Volume Constrained Polyhedronizations of Point Sets in 3-Space

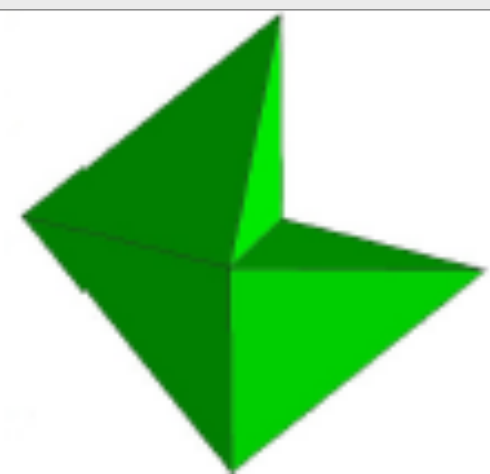
Jiju Peethambaran$ Amal Dev* Ramanathan Muthuganapathy$

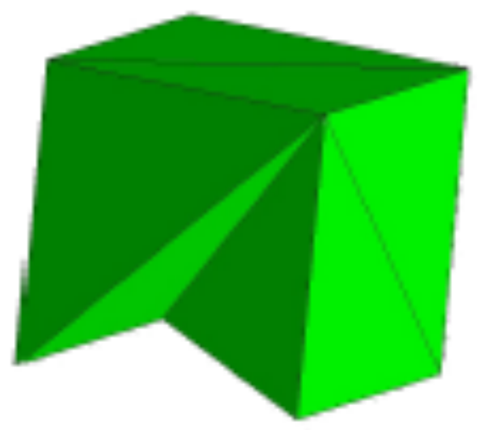
$Advanced Geometric Computing Lab, Department of Engineering Design, IIT Madras, India

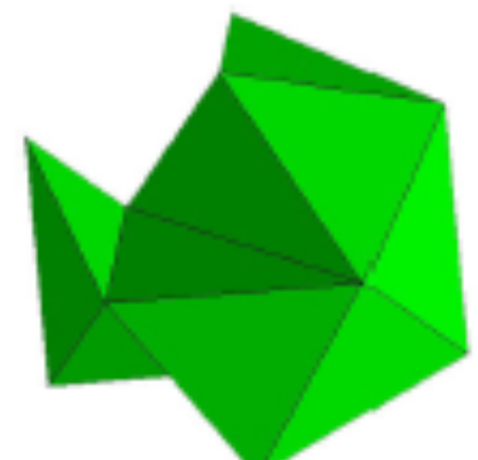
*Department of Computer Science & Engineering, MG University, Kerala, India

Minimum Volume Polyhedronization of Platonic Point sets


Tetrahedral (Optimal)


Octahedral (Optimal)


Cube (Optimal)


Icosahedral (Approximate)


Dodecahedral (Approximate)

Polyhedronization

•Definition

"Given a finite set of points in R^3, polyhedronization deals with constructing a simple polyhedron such that the vertices of the polyhedron are precisely the given points."

•Applications

- Molecular polyhedron structure synthesis.
- Boundary representation of input points in Computer Graphics, Computer Vision & Distance Image Processing.

Algorithm

•RAA_MINVP-Randomized Approximation Algorithm

Let $S=\{p_0,p_1,\ldots,p_{(n-1)}\}$ denotes the point set.

Initialization

Select four points uniformly at random from *S* and form an initial tetrahedron *P*.

Iterations

In each iteration, it chooses one point *q* uniformly at random from *S**P*. Determines the position of *q* relative to the previous polyhedron *P* and does one of the following.

1. *q* lies interior to *P*? ->exclude from *P*, the largest volume tetrahedron that *q* makes with any of the visible faces of *P*.
2. *q* lies exterior to *P*?-> add to *P*, the smallest volume tetrahedron that *q* forms with any of the visible faces of *P*.
3. *q* lies on an edge of *P*?-> divide the adjacent faces of that edge into four new faces by including *q* as the common vertex of all the four faces.
4. *q* lies on a face of *P*?-> divide the face into three new faces by including *q* as the common vertex of all the three faces.

Termination Once the iterations are completed, algorithm returns the final polyhedron (The set of faces).

•RAA_MAXVP

The initial polyhedron is the convex hull of *S*. The iterations are pretty much similar to the iterations of RAA_MINVP. Both differs only in steps 1 & 2.

- *q* lies interior to *P*? ->exclude from *P*, the smallest volume tetrahedron that *q* makes with any of the visible faces of *P* and vice versa.

References

[FP93] FEKETE S. P., PULLEYBLANK W. R.: Area optimization of simple polygons. In *Proc. 9th Annu. ACM Sympos. Computational Geometry. (1993), pp. 173–182.*

Problem Statement

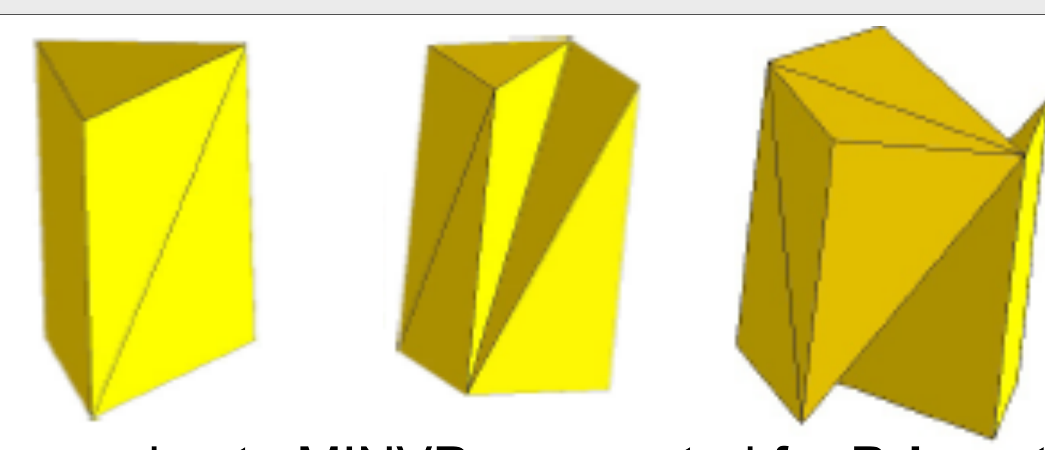
•FACE problem by S.P Fekete [FP93]

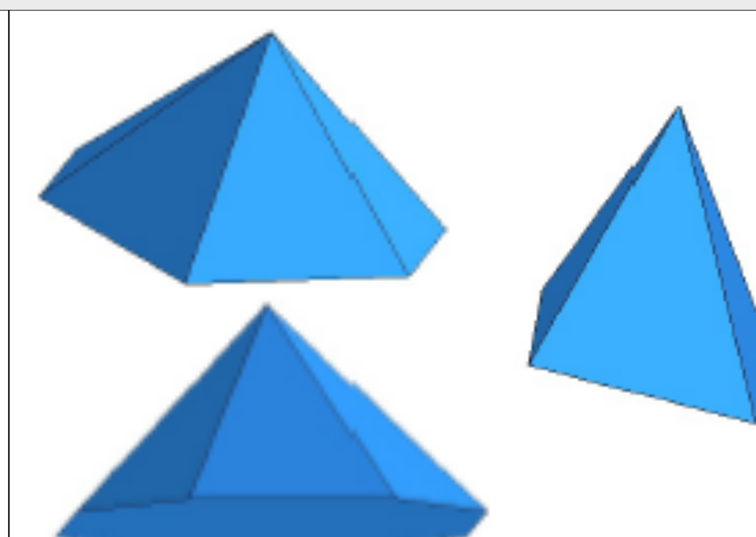
"Let 2 ≤ *d and 1* ≤ *k* ≤ *d.* Given a finite set *S* of points in *d*-dimensional Euclidean space. Among all simple polyhedra that are feasible for vertex set *S*, find one with the smallest volume of its *k*-dimensional faces."

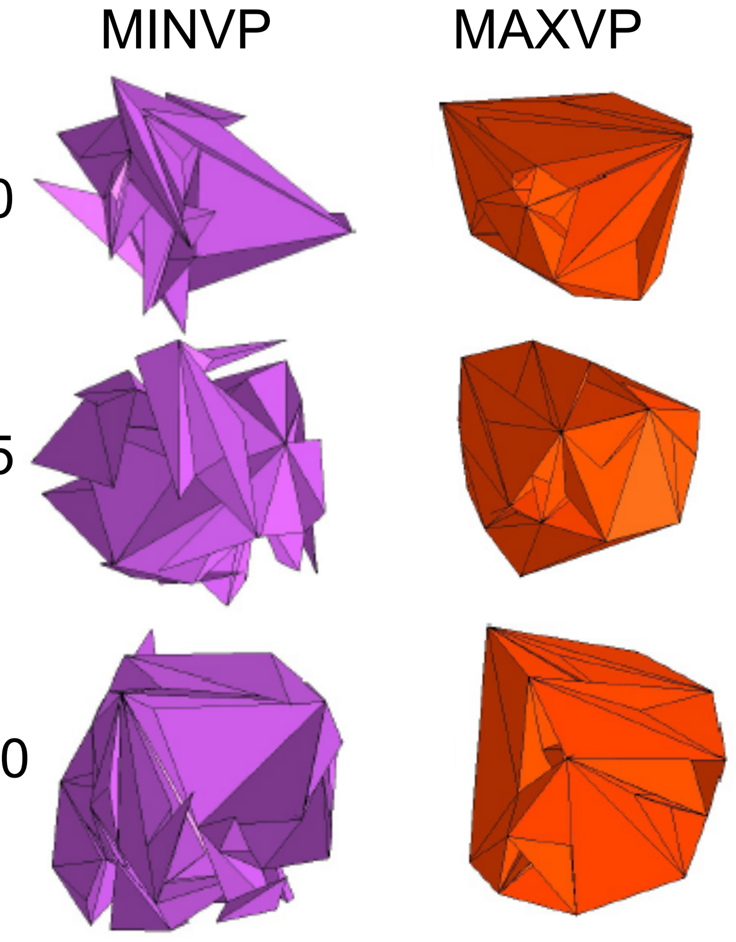
•Minimal(Maximal) Volume Polyhedronization (MINVP (MAXVP))

"Given a finite set *S* of *n* points in R^3, find the simple polyhedron with the smallest (largest) volume from all the simple polyhedra (having triangular faces) that are feasible for the vertex set *S*."

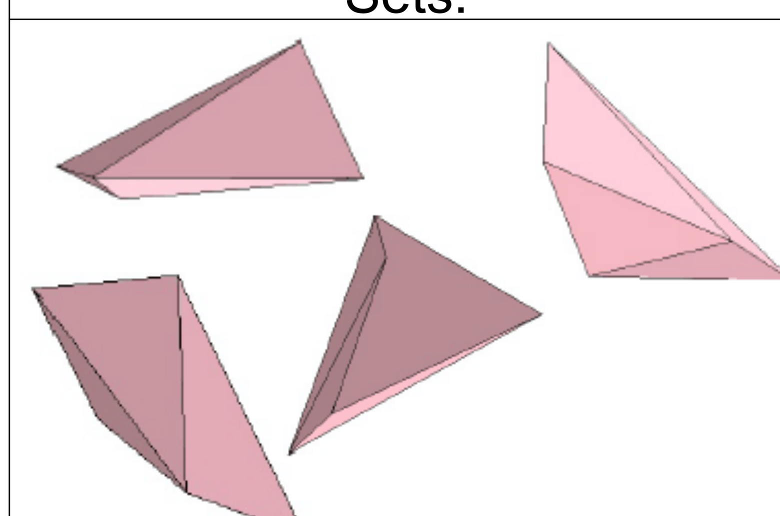
Results


Approximate MINVPs generated for **Prismatic** Point Sets.


MINVPs and/or MAXVPs generated for **Pyramid** Point Sets.



Approximate MINVPs & MAXVPs generated for Point Sets of different sizes.


Optimal MINVPs generated by RAA_MINVP algorithm for point sets of size 5. The results are verified using brute force approach.

Future Work

•To address the following questions:

- What are the performance guarantees of both the algorithms?
- Does there exist an input configuration for which the approach fails for every possible ordering of points?

Suggestions/Comments?-please mail to- emry01@gmail.com, jijupnair2000@yahoo.co.in