M2 internship project

**An optimality measure for nonconvex optimization based on the smoothed gap**

Supervision
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Description

The self-centered smoothed gap is a new measure of optimality [1] which helps detect early epsilon-solutions of convex contrained optimization problems. Indeed, its value is 0 if and only if we are evaluating it at (x*, y*), x* being an optimal solution and y* being a Lagrange multiplier.

Here, we would like to extend these results to non-convex problems with several open questions :
- If the functions are weakly convex, does there exist a smoothing parameter such that the smoothed duality gap at (x*, y*) is 0 if and only if x* is primal-optimal and y* is dual-optimal ?
- When there is no global saddle point, is z* a stationary point of the Lagrangian function if and only if the smoothed gap is minimal?
- More generally, what meaning can we give to the local minima of the smoothed gap in terms of the primal and dual problem?

The smoothed duality gap is strongly linked with the Augmented Lagrangian and this internship may help understand better duality in nonconvex optimization [2].

[1] Walwil, I., & Fercoq, O. (2024). The Smoothed Duality Gap as a Stopping Criterion. arXiv preprint arXiv:2403.12579.
[2] Yalcin, G. D., & Kasimbeyli, R. (2020). On weak conjugacy, augmented Lagrangians and duality in nonconvex optimization. *Mathematical Methods of Operations Research*, *92*(1), 199-228.

Candidate profile

• Student enrolled in a master 2 program in AI, Data Science or optimization
• Programming skills in Python
• Very good command of English or French

Application

Please send to the supervisor:
- Curriculum Vitae
- Personalized motivation letter that explains interest of the candidate in the subject (can be directly in the body of the email)
- Grade reports for recent years