Shape Reconstruction in 2D: From Theory to Practice

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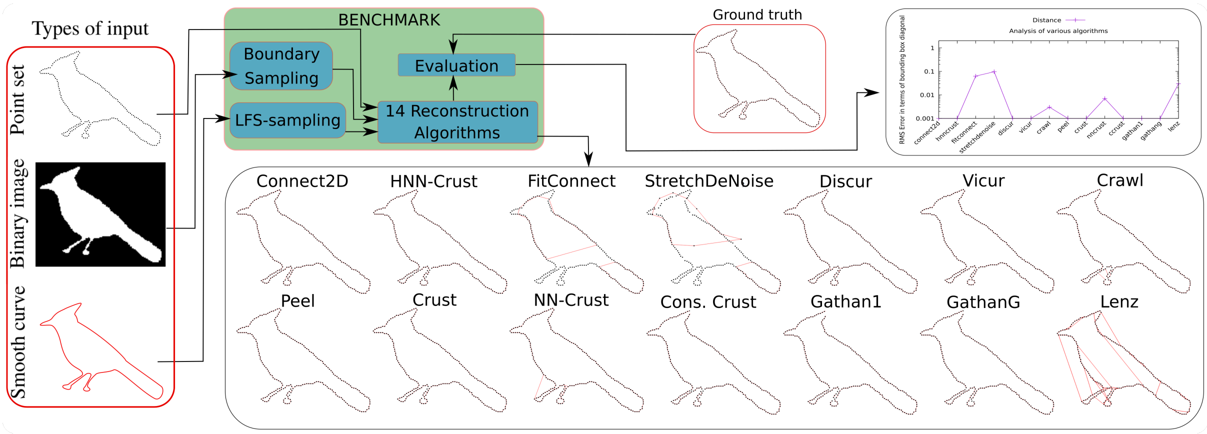

Figure 1: In this tutorial, we present a large number of curve reconstruction algorithms and compare 14 of these with quantitative and qualitative analysis. As inputs, we take unorganized points, samples on the boundary of binary images or smooth curves, and evaluate with ground truth, plus area samples.

Abstract

Shape reconstruction from unstructured points in a plane is a fundamental problem with many applications that has generated research interest for decades. Involved aspects like handling open, sharp, multiple and non-manifold outlines, run-time and provability as well as potential extension to 3D for surface reconstruction have led to many different algorithms. This multitude of reconstruction methods with quite different strengths and focus makes it a difficult task for users to choose a suitable algorithm for their specific problem. In this tutorial, we present the development history of algorithms, together with their related proximity graphs, all in detail. Then, we show algorithms targeted at specific problem classes, such as reconstructing from noise, outliers, or sharp corners. We will also include the latest developments in the field, namely based on Voronoi balls and the sphere-of-influence graph. Examples of the evaluation will show how its results can guide users to select an appropriate algorithm for their input data. We will also explain how to integrate new algorithms into our benchmark framework. Region reconstruction will be shown as an additional field closely related to boundary reconstruction.

CCS Concepts

• ***Computing methodologies*** → ***Point-based models; Mesh geometry models;***

1. Introduction

Reconstruction of curves from unorganized points is a fundamental task in computer graphics/vision, with many applications such as silhouette or slice reconstruction of 3D models in reverse engineering, connecting feature points in medical imaging or facial recognition, all of these possessing varying artifacts and requirements.

In this tutorial we aim to tell the story of the development of curve reconstruction algorithms, based on proximity graphs, further specialized algorithms, and our taxonomy.

We show the evaluation of the different methods based on input and output criteria helps users to select a suitable algorithm for specific problems, and we guide users through the process of inte-

Session title	Presenter	Duration
Intro & Proximity graphs	Stefan Ohrhallinger	0:00 - 0:25
History of algorithms	Stefan Ohrhallinger	0:25 - 0:50
Questions & Answers	*Stefan Ohrhallinger*	*0:50 - 0:55*
Specialized algorithms	Amal Dev Parakkat	0:55 - 1:20
Break: 15 Minutes		*1:20 - 1:35*
Benchmark & Demo	Amal Dev Parakkat	1:35 - 2:00
Questions & Answers	*Amal Dev Parakkat*	*2:00 - 2:05*
HVS-based algorithms	Jiju Peethambaran	2:05 - 2:30
Region reconstruction	Jiju Peethambaran	2:30 - 2:55
Questions & Answers	*Jiju Peethambaran*	*2:55 - 3:00*

Table 1: Sessions with title, presenter and duration.

grating new algorithms into our open source benchmark framework using a comprehensive test data set (`https://gitlab.com/stefango74/curve-benchmark`).

2. Necessary Background and Target Audience

We expect the members of the audience to have a basic background in computer graphics, algorithm analysis and programming. Familiarity with the fundamental concepts in computational geometry and topology will be an advantage.

This course is suitable for graduate students, who want to get an overview of the shape reconstruction field and identify open problems and conduct follow-up research. This course will also be appreciated by computational geometers who want to learn more about the theory and practicalities of shape reconstruction. The course will also cater to graphics developers and non-graphics researchers who work on applications that demand shape or boundary reconstruction but are less familiar with this field.

3. Previous Offerings

This tutorial has not been offered previously. However, the tutorial is heavily based on a recent STAR paper titled "2D Points Curve Reconstruction Survey and Benchmark" [OPP*21] presented at Eurographics 2021, see the link `https://www.youtube.com/watch?v=0lPybx2418s` for the video presentation. Compared to the EG 21 STAR presentation, we have added a few additional topics, e.g., region reconstruction in 2D, more details on how the specific algorithms work, three recently developed algorithms, and details on the proximity graphs underlying some of the algorithms.

4. Tutorial Outline

4.1. Intro & Proximity Graphs

We will show some motivating examples with their varying requirements. These requirements result in specific challenges of input data such as sampling density, noise or outliers, but also intersections, and open- or closedness. We show why these configurations cannot be handled well by any single algorithm, rather that there exist specific tools addressing all of these challenges. Helping users to choose a suitable algorithm for their problem also depends on desired output properties such as manifoldness, open curves, sharp corners, guarantees on time and sampling conditions. We will list 14 curve reconstruction algorithms available as open source that were evaluated thoroughly in our benchmark to highlight their respective strengths and weaknesses. We categorize a total of 36 curve reconstruction algorithms and relate their historical development, in seven categories as follows (some examples referenced):

- Graph-based algorithms [EKS83, PM15a, PM16]
- Algorithms relying on feature size criteria [ABK98, Rup93, DK99]
- Curve fitting based algorithms for noisy points [Lee00, OW18a, OW18b]
- Algorithms designed for handling sharp corners [DW02]
- Traveling salesman based methods [Gie99, AMS00]
- Algorithms that can handle self-intersections [DGCSAD11, PMM18]
- Algorithms based on human visual system [NZ08]

Additionally to the content in the STAR, we add three new algorithms, plus region reconstruction algorithms. Finally, we explain basic definitions on curves, and sampling.

Proximity graphs are a basis for many reconstruction algorithms, most of them form a subset of the Delaunay triangulation. We will show how *Euclidean Minimum Spanning Tree* (EMST), *Relative Neighborhood Graph* (RNG), *Gabriel Graph* (GG) relate to each other and to the *Delaunay Triangulation* (DT). Additionally, we show the *Shape-Hull Graph* and the *Minimum Boundary Complex*. In addition to the content in the STAR, we show which reconstruction algorithms use them, e.g., as an initial guess, how they can be generalized, plus the *Sphere-of-Influence Graph* which is used for a novel reconstruction algorithm.

4.2. History of Algorithms

We explain the concepts of the following historical development and their relations among themselves and to proximity graphs:

- α-shapes [EKS83] as a generalization of the convex hull
- The ball-pivoting algorithm [BMR*99] based on α-shapes
- The β-skeleton [KR85] based on a proximity graph
- The γ-neighborhood graph [Vel92] unifying several graphs
- A Voronoi-based minimum tree length algorithm [OBW87]
- EMST-based proved reconstruction of open curves [FMG94]
- r-regular shapes proved and based on curvature [Att97]
- EMST-based reconstruction for noisy and sparse points [OM11]
- Proved inflating/sculpting based on a proximity graph [OM13]
- Shape-hull graph based curve reconstruction [PM15b]
- Incremental Voronoi pole classification [PPT*19]
- *Crawl* iteratively adding shortest edges [PM16]
- *Peeling* longest edges [PMM18] on the DT
- *Crust* [ABE98] filters the DT with guarantees
- *Anti-Crust* [Gol99] improves it to a single step
- *NN-Crust* [DK99] relaxes its sampling condition
- *Conservative Crust* [DMR99] filters the GG
- Probing with a tear shape [Len06] for self-intersections
- Traveling Salesman for multiple curves [Hiy09]
- *HNN-Crust* as a simple, proved with best guarantee [OMW16]
- Filtering DT based on Voronoi ball configuration

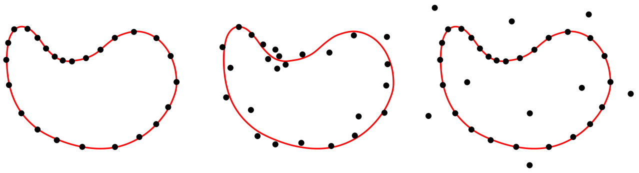

Figure 2: Sample points (left) with noise (middle) and outliers (right)

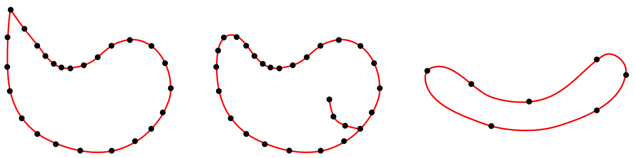

Figure 3: Curve with sharp corner (left), non-manifold curve (middle), sparsely sampled curve (right)

4.3. Specialized Algorithms

Though the simple reconstruction problem itself is difficult enough, the issues brought in by the inputs/sampling make it even more challenging. A few of such important challenges include:

- Intrinsic curve properties: Sometimes the intrinsic properties of the curve, such as self-intersections (or non-manifold curves) and sharp corners, lead to additional issues. These issues are mainly because of the inability of the well-known ε-sampling model (to which many of the algorithms relate) to capture such properties.
- Artifacts accompanied by sampling: Based from the errors induced by sensors in the 3D counterpart of the curve reconstruction problem (namely surface reconstruction), the input samples can deviate from their expected positions leading to so-called outliers and noise. This means that instead of having a set of good samples lying exactly on the curve, we have distorted data with some unwanted additional points (called outliers) or displaced samples (called noise). Such noise or outliers make many classical algorithms fail and hence expedited researchers to develop specialized algorithms. Another related challenge is identifying the underlying curve from as few as possible sample points. This can be considered an aftereffect of missing data or local-feature-independent sparse sampling.

Figures 2 and 3 show sample cases depicting a few of these challenges.

To evaluate the efficiency of various algorithms on different kinds of inputs, we created a large dataset containing inputs having multiple features. This dataset includes classical data extracted from various papers, samples extracted from silhouette images, and manually (or by using a curve sampler) generated synthetic data (a few representative data is shown in Figure 4).

4.4. Benchmark & Demo

The benchmark contains 14 algorithms (CRUST [ABE98], NNCRUST [DK99], CCRUST [DMR99], GATHAN [DW01], GATHANG [DW02], LENZ [Len06], CONNECT2D [OM13], CRAWL [PM16], HNNCRUST [OMW16], FITCONNECT [OW18a], STRETCHDENOISE [OW18b], PEEL [PMM18], DISCUR [ZNYL08] and VICUR [NZ08]) and thousands of point sets (classic, image-based, and synthetically made) along with ground truth information.

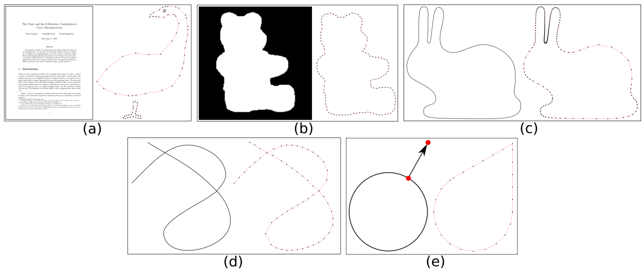

Figure 4: Examples of different types of test data. (a) Classical data collected from different papers, (b) Points sampled from a binary image boundary, (c) LFS-sampling from a cubic Bézier curve, (d) Points sampled from a synthetic curve, (e) Synthetic data generated by extruding sharp corners from circles.

To allow the users to create samples from new inputs, we provide sampling tools to generate samples from B'ezier curves and images. Furthermore, the benchmark includes an interactive interface that allows users to input the unknown ground truth.

Together with these 14 algorithms, different data sets and sampling tools, our benchmark also contains a set of test scripts that facilitates quantitative and qualitative evaluation of various curve reconstruction algorithms. These test scripts allow the users to specify the algorithms and data sets (on which they must be evaluated), and the final evaluated data is written as graphs for easy appraisal.

For quickly evaluating different feature specific aspects of an algorithm, we classified the inputs and appropriately included them as separate scripts, namely:

- run-sampling.sh: ε-sampled [ABK98] test data
- run-noisy.sh: perturbed with uniform noise
- run-lfsnoise.sh: perturbed with lfs-based noise
- run-outliers.sh: added outlier points
- run-manifold.sh: whether reconstruction is a manifold
- run-sharp-corners.sh: sharp feature curves
- run-open-curves.sh: open curves
- run-multiple-curves.sh: multiply connected curves
- run-intersecting.sh: curves with intersections

These scripts not only write the results into image files for easy visual inspection but also evaluate how good they approximate the ground truth (expressed in terms of RMSE error as defined in [OPP*21]). A set of sample results generated using our benchmark can be seen in Figures 5 and 6.

4.5. Visual Perception of Shapes

A few shape reconstruction algorithms rely on a subset of Gestalt laws of perception which describe how humans perceive visual elements. In this section, we will present various algorithms designed based on Gestalt laws of proximity, continuation and closure. Example algorithms include DISCUR [ZNYL08], VICUR [NZ08] and

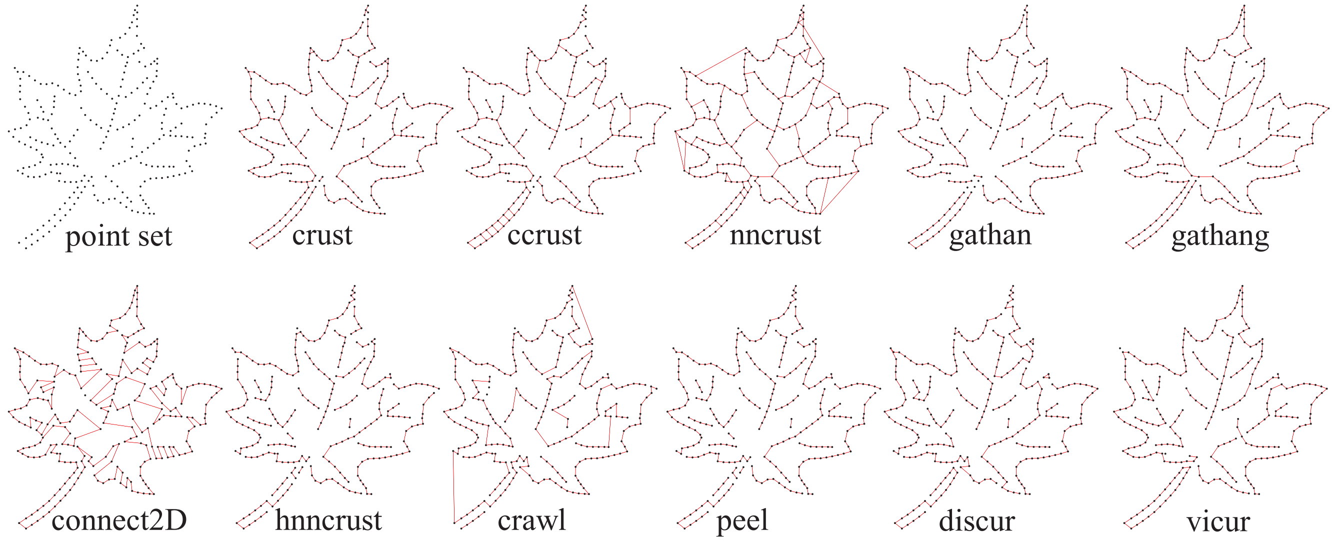

Figure 5: A sample qualitative comparison of different algorithms on the leaf input that contains sharp-corners, non-manifold edges, multiple and open curves created by our scripts.

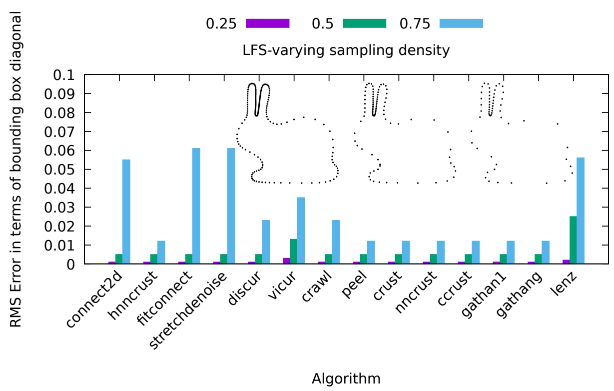

Figure 6: A sample RMS Error graph of reconstructed curves from ground truth for a cubic Bézier curve sampled with = 0.25, 0.5 and 0.75 generated by run-sampling.sh script. The point sets sampled from the bunny curve are shown in the figure.

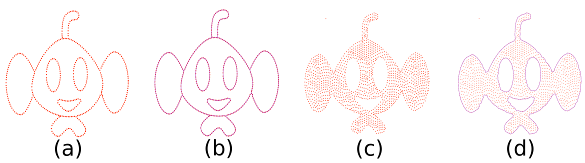

Figure 7: Two common types of inputs to the 2D reconstruction algorithms. (a) Boundary sample (b) Reconstructed curves (c) Area sample (d) Reconstruction from area samples.

Connect2D [OM13]. The discussion will also include the region reconstruction algorithms based on Gestalt laws. Region reconstruction is the problem of constructing the polygonal boundary of a set of points distributed over a 2D region or sampled from a 2D object, commonly referred to as *area samples*. Figure 7 illustrates the distinction between *boundary samples* and *area samples* and the corresponding reconstruction results. We will provide an introduction to the region reconstruction problem along with the motivations and challenges and how the solutions to this problem in general utilize Gestalt laws of visual perception.

4.6. Region Reconstruction in 2D

In this section, the discussion will focus on various sampling models, e.g., r-regular sampling, and important theoretical results on region reconstruction. We will also present the unified algorithms [MPM15, DKWG08, GDJ*11, TPM20, TPM21] that handle sampled boundaries, as well as areas. Since there is a considerable body of literature on reconstruction from area samples, we will provide only the most important algorithms and results in this section. A discussion on various sampling tools for curve and region reconstruction, different metrics used for evaluating the shape reconstruction algorithms and available datasets [OPP*21] will also be provided. The section will conclude with a discussion on future directions (including [PMC19]), presenting the open problems around 2D shape reconstruction. Though this is a mature field, we still consider some directions to be worthwhile for future work, mostly building onto the fundamental results published so far. Some potential topics include sampling conditions for non-smooth and self-intersecting curves, deep learning for curve generation, 3D curve reconstruction and curves from hand-drawn sketches.

5. Presenters

Stefan Ohrhallinger is a Postdoc researcher at the Institute of Visual Computing and Human- Centered Technology at TU Wien, Austria. In 2013 he obtained his PhD from Concordia University, Montréal, Canada. Since October 2012 he is a research associate at TU Wien, working mainly on surface re-

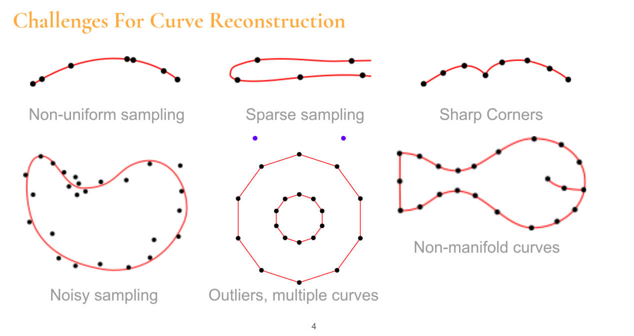

Figure 8: Course Notes: Session "Intro & Proximity graphs", Basics.

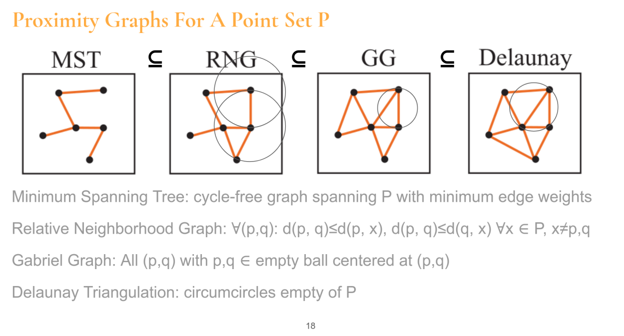

Figure 9: Course Notes: Session "Intro & Proximity graphs", Relationships.

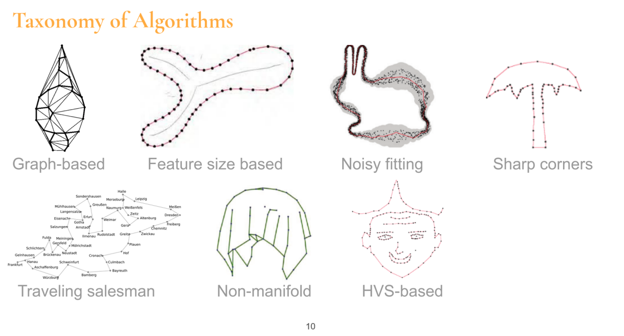

Figure 10: Course Notes: Session "History of algorithms", Taxonomy.

construction, geometry processing and point-based graphics. He has first-authored six peer-reviewed papers in the domain of curve and surface reconstruction. Website: `https://www.cg.tuwien.ac.at/staff/StefanOhrhallinger.html` E-Mail: ohrhallinger@cg.tuwien.ac.at

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Jiju Peethambaran is currently an assistant professor in the Department of Math and Computing Science, Saint Mary's University, Halifax, Canada. He received his Ph.D. degree in computational geometry from Indian Institute of Technology Madras, Chennai, India. He has held postdoc positions at University of Victoria, Canada and University of Calgary, Canada. He has (co)authored peer reviewed articles on/related to shape and curve reconstruction from 2D/ 3D point sets and its applications. His current research interests include Computer graphics, 3D computer vision, Geometric deep learning, and related applications including LiDAR-based modeling of real-world scenes. Website: `http://cs.smu.ca/~jiju` E-Mail: jiju.poovvancheri@smu.ca

6. Sample Course Notes[Optional]

6.1. Intro & Proximity graphs

See Figures 8 and 9 for example slides.

6.2. Explicit Reconstruction

See Figure 10 for an example slide.

6.3. Visual Perception of Shapes

See Figure 11 for an example slide.

6.4. Benchmark

See Figure 12-13 for example slides.

References

[ABE98] AMENTA N., BERN M. W., EPPSTEIN D.: The crust and the beta-skeleton: Combinatorial curve reconstruction. *Graphical Models and Image Processing 60*, 2 (1998), 125–135. 2, 3

[ABK98] AMENTA N., BERN M., KAMVYSSELIS M.: A new voronoi-based surface reconstruction algorithm. In *Proceedings of the 25th annual conference on Computer graphics and interactive techniques* (1998), ACM, pp. 415–421. 2, 3

[AMS00] ALTHAUS E., MEHLHORN K., SCHIRRA S.: Experiments on curve reconstruction. In *Proc. 2nd Workshop Algorithm Eng. Exper* (2000), pp. 103–114. 2

[Att97] ATTALI D.: r-regular shape reconstruction from unorganized points. In *Symp. on Computational Geometry* (1997), pp. 248–253. 2

[BMR*99] BERNARDINI F., MITTLEMAN J., RUSHMEIER H., SILVA C., TAUBIN G.: The ball-pivoting algorithm for surface reconstruction. *IEEE transactions on visualization and computer graphics 5*, 4 (1999), 349–359. 2

[DGCSAD11] DE GOES F., COHEN-STEINER D., ALLIEZ P., DESBRUN M.: An optimal transport approach to robust reconstruction and simplification of 2d shapes. In *Computer Graphics Forum* (2011), vol. 30, Wiley Online Library, pp. 1593–1602. 2

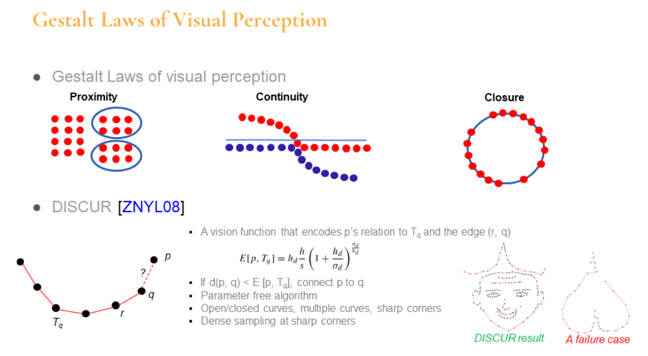

Figure 11: Course Notes: Session "Visual Perception of Shapes", Gestalt Laws for Shape Perception.

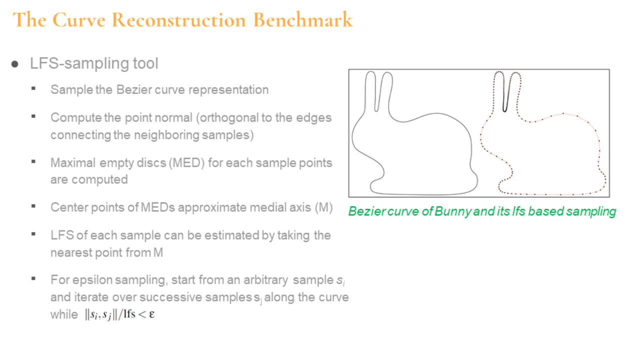

Figure 13: Course Notes: Session "Region Reconstruction in 2D", Local Feature Size based sampler.

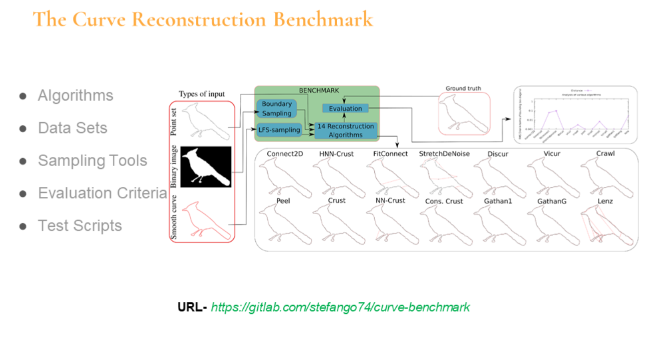

Figure 12: Course Notes: Session "Benchmark & Demo", Overview of the curve reconstruction benchmark.

[DK99] Dey T. K., Kumar P.: A simple provable algorithm for curve reconstruction. In *Proc. 10th ACM-SIAM SODA '99* (1999), pp. 893–894. 2, 3

[DKWG08] Duckham M., Kulik L., Worboys M., Galton A.: Efficient generation of simple polygons for characterizing the shape of a set of points in the plane. *Pattern Recognition 41*, 10 (2008), 3224 – 3236. 4

[DMR99] Dey T. K., Mehlhorn K., Ramos E. A.: Curve reconstruction: Connecting dots with good reason. *In Proc. 15th ACM Symp. Comp. Geom 15* (1999), 229–244. 2, 3

[DW01] Dey T. K., Wenger R.: Reconstructing curves with sharp corners. *Computational Geometry 19*, 2-3 (2001), 89 – 99. 3

[DW02] Dey T. K., Wenger R.: Fast reconstruction of curves with sharp corners. *Int. J. Comp. Geom. Appl. 12*, 5 (2002), 353 – 400. 2, 3

[EKS83] Edelsbrunner H., Kirkpatrick D. G., Seidel R.: On the shape of a set of points in the plane. *IEEE Trans. Inf. Theor. IT-29*, 4 (1983), 551–559. 2

[FMG94] Figueiredo L. H. d., Mirandas Gomes J. d.: Computational morphology of curves. *Vis. Comp. 11*, 2 (1994), 105–112. 2

[GDJ*11] Gheibi A., Davoodi M., Javad A., Panahi F., Aghdam M., Asgaripour M., Mohades A.: Polygonal shape reconstruction in the plane. *Computer Vision, IET 5*, 2 (March 2011), 97 –106. 4

[Gie99] Giesen J.: Curve reconstruction in arbitrary dimension and the traveling salesman problem. In *Proc. 8th DCGI '99* (1999), pp. 164–176. 2

[Gol99] Gold C.: Crust and anti-crust: a one-step boundary and skeleton extraction algorithm. In *Proc. of the 15th ann. Symp. on Computational geometry* (New York, NY, USA, 1999), SCG '99, ACM, pp. 189–196. 2

[Hiy09] Hiyoshi H.: Optimization-based approach for curve and surface reconstruction. *Comput. Aided Des. 41* (May 2009), 366–374. 2

[KR85] Kirkpatrick D. G., Radke J. D.: A framework for computational morphology. *Computational Geometry* (1985), 217–248. 2

[Lee00] Lee I.-K.: Curve reconstruction from unorganized points. *Computer Aided Geometric Design 17*, 2 (2000), 161 – 177. 2

[Len06] Lenz T.: How to sample and reconstruct curves with unusual features. In *Proceedings of the 22nd European Workshop on Computational Geometry (EWCG)* (Delphi, Greece, March 2006). 2, 3

[MPM15] Methirumangalath S., Parakkat A. D., Muthuganapathy R.: A unified approach towards reconstruction of a planar point set. *Computers & Graphics 51* (2015), 90 – 97. International Conference Shape Modeling International. 4

[NZ08] Nguyen T. A., Zeng Y.: Vicur: A human-vision-based algorithm for curve reconstruction. *Robotics and Computer-Integrated Manufacturing 24*, 6 (2008), 824 – 834. FAIM 2007, 17th International Conference on Flexible Automation and Intelligent Manufacturing. 2, 3

[OBW87] O'Rourke J., Booth H., Washington R.: Connect-the-dots: a new heuristic. *Computer Vision, Graphics, and Image Processing 39*, 2 (1987), 258–266. 2

[OM11] Ohrhallinger S., Mudur S. P.: Interpolating an unorganized 2d point cloud with a single closed shape. *Computer-Aided Design 43*, 12 (2011), 1629–1638. 2

[OM13] Ohrhallinger S., Mudur S.: An efficient algorithm for determining an aesthetic shape connecting unorganized 2d points. In *Comp. Graph. Forum* (2013), vol. 32, Wiley Online Library, pp. 72–88. 2, 3, 4

[OMW16] Ohrhallinger S., Mitchell S. A., Wimmer M.: Curve reconstruction with many fewer samples. In *Computer Graphics Forum* (2016), vol. 35, Wiley Online Library, pp. 167–176. 2, 3

[OPP*21] Ohrhallinger S., Peethambaran J., Parakkat A. D., Dey T. K., Muthuganapathy R.: 2d points curve reconstruction survey and benchmark. *Computer Graphics Forum 40*, 2 (2021), 611–632. 2, 3, 4

[OW18a] Ohrhallinger S., Wimmer M.: Fitconnect: Connecting noisy 2d samples by fitted neighbourhoods. In *Computer Graphics Forum* (2018), Wiley Online Library. 2, 3

[OW18b] Ohrhallinger S., Wimmer M.: Stretchdenoise: Parametric curve reconstruction with guarantees by separating connectivity from residual uncertainty of samples. In *Pacific Graphics* (2018), Wiley Online Library. 2, 3

[PM15a] PEETHAMBARAN J., MUTHUGANAPATHY R.: A non-parametric approach to shape reconstruction from planar point sets through delaunay filtering. *Computer-Aided Design 62* (2015), 164 – 175. 2

[PM15b] PEETHAMBARAN J., MUTHUGANAPATHY R.: Reconstruction of water-tight surfaces through delaunay sculpting. *Computer-Aided Design 58* (2015), 62 – 72. Solid and Physical Modeling 2014. 2

[PM16] PARAKKAT A. D., MUTHUGANAPATHY R.: Crawl through neighbors: A simple curve reconstruction algorithm. In *Computer Graphics Forum* (2016), vol. 35, Wiley Onl. Library, pp. 177–186. 2, 3

[PMC19] PARAKKAT A. D., MEMARI P., CANI M.-P.: Layered reconstruction of stippling art. In *ACM SIGGRAPH 2019 Posters* (New York, NY, USA, 2019), SIGGRAPH '19, Association for Computing Machinery. 4

[PMM18] PARAKKAT A. D., METHIRUMANGALATH S., MUTHUGANAPATHY R.: Peeling the longest: A simple generalized curve reconstruction algorithm. *Computers & Graphics 74* (2018), 191 – 201. 2, 3

[PPT*19] PEETHAMBARAN J., PARAKKAT A., TAGLIASACCHI A., WANG R., MUTHUGANAPATHY R.: Incremental labelling of voronoi vertices for shape reconstruction. *Computer Graphics Forum 38*, 1 (2019), 521–536. 2

[Rup93] RUPPERT J.: A new and simple algorithm for quality 2-dimensional mesh generation. In *Proceedings of the fourth annual ACM-SIAM Symposium on Discrete algorithms* (Philadelphia, PA, USA, 1993), SODA '93, Soc. for Industr. and Appl. Math., pp. 83–92. 2

[TPM20] THAYYIL S. B., PARAKKAT A. D., MUTHUGANAPATHY R.: An input-independent single pass algorithm for reconstruction from dot patterns and boundary samples. *Computer Aided Geometric Design 80* (2020), 101879. 4

[TPM21] THAYYIL S. B., PEETHAMBARAN J., MUTHUGANAPATHY R.: A sampling type discernment approach towards reconstruction of a point set in r2. *Computer Aided Geometric Design 84* (2021), 101953. 4

[Vel92] VELTKAMP R. C.: The γ-neighborhood graph. *Computational Geometry 1*, 4 (1992), 227–246. 2

[ZNYL08] ZENG Y., NGUYEN T. A., YAN B., LI S.: A distance-based parameter free algorithm for curve reconstruction. *Comput. Aided Des. 40*, 2 (2008), 210–222. 3