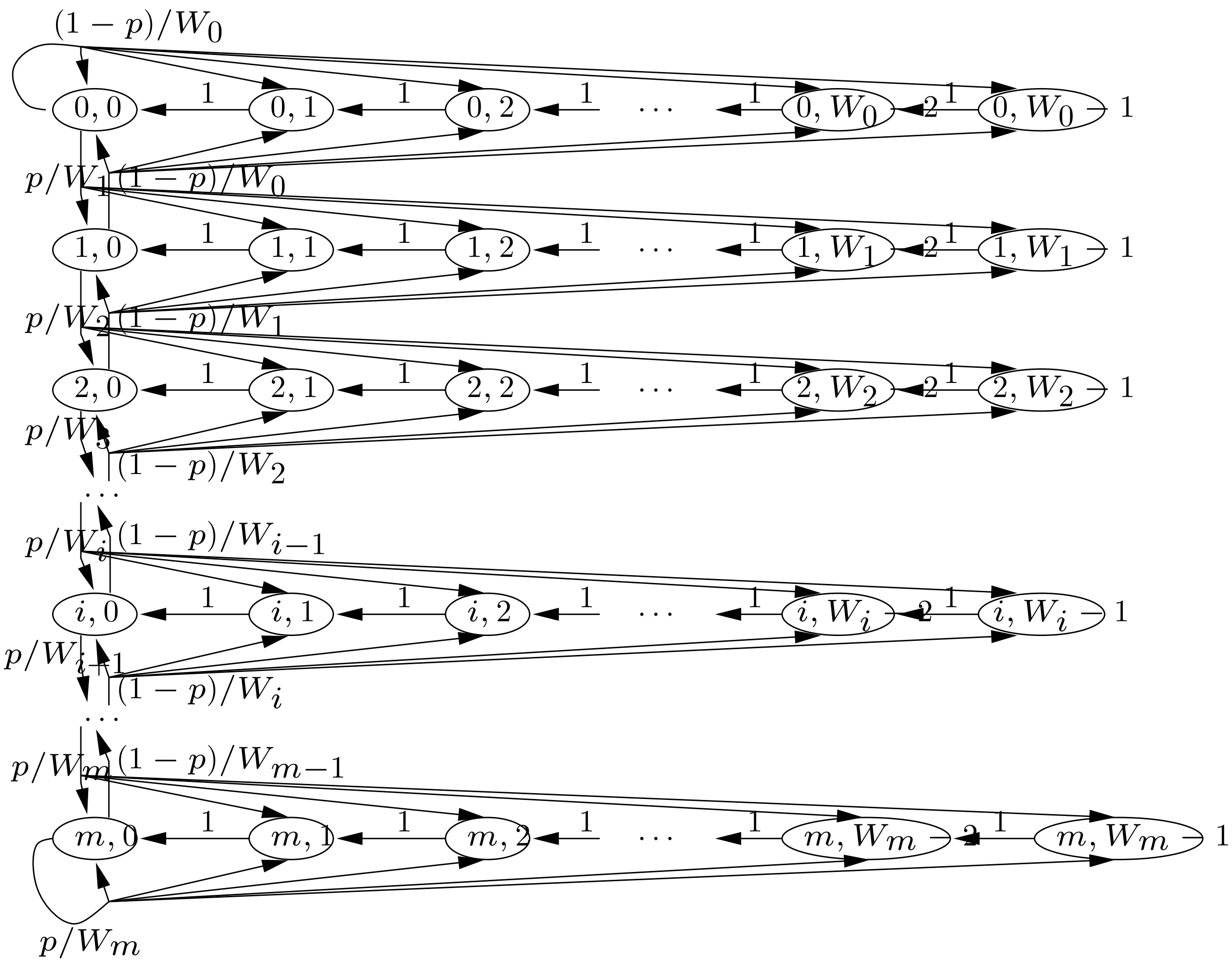
<u>Notations</u>

Multiplicative mode: $W_i = \eta^i CWmin$

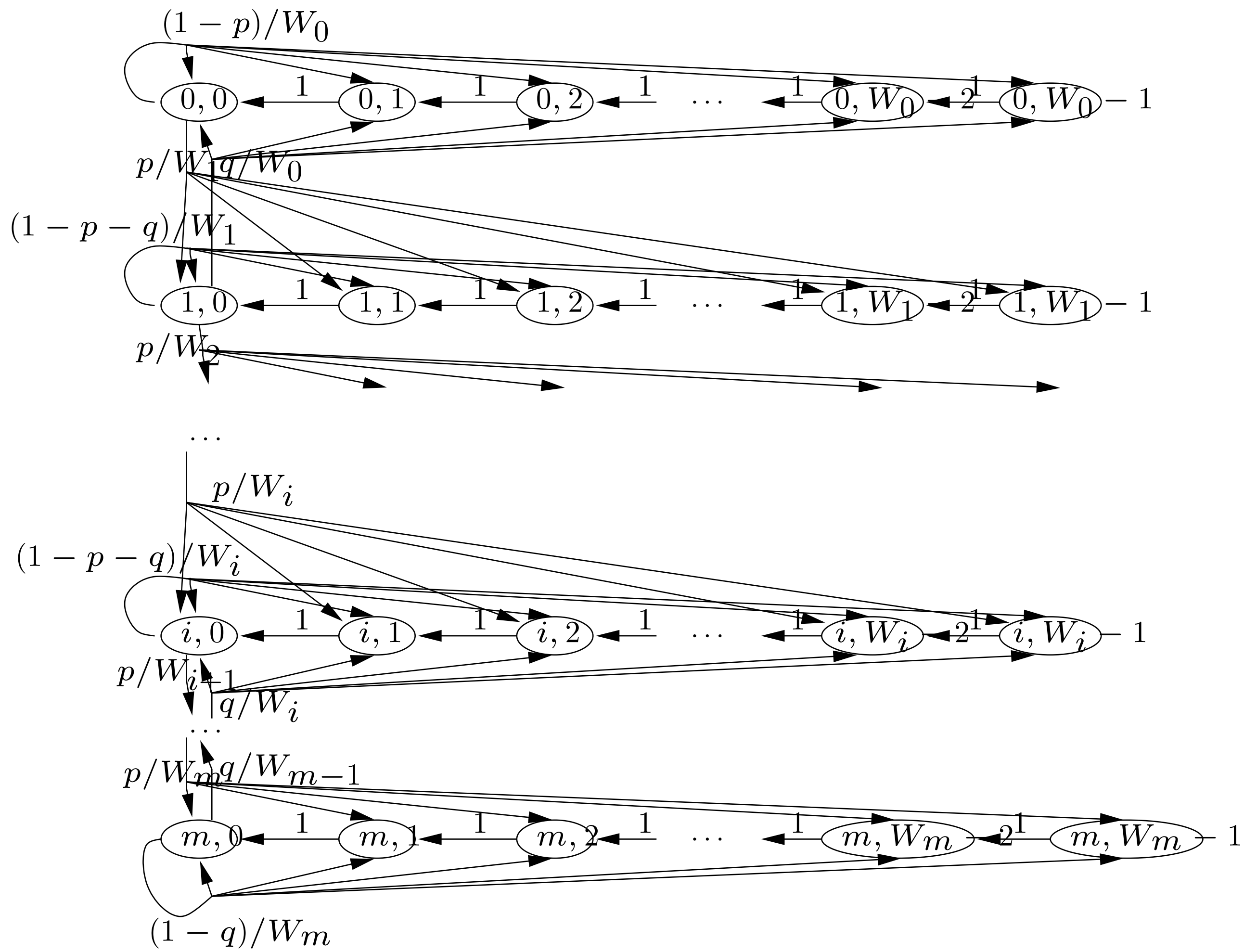
Additive mode: $W_i = i \cdot \omega + CWmin$

p: probability of failure

Chain in the multiplicative case



Chain in the additive case



Steady-state formulas

$$\pi_{i,k}=\lim_{t\to\infty} P[s(t)=i,b(t)=k],\ i\in\{0,\ldots,m\}\ \ k\in\{0,\ldots,W_i-1\}.$$

Multiplicative case

$$\pi_{m-j,0}=\left(\frac{1-p}{p}\right)^j \pi_{m,0},\quad 0\le j\le m.$$

$$\pi_{m,0}=\frac{2\cdot p^m}{\frac{(1-p)^{m+1}-p^{m+1}}{1-2p}+W_0\frac{(1-p)^{m+1}-(\eta p)^{m+1}}{1-(\eta+1)p}}.$$

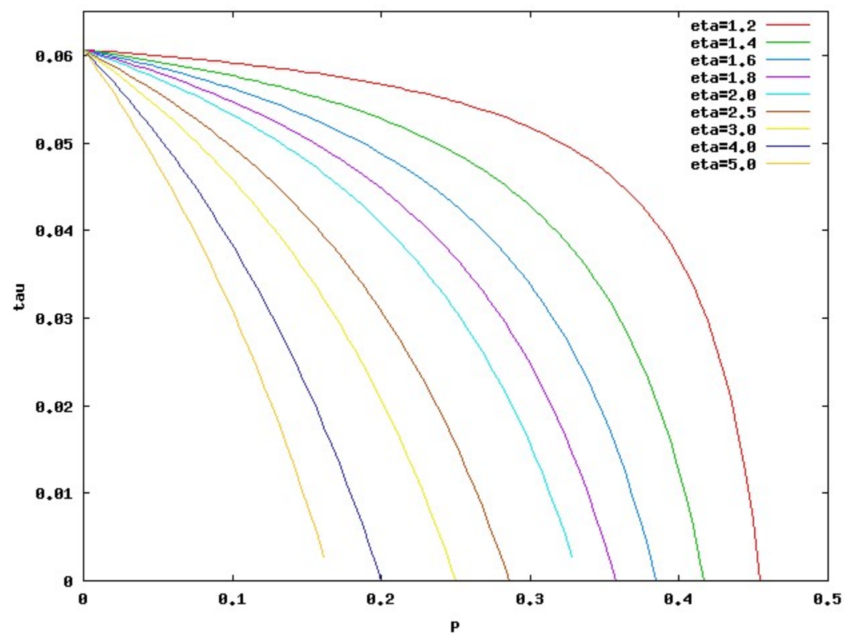
Additive case

$$\pi_{j,0}=\left(\frac{p}{q}\right)^j \pi_{0,0},\quad 0\le j\le m.$$

$$\frac{1}{\pi_{m,0}}=\frac{[(q-p)(W_0+1)+\omega p]q^{m+1}}{2\cdot p^m(q-p)^2}-\frac{[(W_0+m\omega+1)(q-p)+\omega q]p^{m+1}}{2\cdot p^m(q-p)^2}.$$

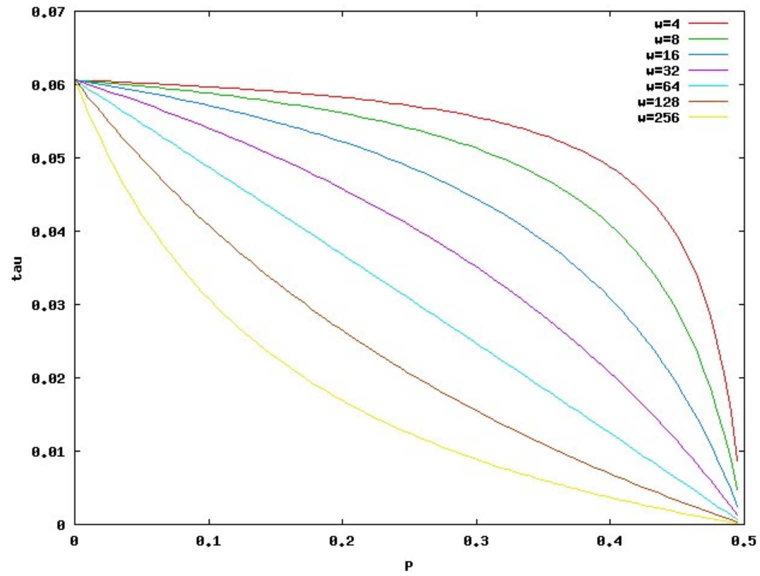
Multiplicative transmission probability

$$\tau = \frac{2}{1 + W_0 \frac{1-2p}{1-(\eta+1)p} \frac{(1-p)^{m+1}-(\eta p)^{m+1}}{(1-p)^{m+1}-p^{m+1}}}.$$

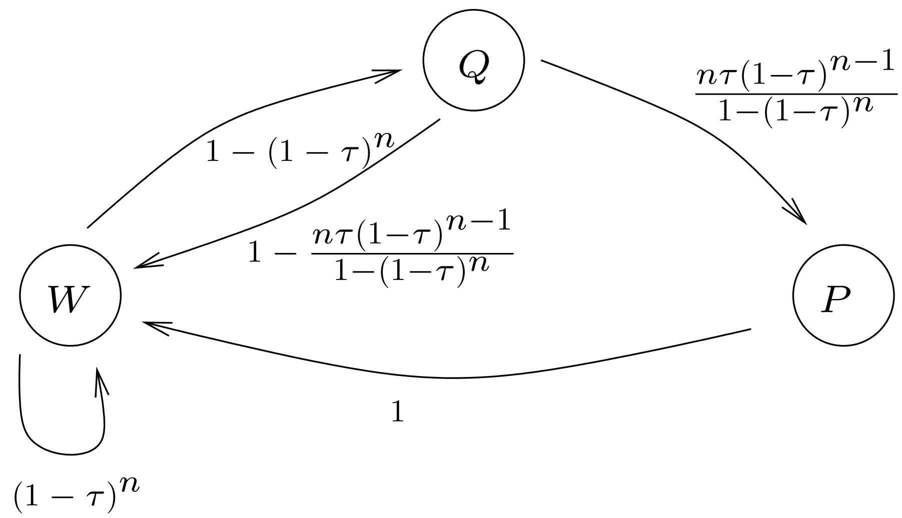


Additive transmission probability

$$\tau = \frac{2}{1 + W_0 + \omega \frac{p}{q-p} \frac{1-(m+1)\left(\frac{p}{q}\right)^m + m\left(\frac{p}{q}\right)^{m+1}}{1-\left(\frac{p}{q}\right)^{m+1}}}.$$



Maximizing the saturation throughput



Optimal saturation throughput formulas

w: Lambert (or omega) function ($w(z)e^{w(z)} = z$)
μ_w: slot duration
μ_q: average duration of a collision
μ_p: average duration of a successful paquet (on top of collision time)

Multiplicative case: $\eta = \dfrac{1}{\dfrac{-\frac{\mu_q}{\mu_w+\mu_q}}{w\left(-\frac{1}{e}\frac{\mu_q}{\mu_w+\mu_q}\right)} - 1}$

Additive case: $q = (1-p)(1-\delta)$, with $\delta = 2 + \dfrac{\frac{\mu_q}{\mu_w+\mu_q}}{w\left(-\frac{1}{e}\frac{\mu_q}{\mu_w+\mu_q}\right)}$

Optimal saturation throughput: $\boxed{Sat^+ = \dfrac{1}{1 - \frac{\mu_q/\mu_p}{w(-\frac{1}{e}\frac{\mu_q}{\mu_w+\mu_q})}}.}$

Assymptotical waiting time ($m = \infty$)

Multiplicative case

If $p \geq 1/(1+\eta^2)$ $E[W] = \infty$.

If $p < 1/(1+\eta^2)$, $E[W] = \frac{1}{6} \frac{\frac{1}{1-(\eta^2+1)p} - \frac{1}{1-2p}}{\frac{1}{1-2p} + W_0 \frac{1-p}{1-(\eta+1)p}}$

Additive case

$$E[W] = \frac{1}{3} \frac{q(q-p)(W_0^2+1) + 2W_0\omega pq + \frac{\omega^2 pq(3p-q)}{q-p}}{(q-p)(W_0+1)+\omega p}.$$

Proposed tuning

Chanel rate: 11Mbytes/s
Average packet size: 1500 bytes.
We set for all the protocols $CW_{min} = 32$, and $CW_{max} = 1024$.

The basic 801.11b scheme with $PF = 2$, $RF = 0$,

Slow CW decrease multiplicative with $\eta = PF = 1/RF$, $\eta = 5.5$ on the basic mode, and $\eta = 2$ with RTS/CTS.

Slow CW decrease additive with $\omega = 32$, $\delta = 0.81910$ on the basic mode, and $\delta = 0.49434$ with RTS/CTS.

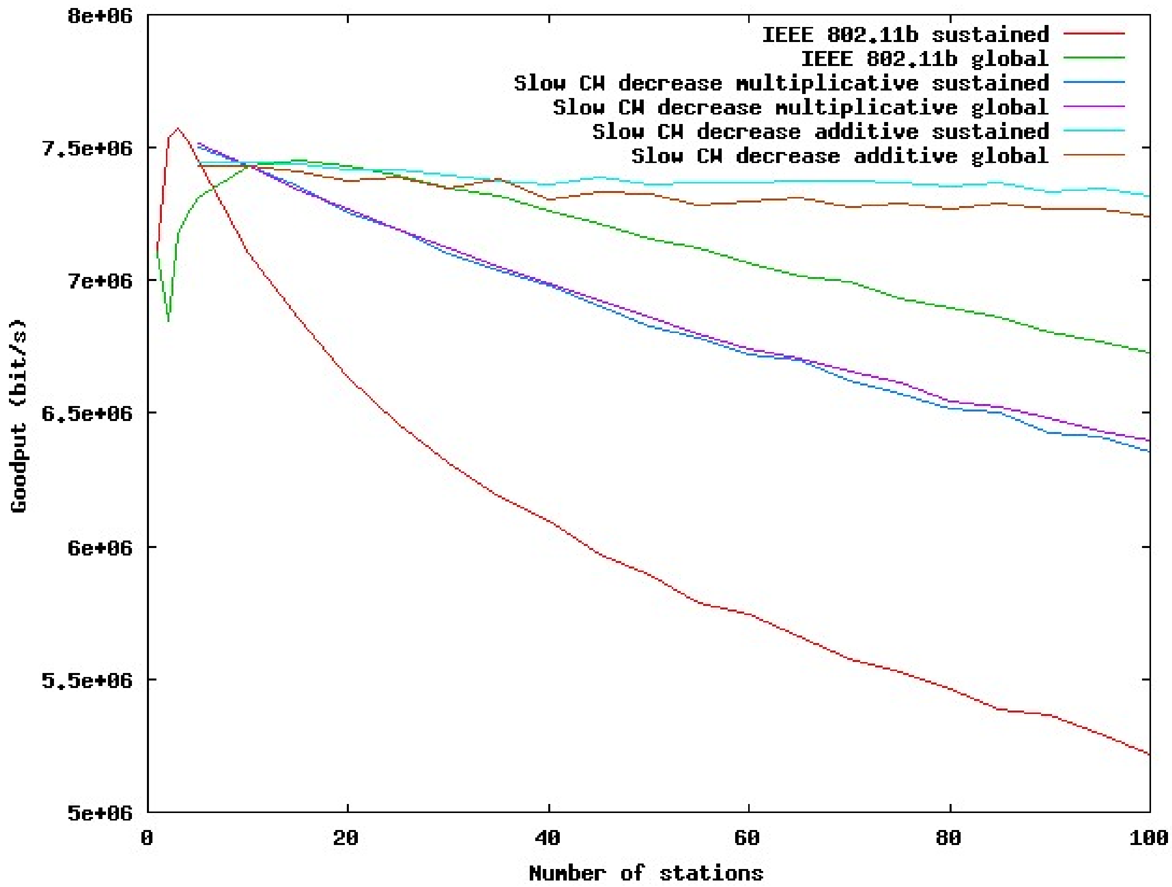
Note (additive case): In case of success, with probability δ, we do not change the CW size, and with probability $1 - \delta$, we decrease it by ω. In case of failure, CW is increased by ω.

Simulation principles

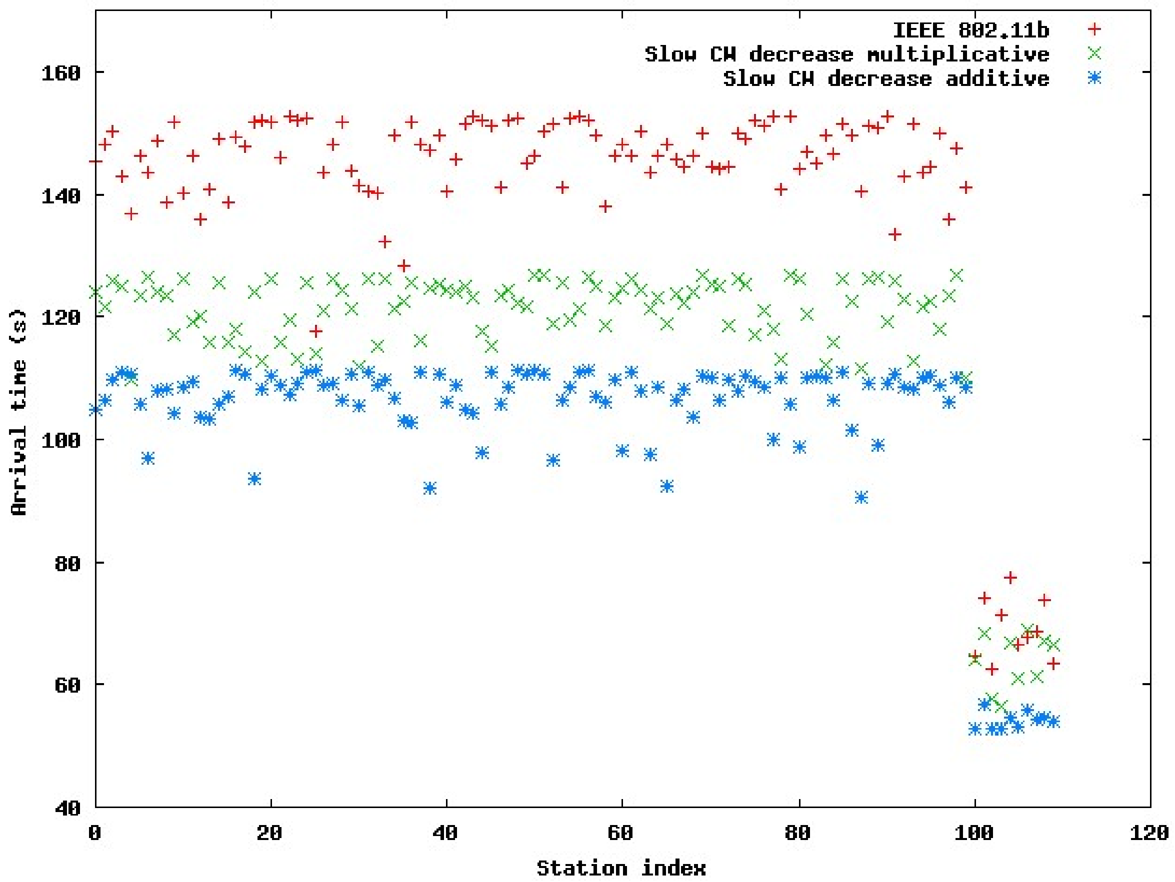
Experience 1: n stations ask simultaneously to transmit each $100/n$ MegaBytes

Experience 2: 100 stations start simultaneously to transmit 1 MegaByte each, and after 50s, 10 stations start to transmit 100 kBytes.

Results (experience 1)



Arrival dates (experience 2)



Conclusions

$\longrightarrow$ we have derived closed formulas for multiplicative and additive slow congestion window decrease.

$\longrightarrow$ we propose accordingly a tuning of the addiditve protocol.

$\longrightarrow$ we obtain in practice optimal spectral efficiency, nearly insensitive to the number of stations.

Further work

- (Real) networking problem
- Hidden station problem