Specifying Concurrent Problems: Beyond Linearizability and up to Tasks

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Joint work with

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Distributed computer scientists excel
at thinking **concurrently**,
and building large distributed systems

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and **building** large distributed systems

Yet, they evade thinking about concurrent problem **specifications**.

Weaver Ants Building Nest from Mango Leaves, Ubon Ratchathani, Thailand

It is infinitely easier and more intuitive for us humans to specify how abstract data structures behave in a sequential setting.

Nir Shavit, CACM 2011

An object

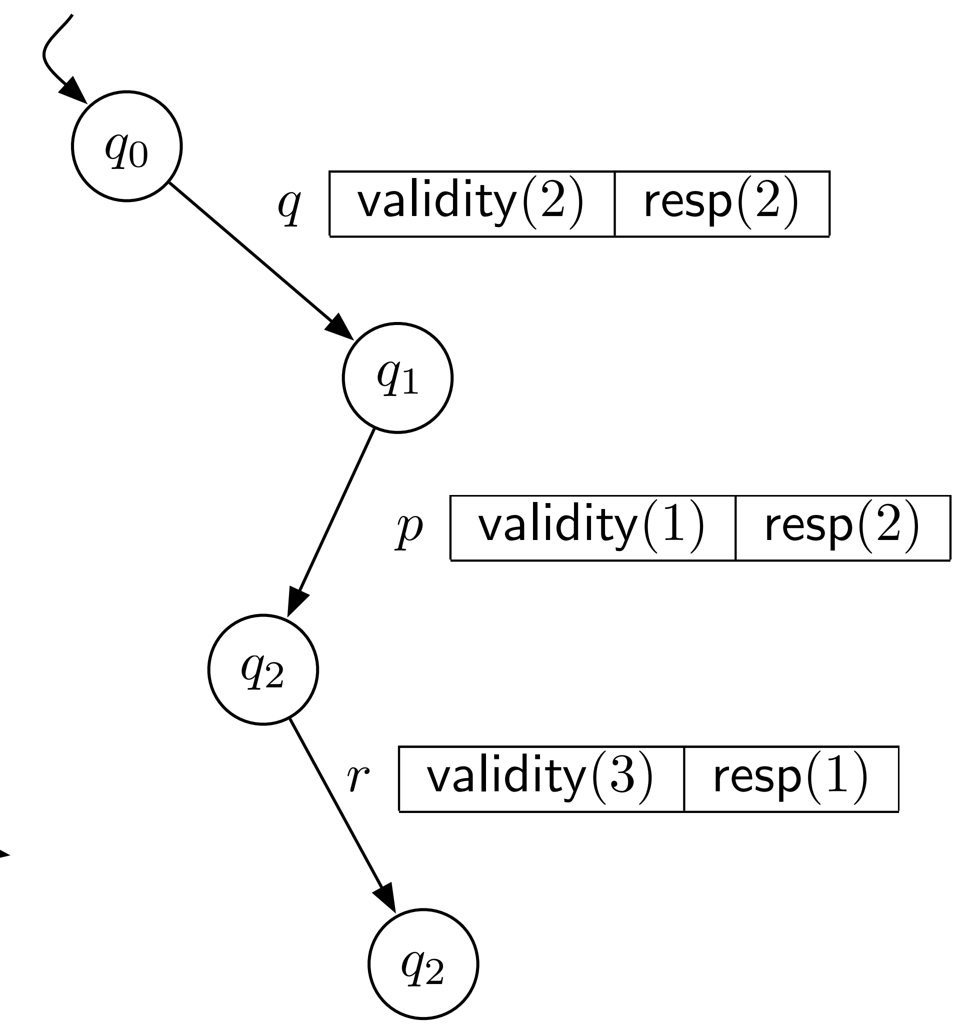
- A central paradigm
- The processes may access it concurrently but specified in terms of a sequential specification, namely…

An object

- an **automaton** describing the outputs the object produces when it is accessed sequentially.
  - Mealy state machine, with transitions of the form

$$\delta(q, in) = (q', r)$$

Example: validity



- Invocations propose input
- responses return values that have been proposed

Sequential specifications are convenient

- The paradigm of a sequentially specified object is very convenient:
    - It provides the notion of a state
    - Specification manual grows linearly with the number of operations

Is an implementation correct?

- Given that an object specifies its behaviour only in sequential executions,
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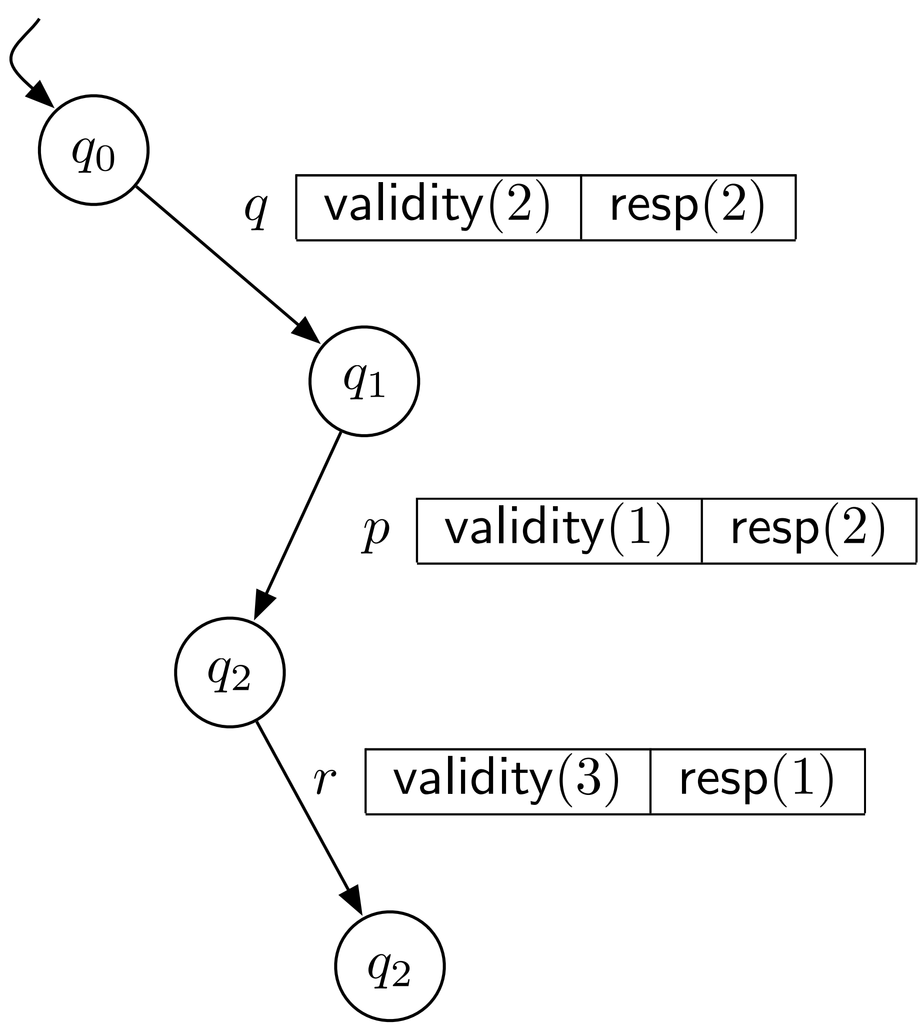

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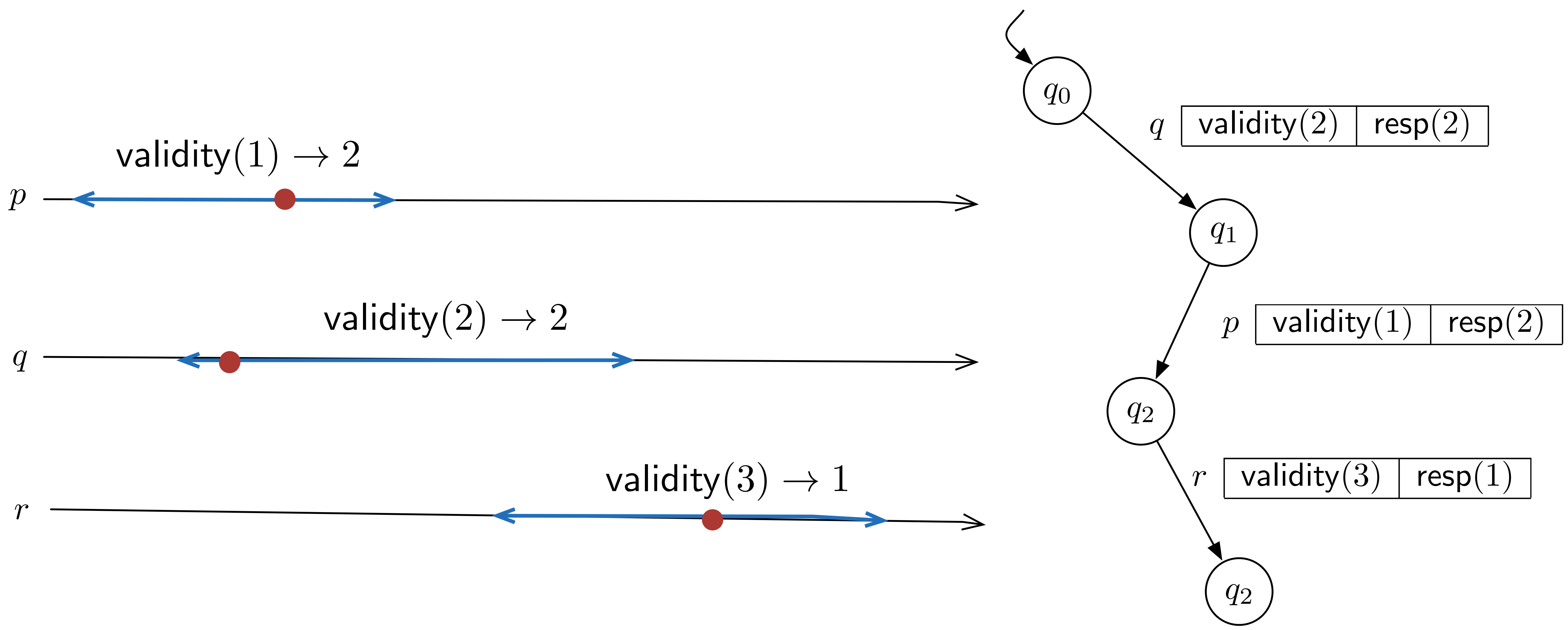
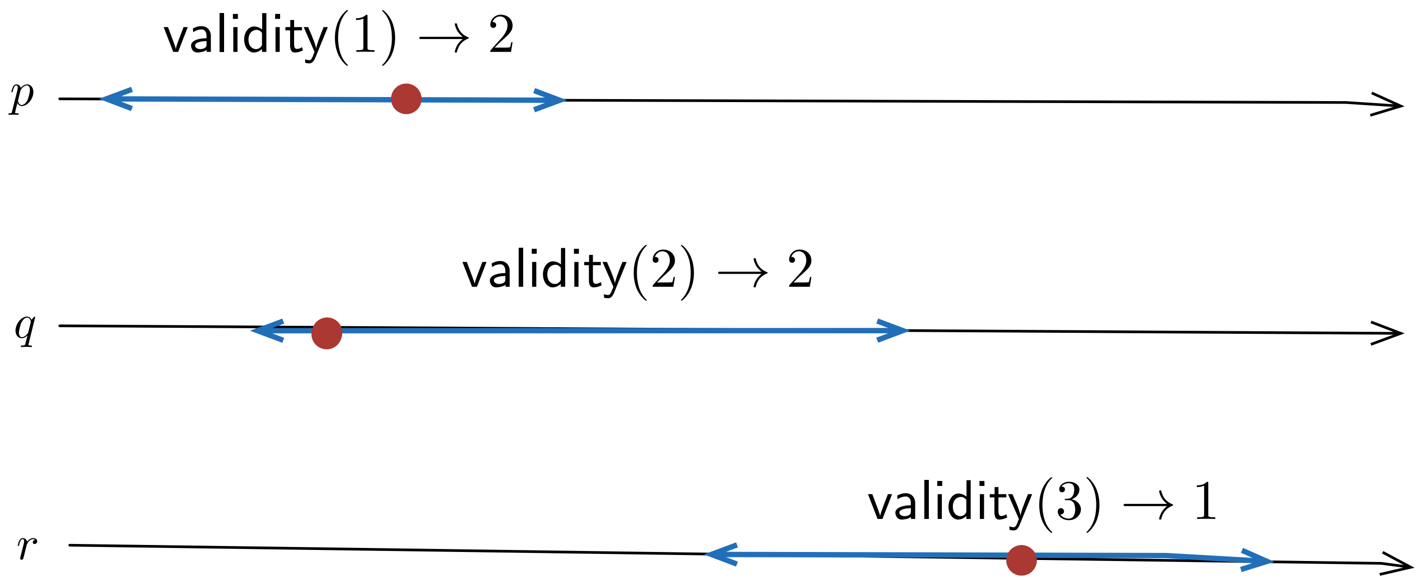
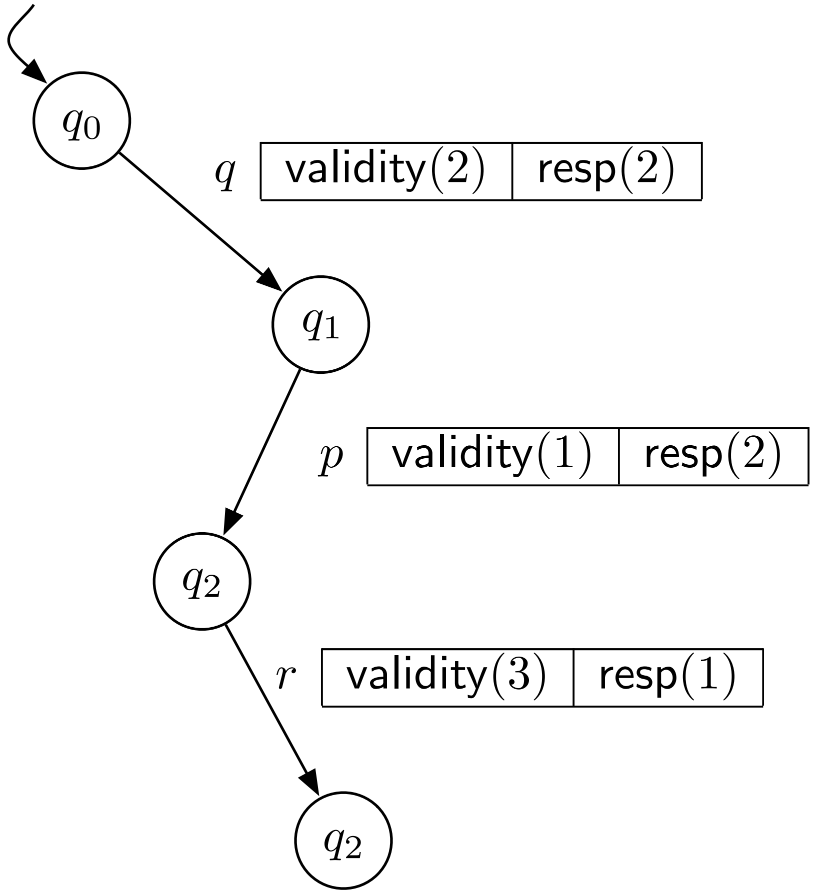


Linearizability

- Operations seem to occur at a point, in between invocation and response,
- i.e., they can be transformed to a valid sequential execution.





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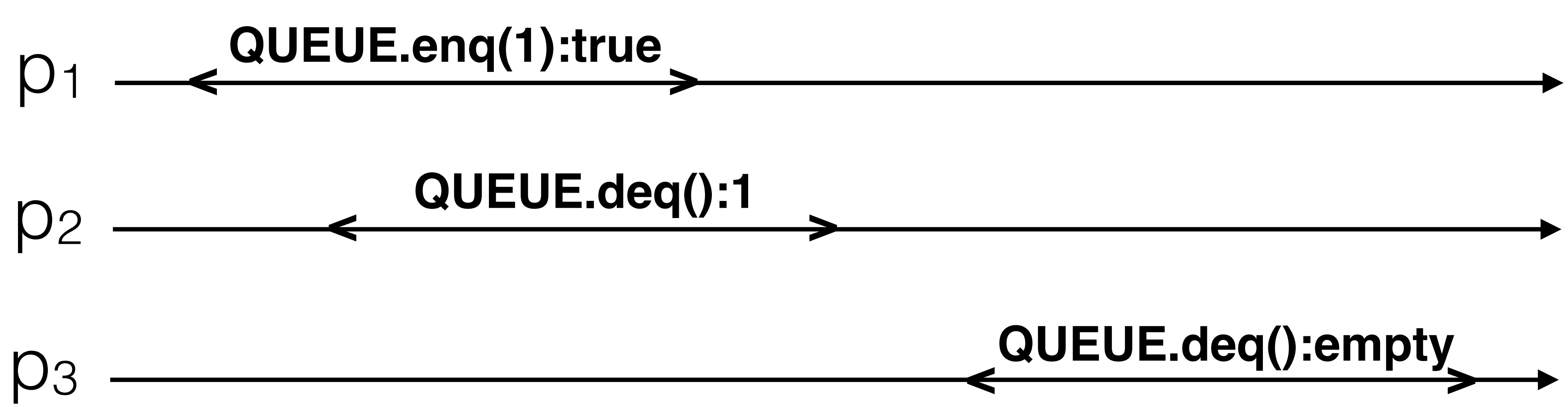

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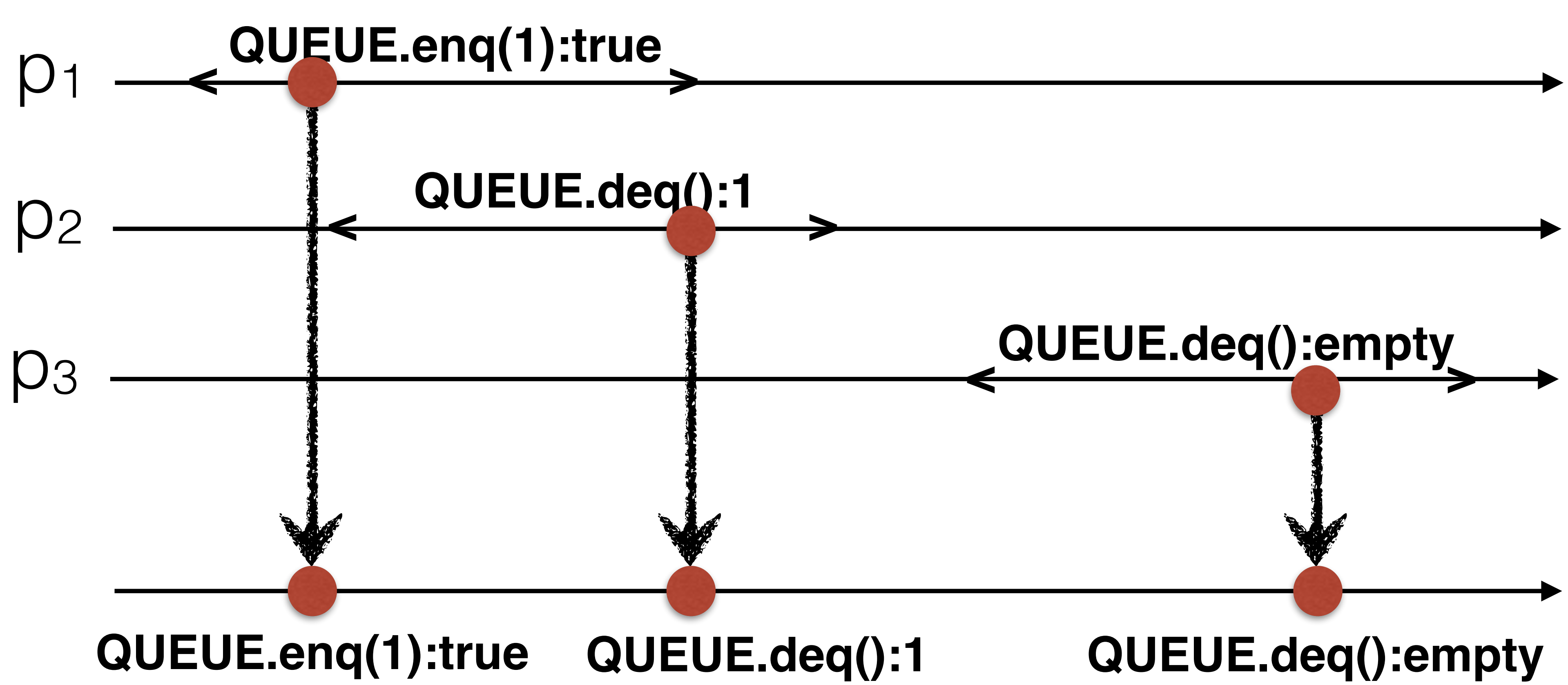

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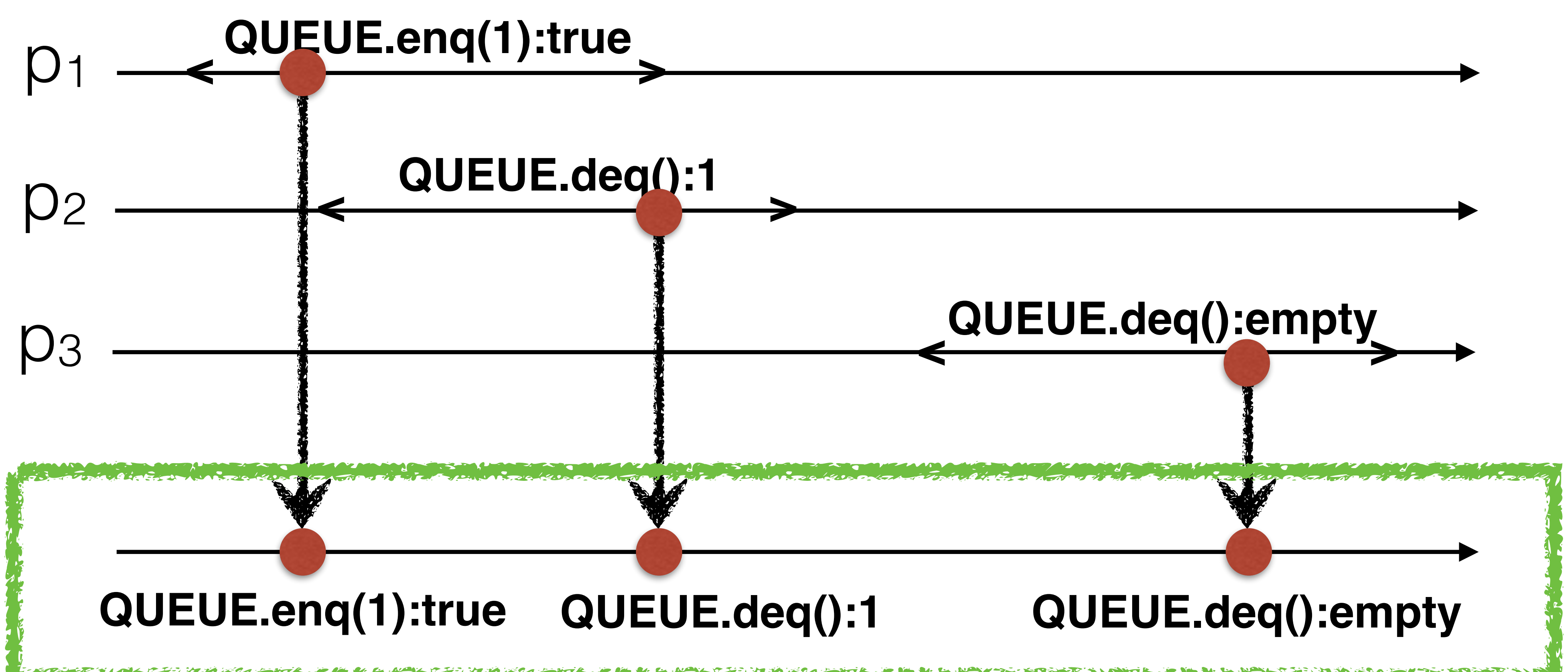
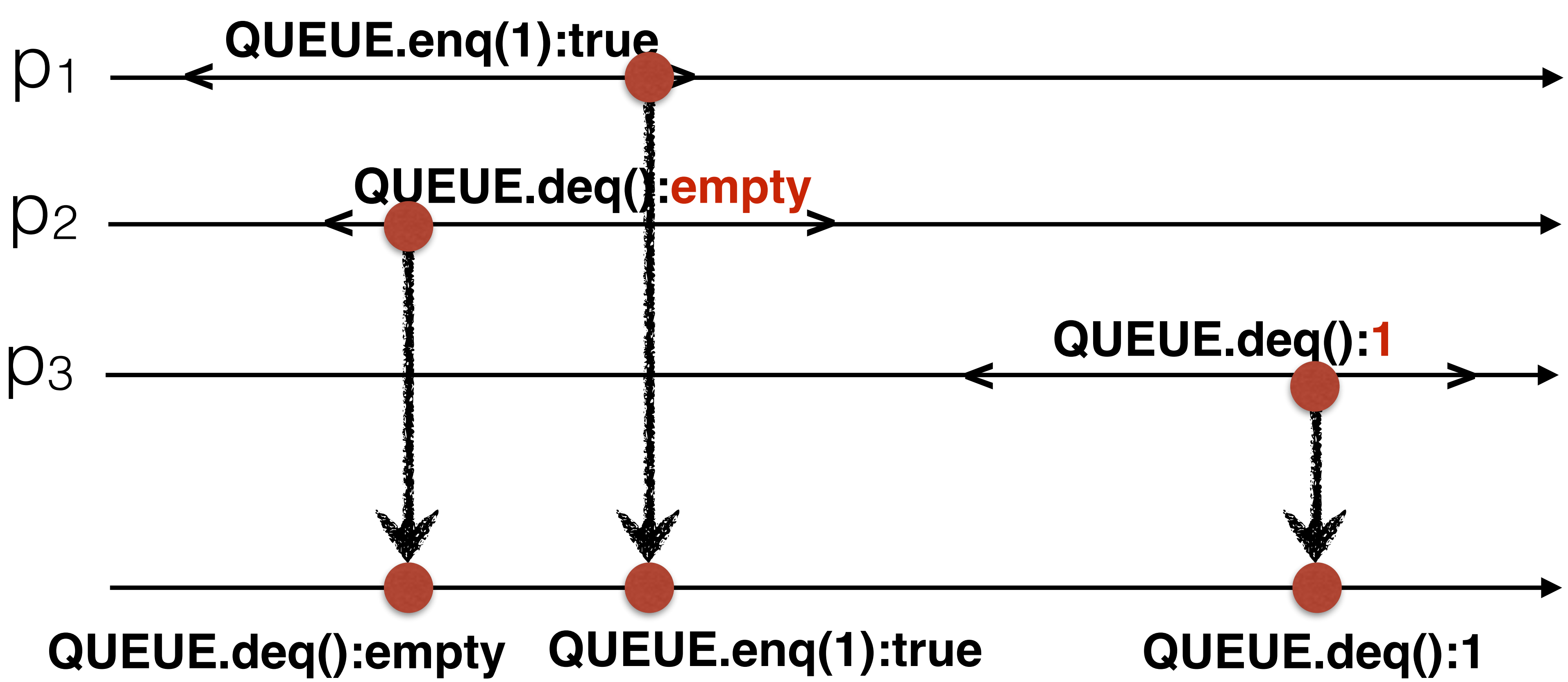


Queue

- Standard correctness criteria.
- **Linearizability:** Operations seem to occur sequentially, i.e., they can be **transform** **alid** sequential execution.



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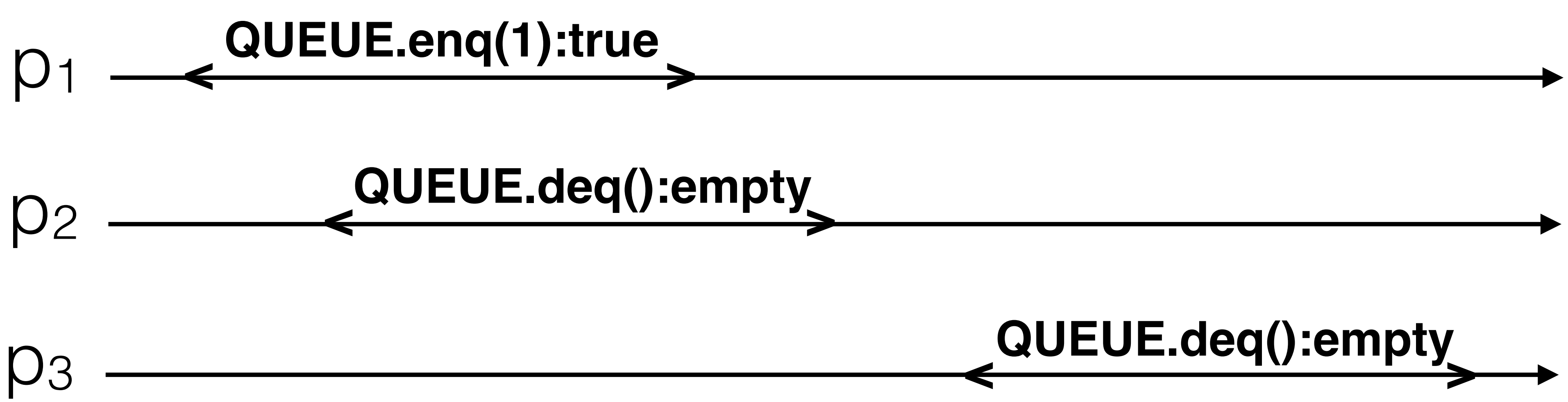


Importance of Linearizability

- Clear specifications. Easy to think sequentially.
- **Good properties** for the development of systems:

  Non-blocking: It never forces the system to block

  Locality: Modular approach. Linearizable implementations compose a linearizable system.

Importance of Linearizability

- Clear specifications. Easy to think sequentially.
- **Good properties** for the development of systems:

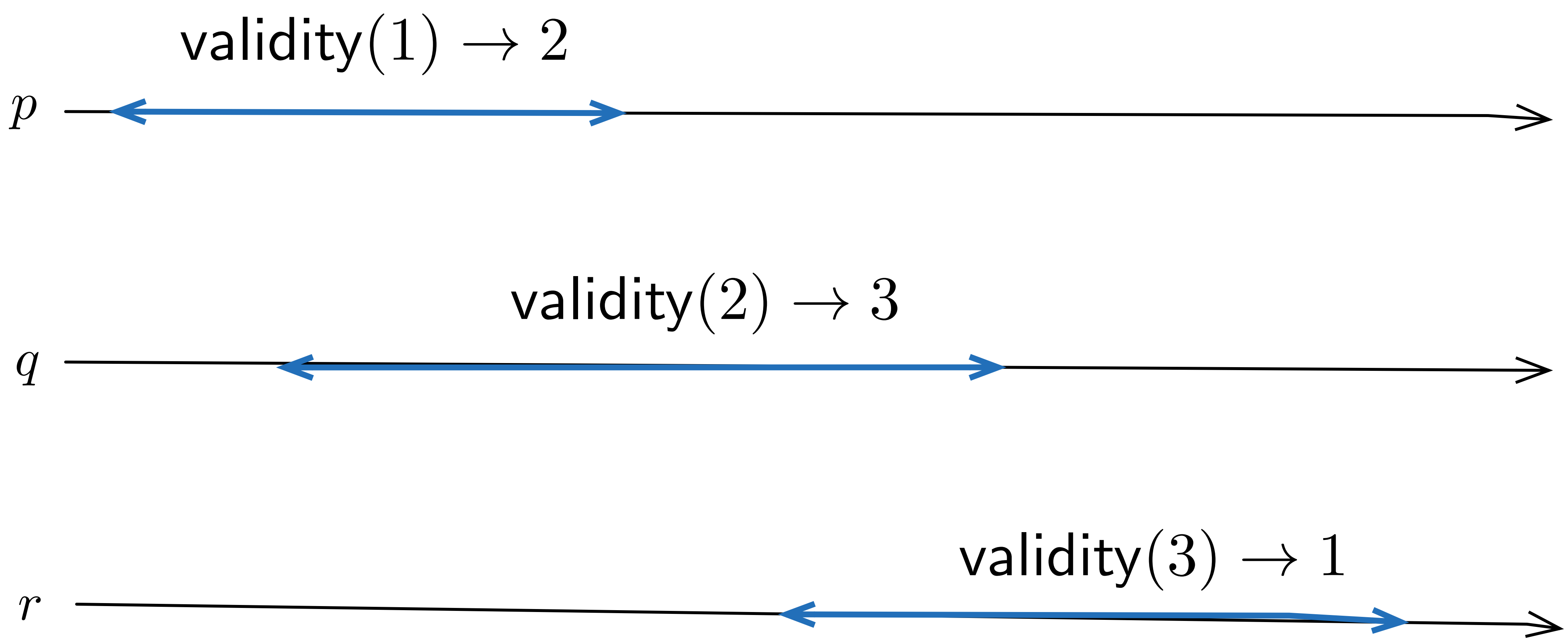
There are limitations!!

Locality: Modular approach. Linearizable implementations compose a linearizable system.

Distributed object

- Are all distributed problems objects?
- No!
- What *is* a distributed object?

Validity object



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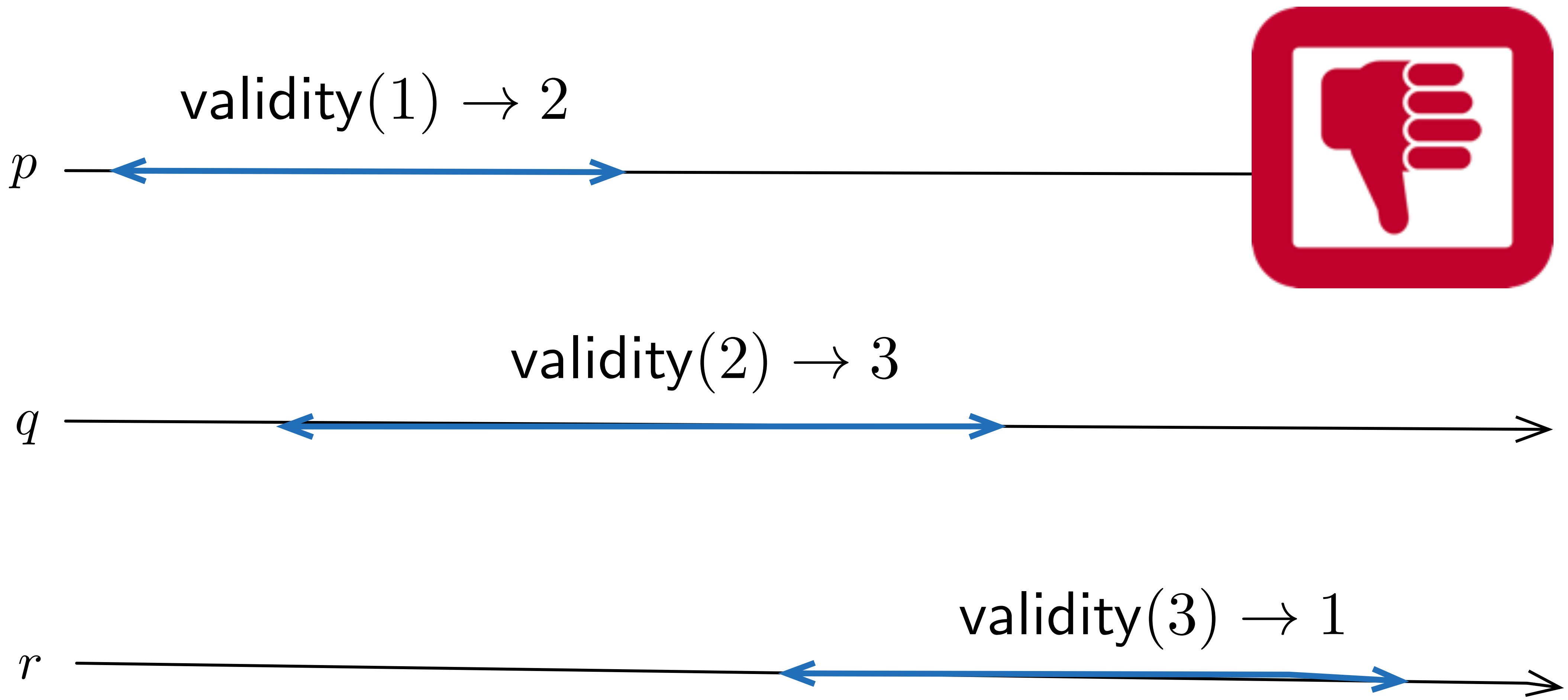

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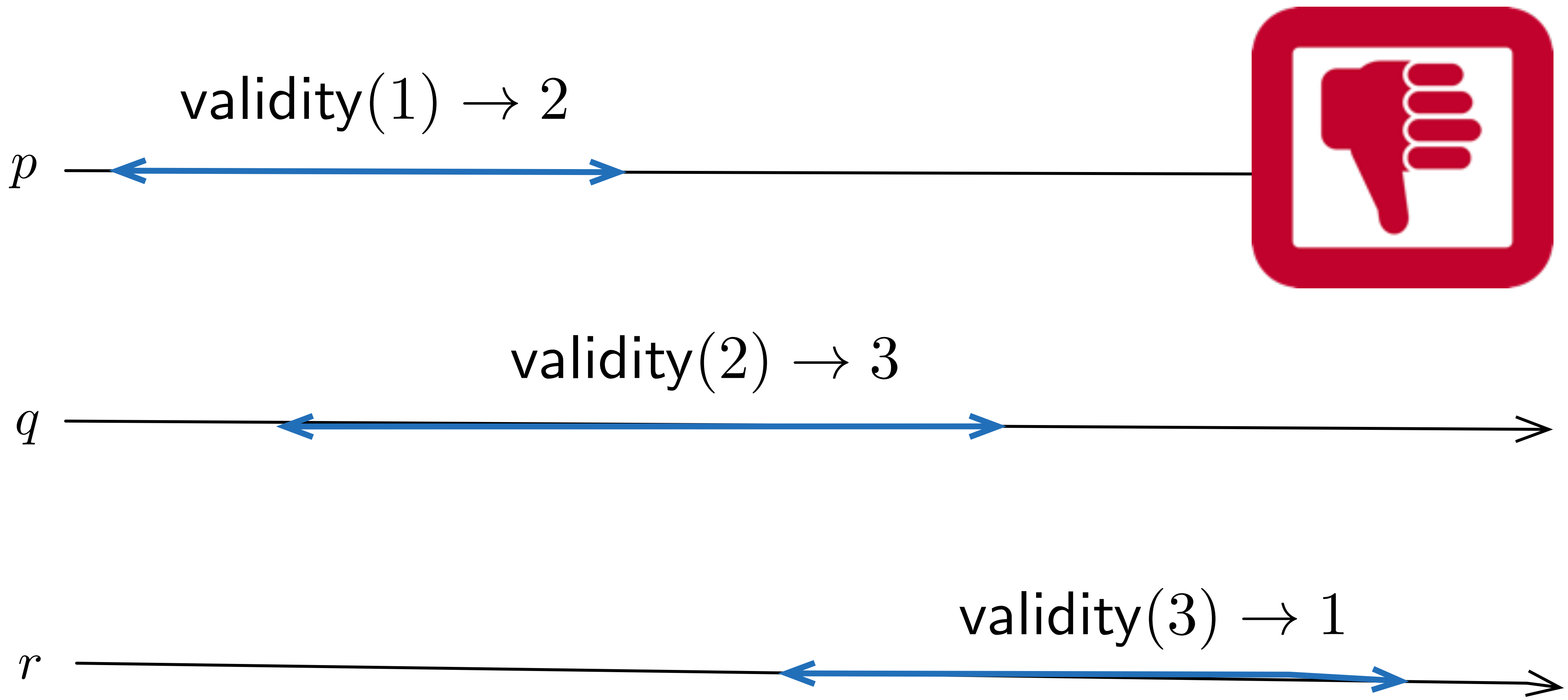

- Ther d on
  read

not linearizable

Snapshot Object

- Shared memory M; one entry per process

  write(i, v): atomically writes v in M[i]

  snapshot(M): takes an atomic snapshot of M

- Has a natural sequential specification

- Several **linearizable** implementations based on **read/write** primitives

Write-Snapshot Object

- In some applications a **snapshot** always goes after a **write**
- New object with a single operation
- **write-snapshot(i, v)**: writes v in M[i] and takes a snapshot of the memory.
- Let’s focus on **one-shot** for this talk
- How do we specify it?

Informal specifications

- **write-snapshot(v)**: writes and takes a snapshot of the memory

- Usual property-based specification:
  1. **Self-inclusion**: each S_i contains i
  2. **Containment**: every S_i, S_j are comparable under containment
  3. **Validity**: if j is S_i in j was written in M[j]

Concurrent-based specifications

- Used in distributed computability (often using topology)
- Main example: k-set agreement and consensus
- Many others, loop agreement, adopt-commit, renaming, etc.
- **propose(x)**: each process has an input x, returns a value y
- Usual property-based specification for k-set agreement:
  1. **Agreement**: at most k different values are returned
  2. **Validity**: an output value y was proposed

More formal: Tasks

- **One-shot** distributed problem
- **Static** approach
- **Task** :
  1. **Input configurations** (simplicial complex)
  2. **Output configurations** (simplicial complex)
  3. **Input/output relation**
- Less explored but fundamental: computability, topological approach, simulations

More formal: Tasks

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More formal: Tasks

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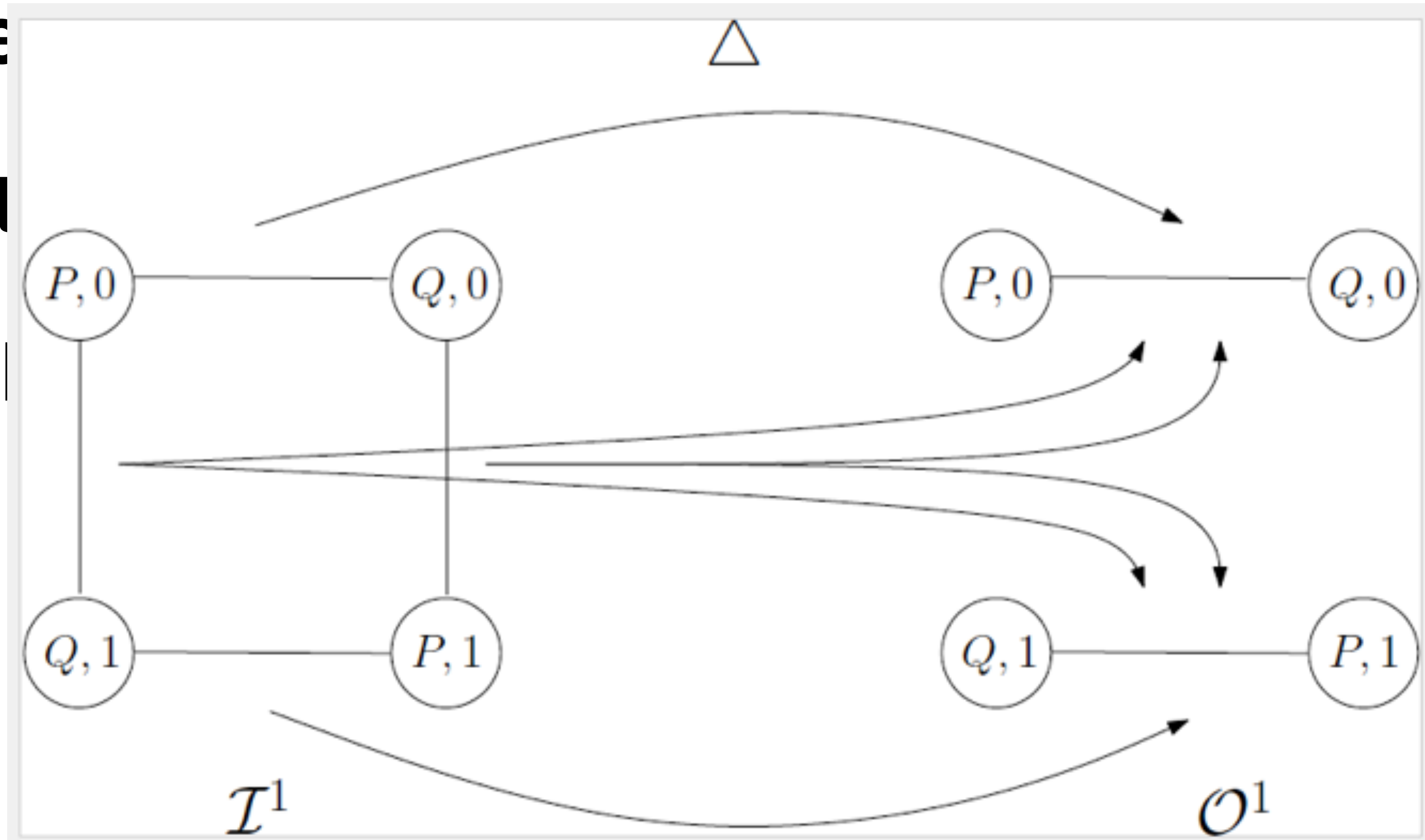

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Tasks tell what might happen in presence of concurrency

Solving Tasks

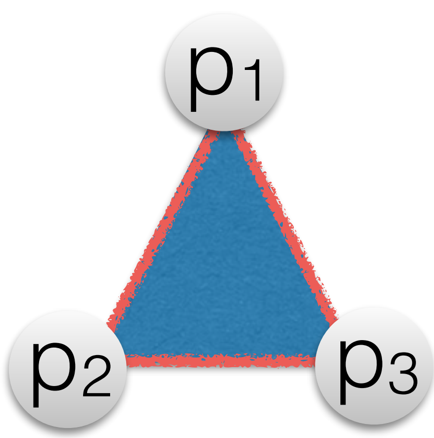
- When does an algorithm **solves** a task ?

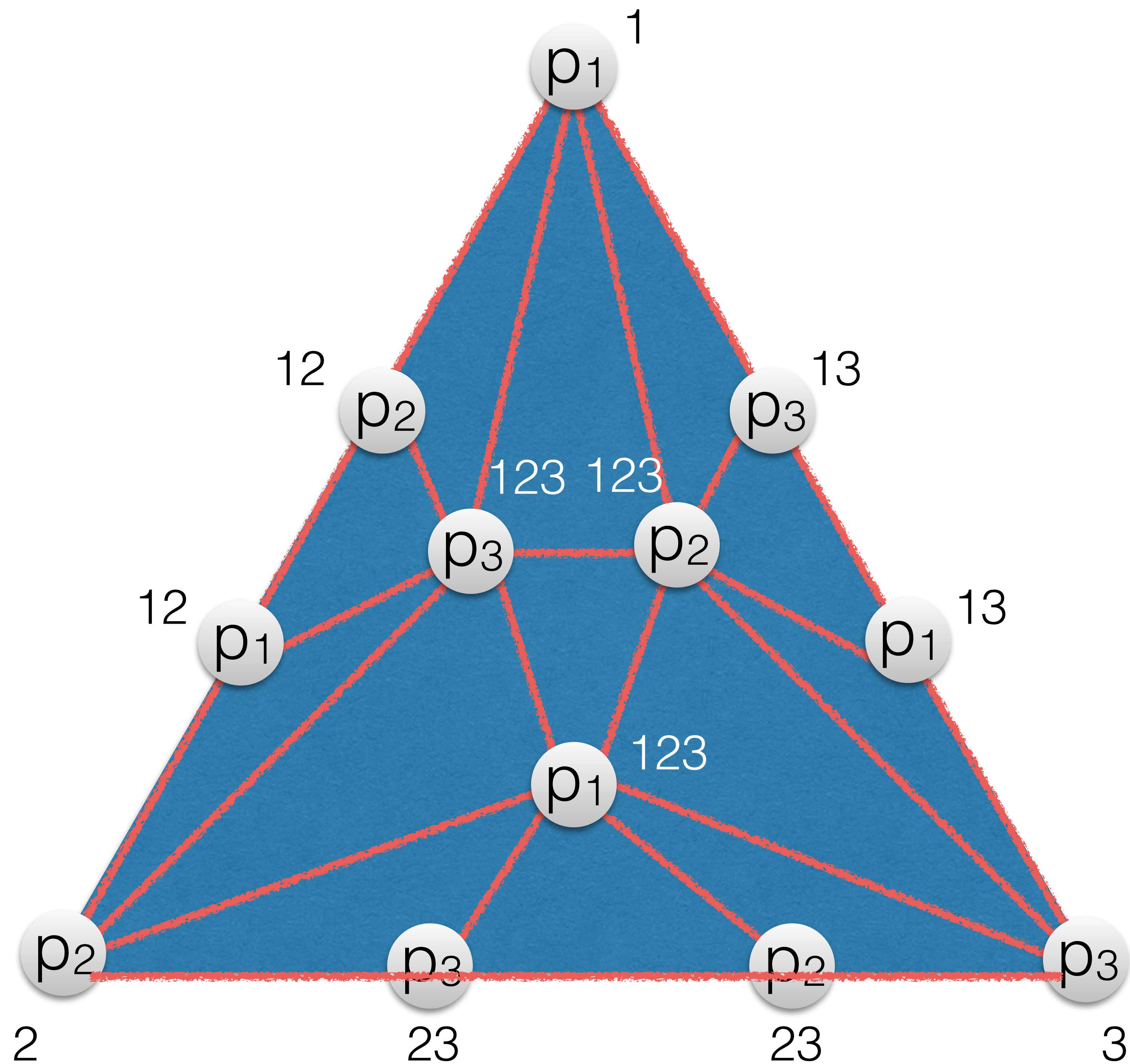
  For each set of participating processes, in every execution, **inputs and outputs** in every execution **agree** with the **mapping** specifying the task

Importance of Tasks

- Basic **computability** unit, distributed equivalent of a function
- Study of **set agreement** and **renaming** lead to a **connection** between **distributed computing** and **topology**
- **but:** Semantic of tasks is not well studied. What are they? Certainly, not sequential objects

Write-Snapshot Task





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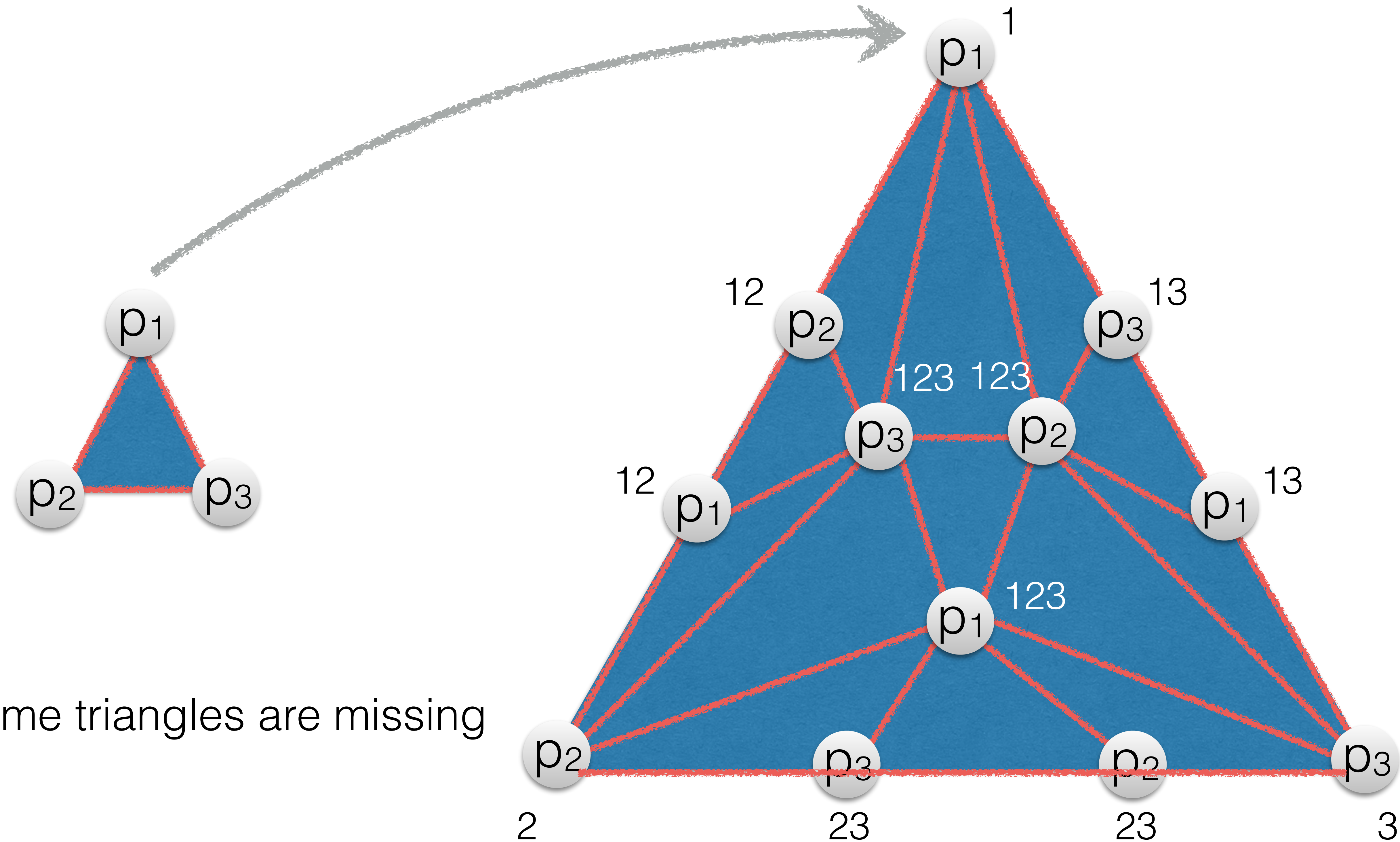

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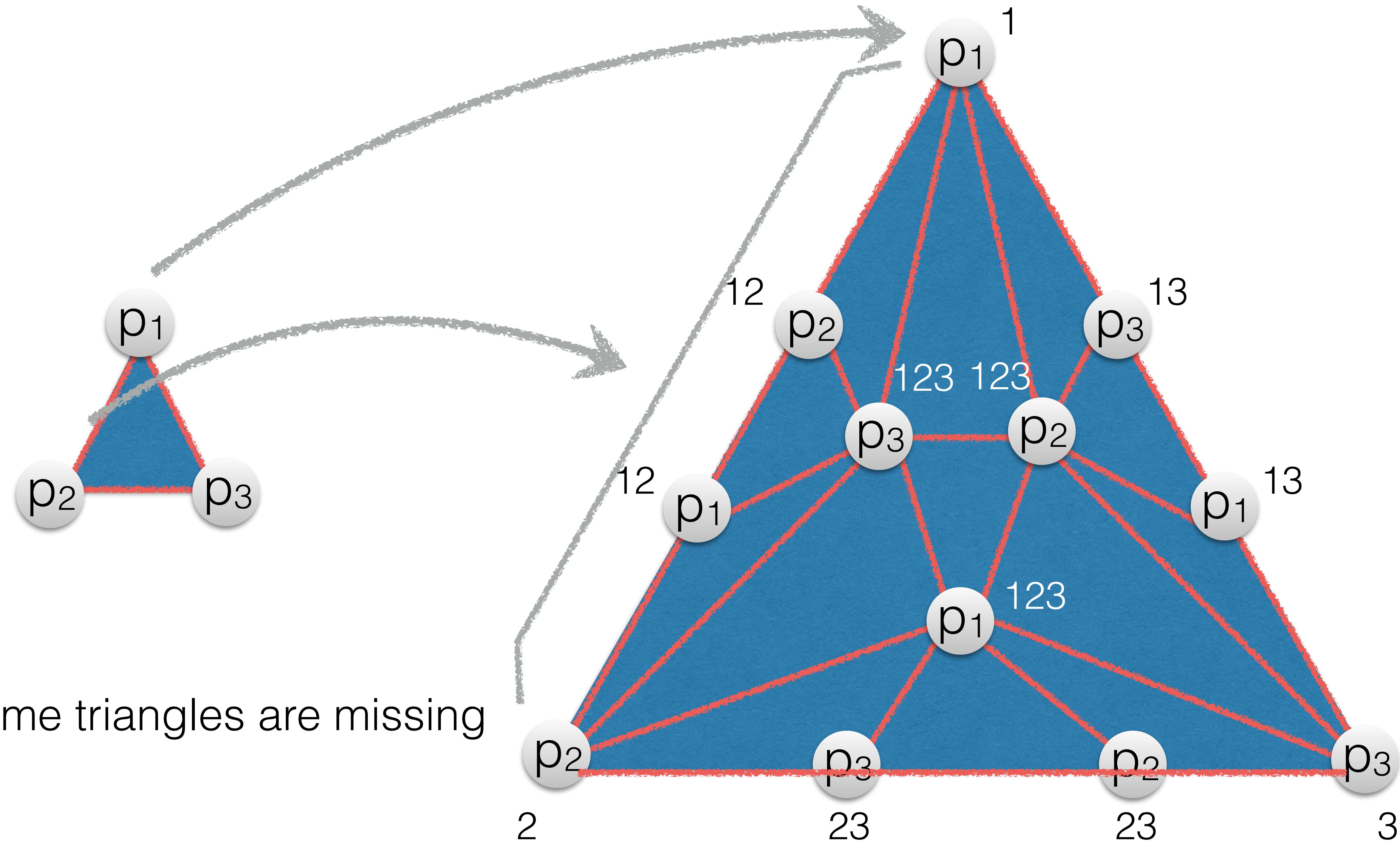

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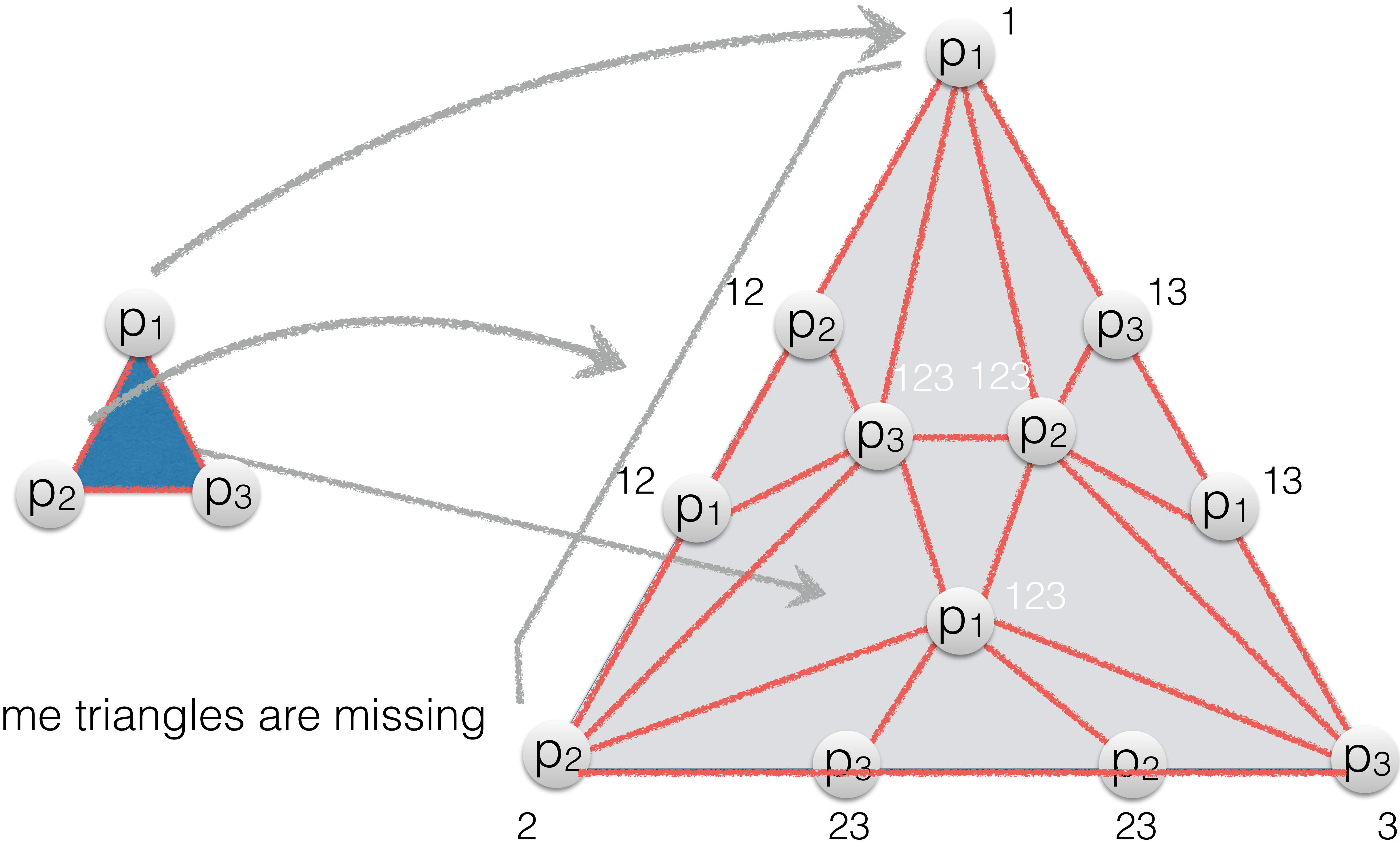

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Write-Snapshot Object

- An implementation based on **read/write**

operation write_snapshot(i) **is** % issued by p_i
$MEM[i] \leftarrow i$;
$new_i \leftarrow \cup_{1 \leq j \leq n}\{MEM[j]$ such that $MEM[j] \neq \bot\}$;
repeat $old_i \leftarrow new_i$;
$new_i \leftarrow \cup_{1 \leq j \leq n}\{MEM[j]$ such that $MEM[j] \neq \bot\}$
until $(old_i = new_i)$ **end repeat**;
return(new_i).

Write-Snapshot Object

- An implementation based on **read/write**

Is it linearizable?
Is there a sequential specification?

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NO!!

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(05) **until** $(old_i = new_i)$ **end repeat**;

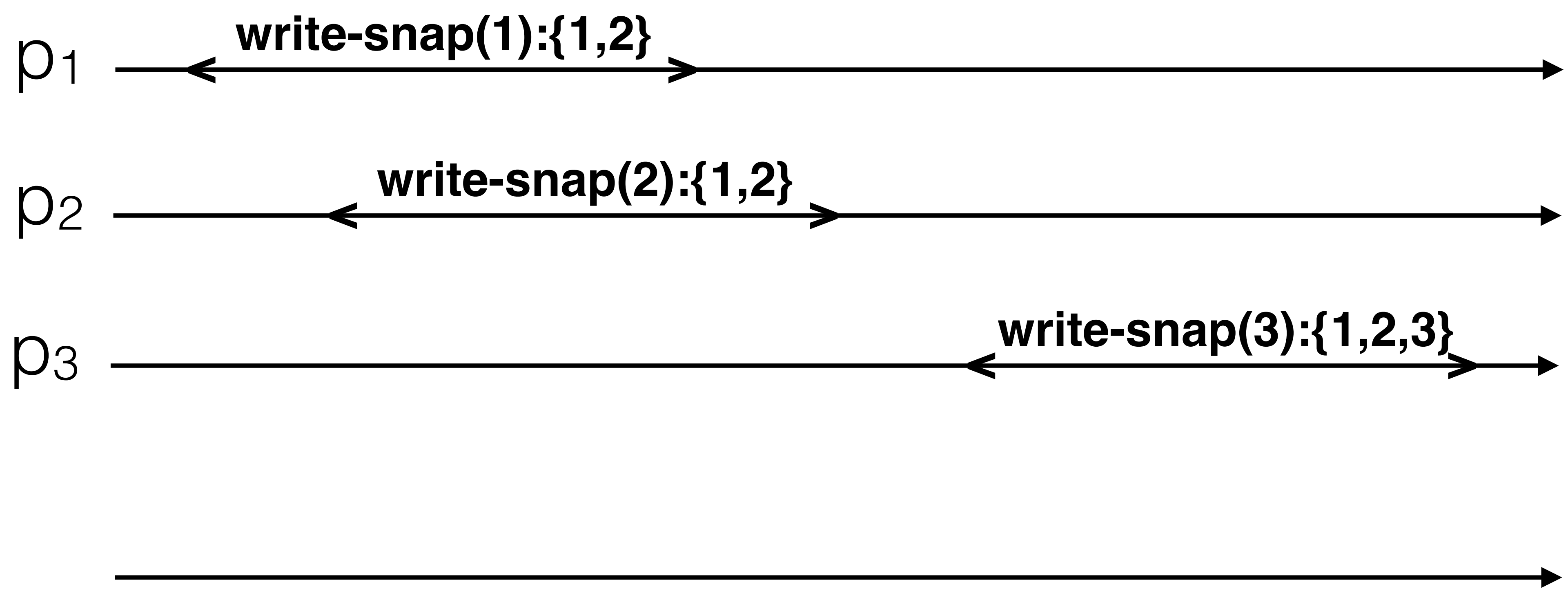
(06) return(new_i).

Write-Snapshot Object

- There is no sequential specification
- If there is such an specification, in each execution of a **read/write** linearizable implementation, there is a **‘first’** process
- Solve **Test&Set** from any such **read/write** implementation. A contradiction!!
- What is going on?

Write-Snapshot Object

- *Tasks can model executions that sequential specs cannot:*



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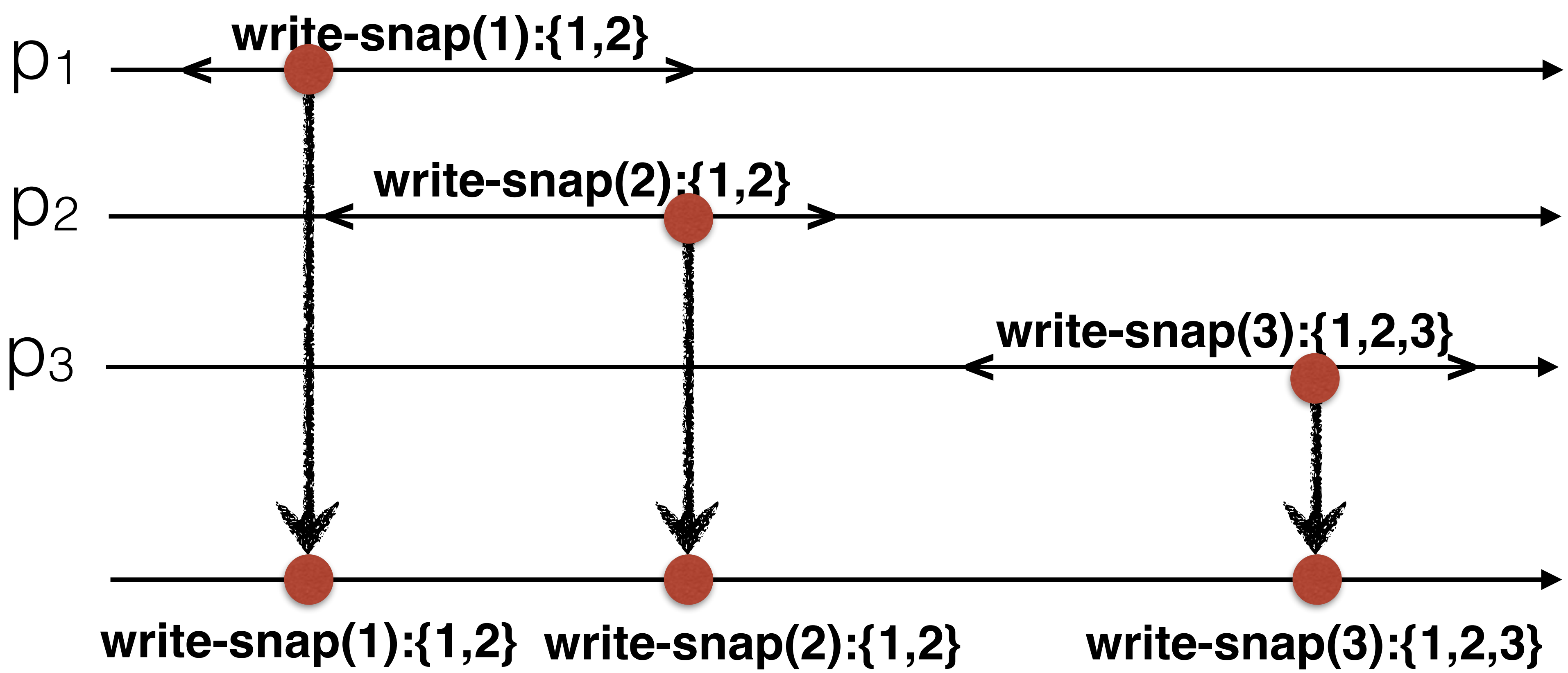

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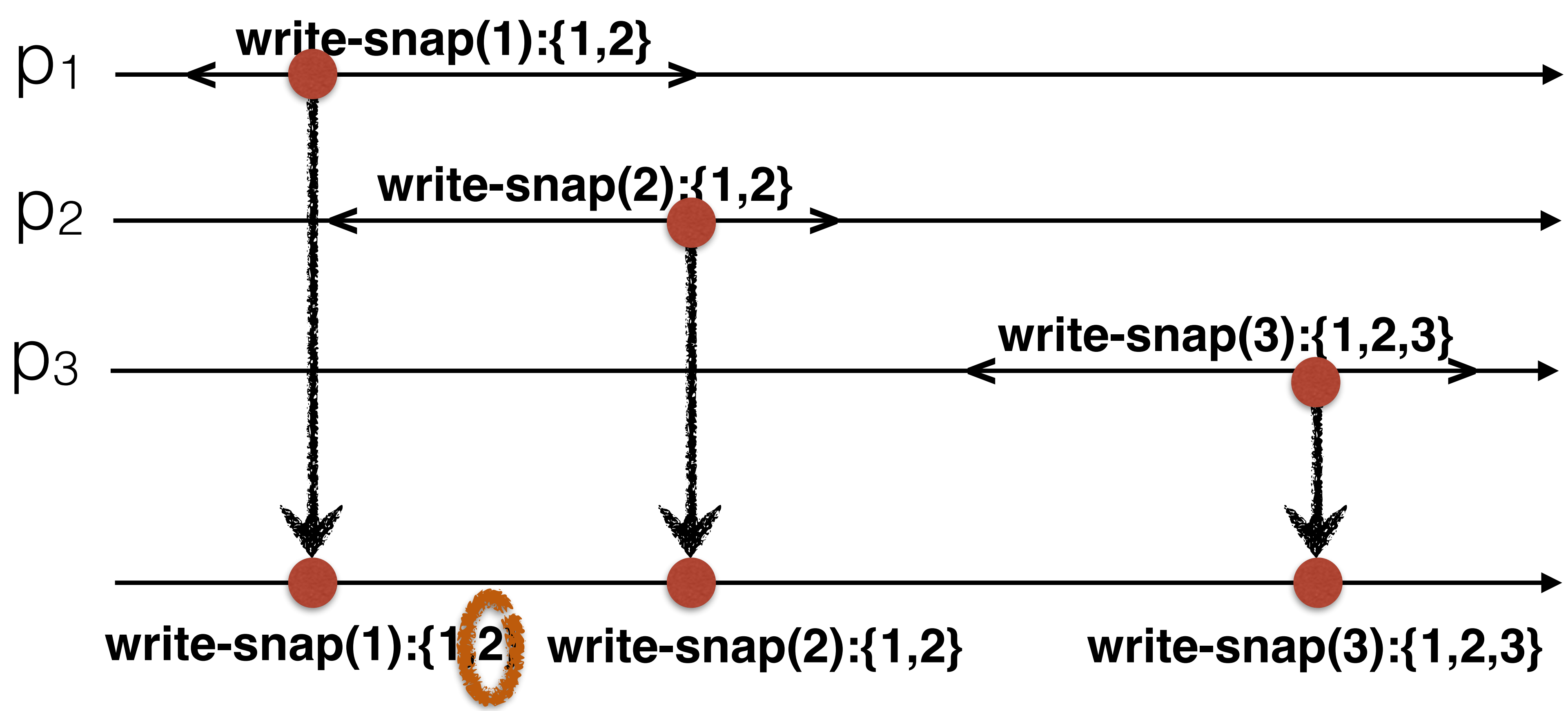

Write-Snapshot Object

- Any sequential spec. of **write-snapshot** models a **proper subset of executions**

- The **resulting specification** is **stronger** than the object we want to model

Limitations of Linearizability

- First noted by Neiger BA PODC'94: **NO sequential specification** for **set agreement** and **immediate snapshot** (property-based specification)
- **Set linearizability**
- Similar approach: **concurrency-aware** by Hemed, Rinetzky and Vafeiadis DISC'15
- **Not enough** to specify **write-snapshot**

Examples of non-sequentially specifiable tasks:

1. Adopt-commit (used in Paxos for safety)
2. Conflict-detection (Aspnes-Ellen)
3. Safe-consensus (weaker validity of consensus)
4. Immediate snapshot (Asyn. Computability Theorem)
5. k-set agreement (generalization of consensus)
6. Exchanger (Java object)

Limitations of Tasks

- A **one-shot queue** (or **stack**) **cannot** be **specified** as a **task**.

- **Problem:** Tasks have no mechanism to model memory of automatons

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Linearizability and Tasks are importan but not unified!!

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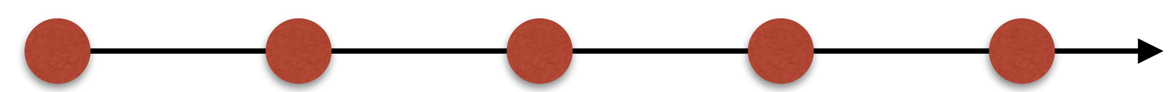
Our contribution: Unify these two styles of specifications

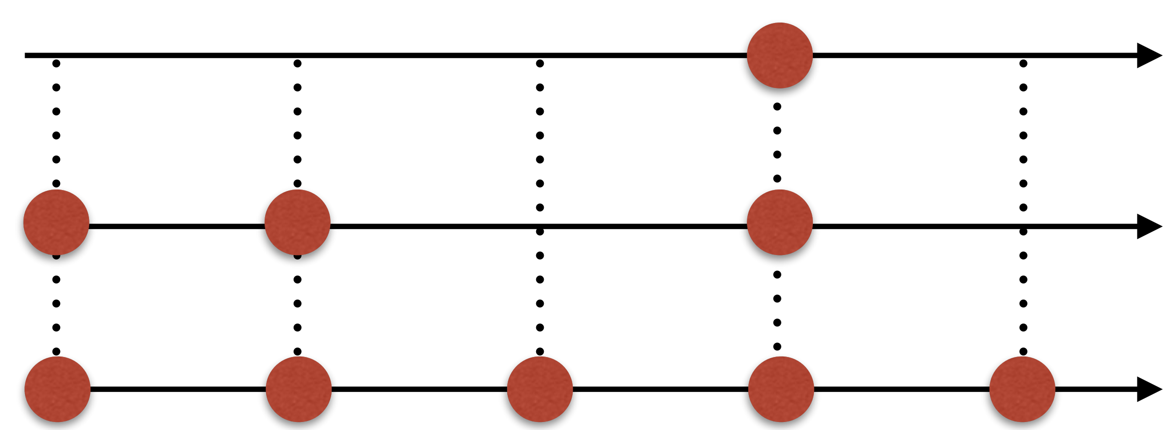
Set Linearizability (Neiger 94)

- Go from dimension 1 to dimension 2:

Sequential



Set sequential



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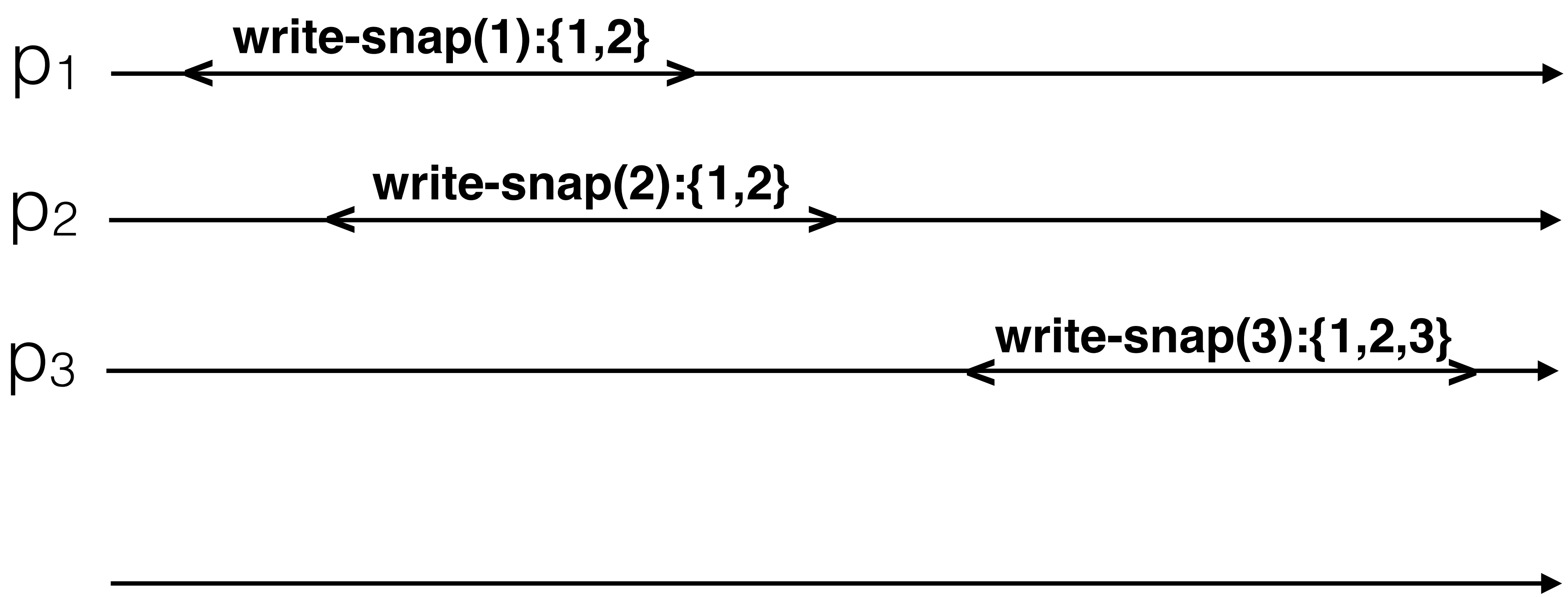

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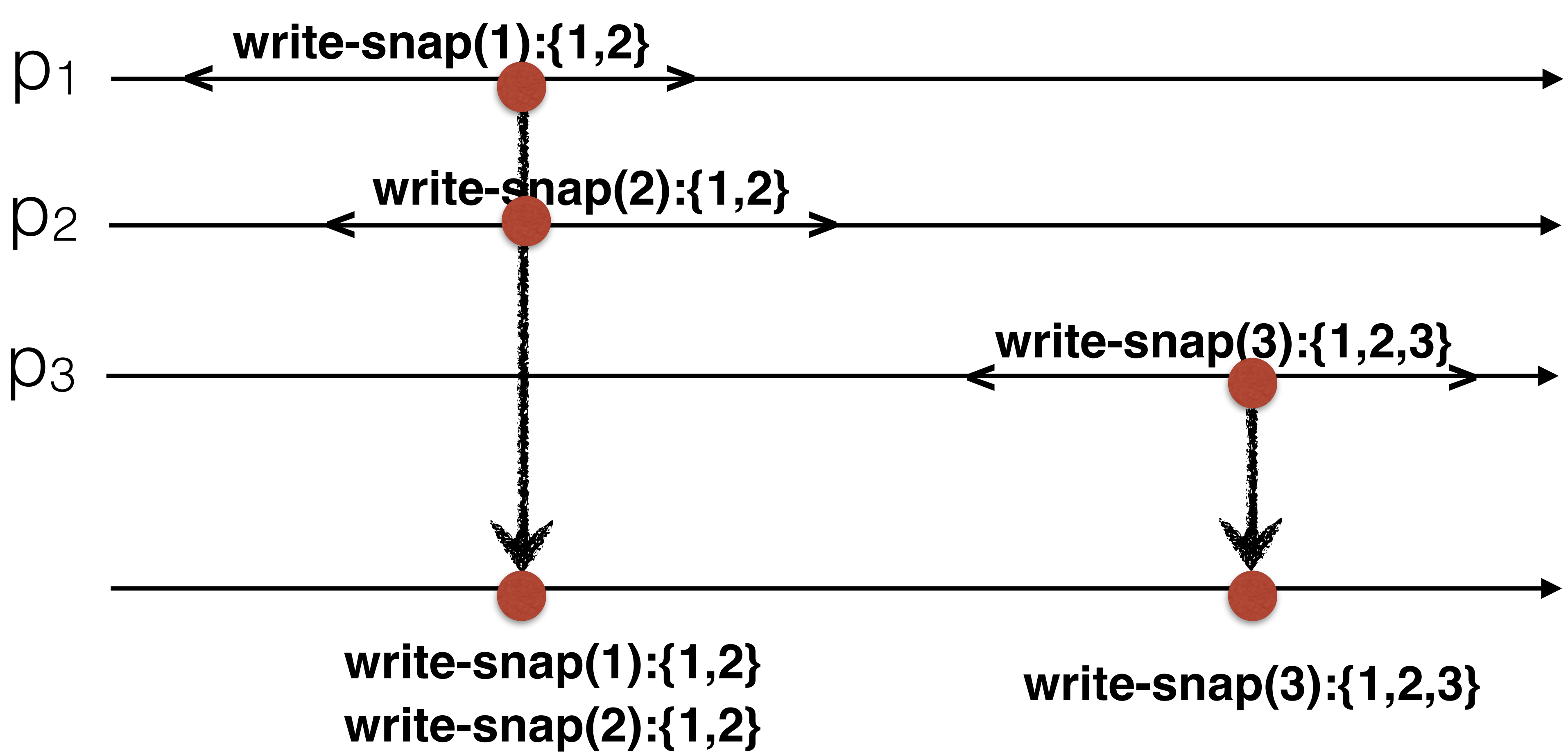

Set sequential automata (Neiger 94)

- Transitions labeled with *sets* of operations and their responses

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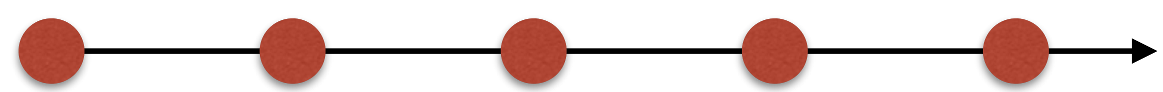

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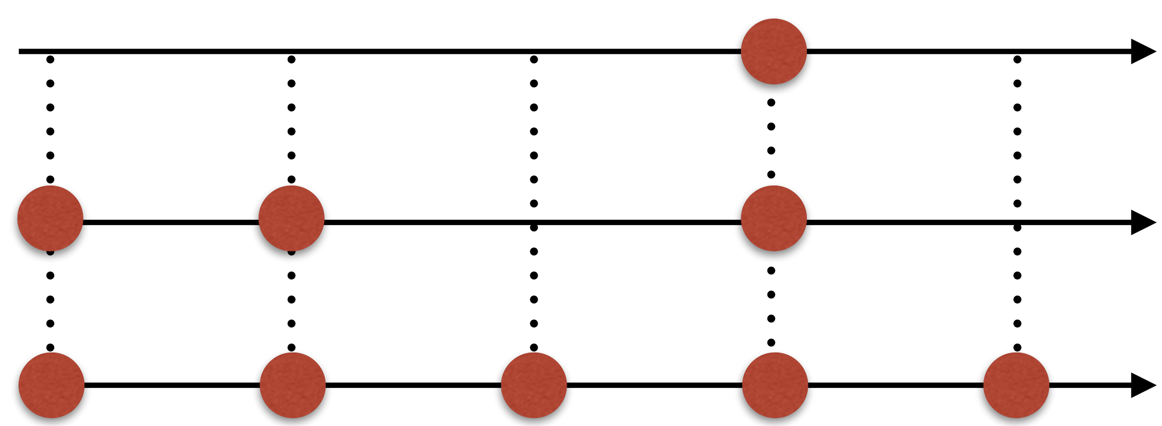

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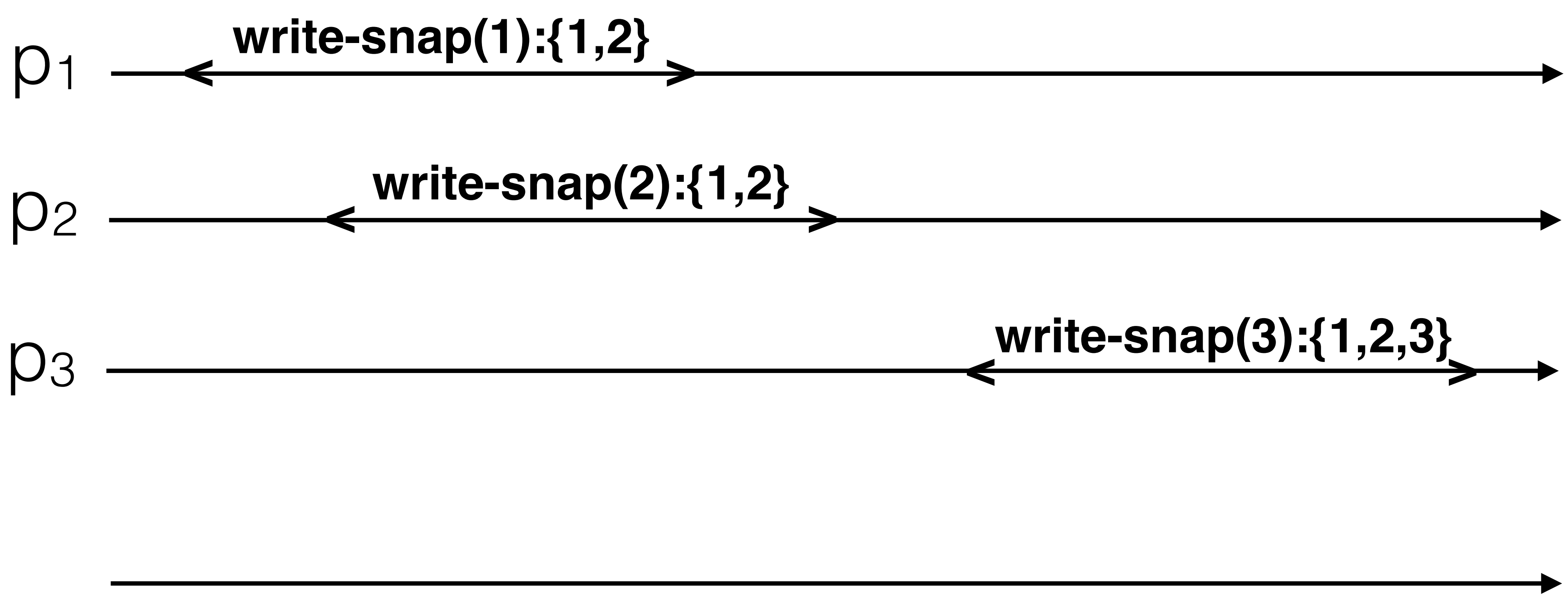

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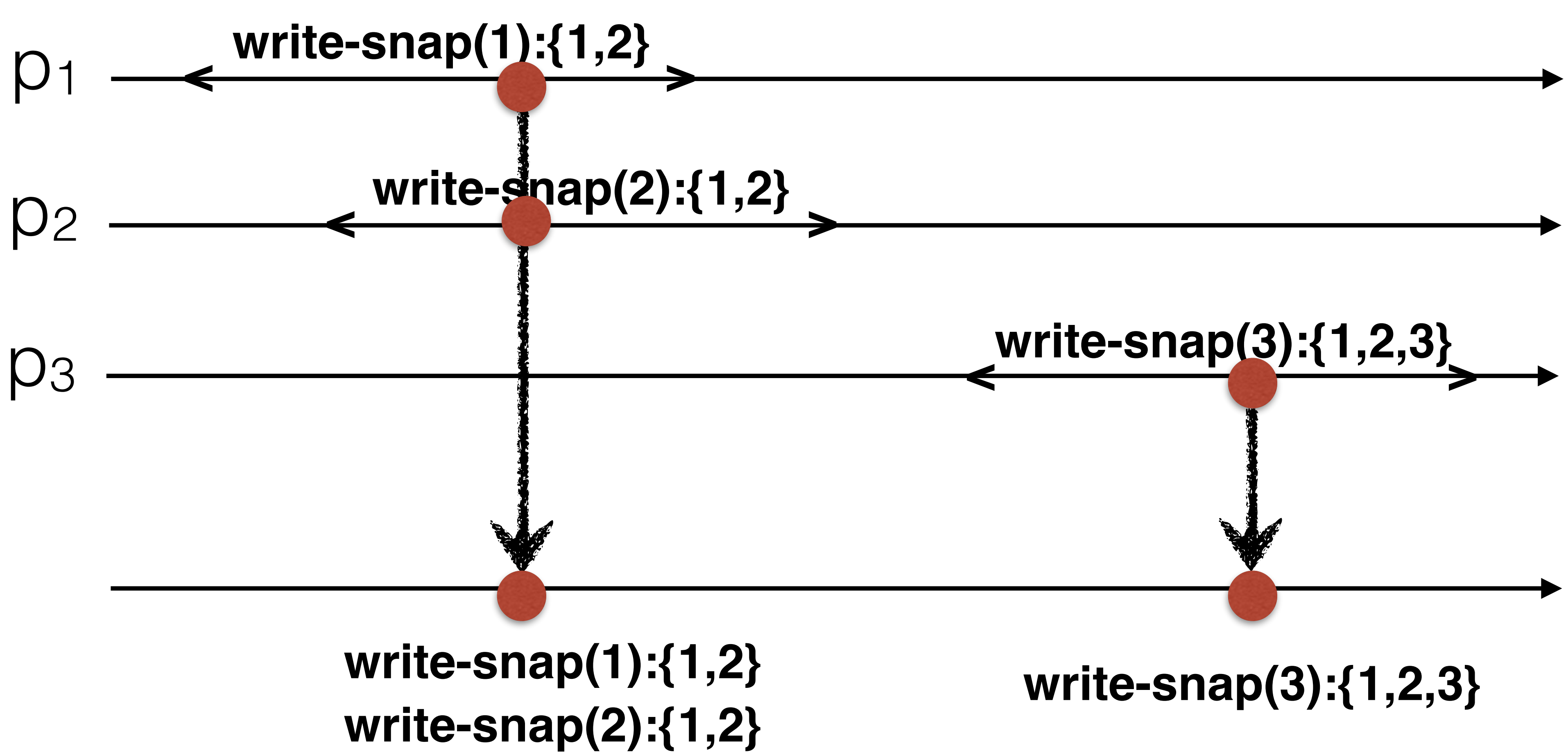
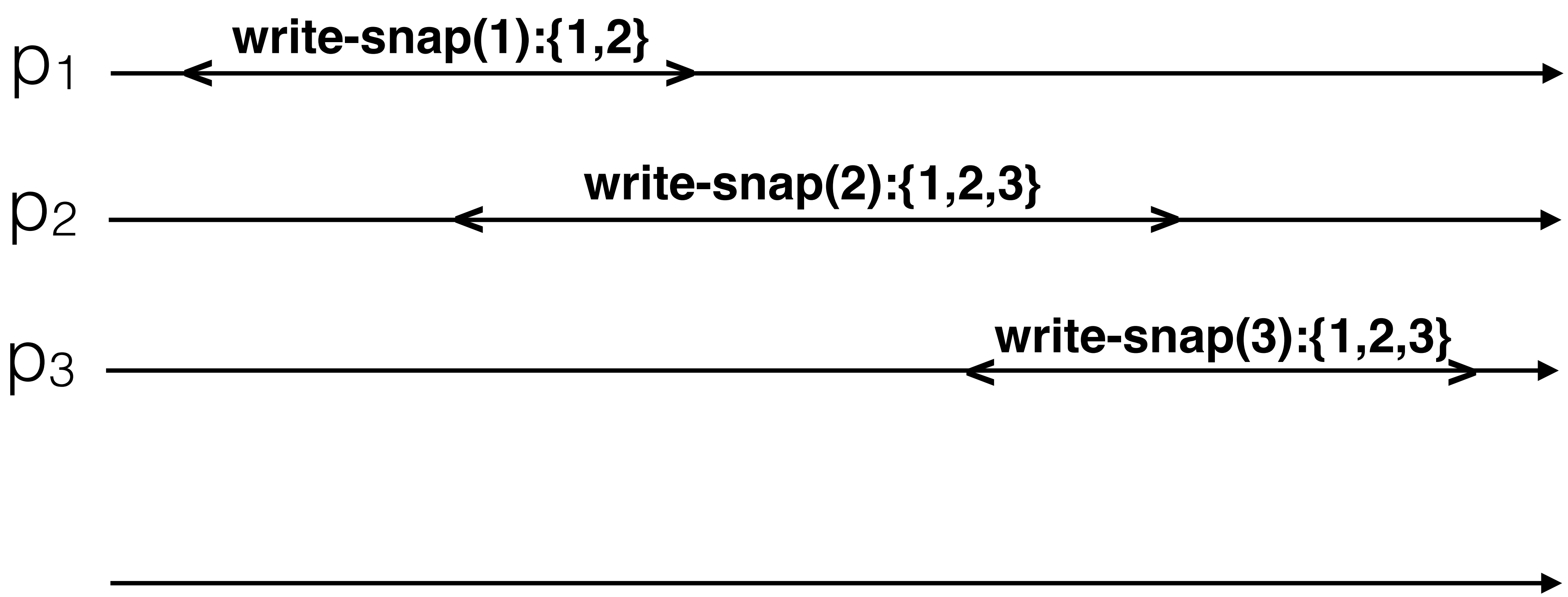


Set linearizability is not enough!!

Limitations of Set Linearizability



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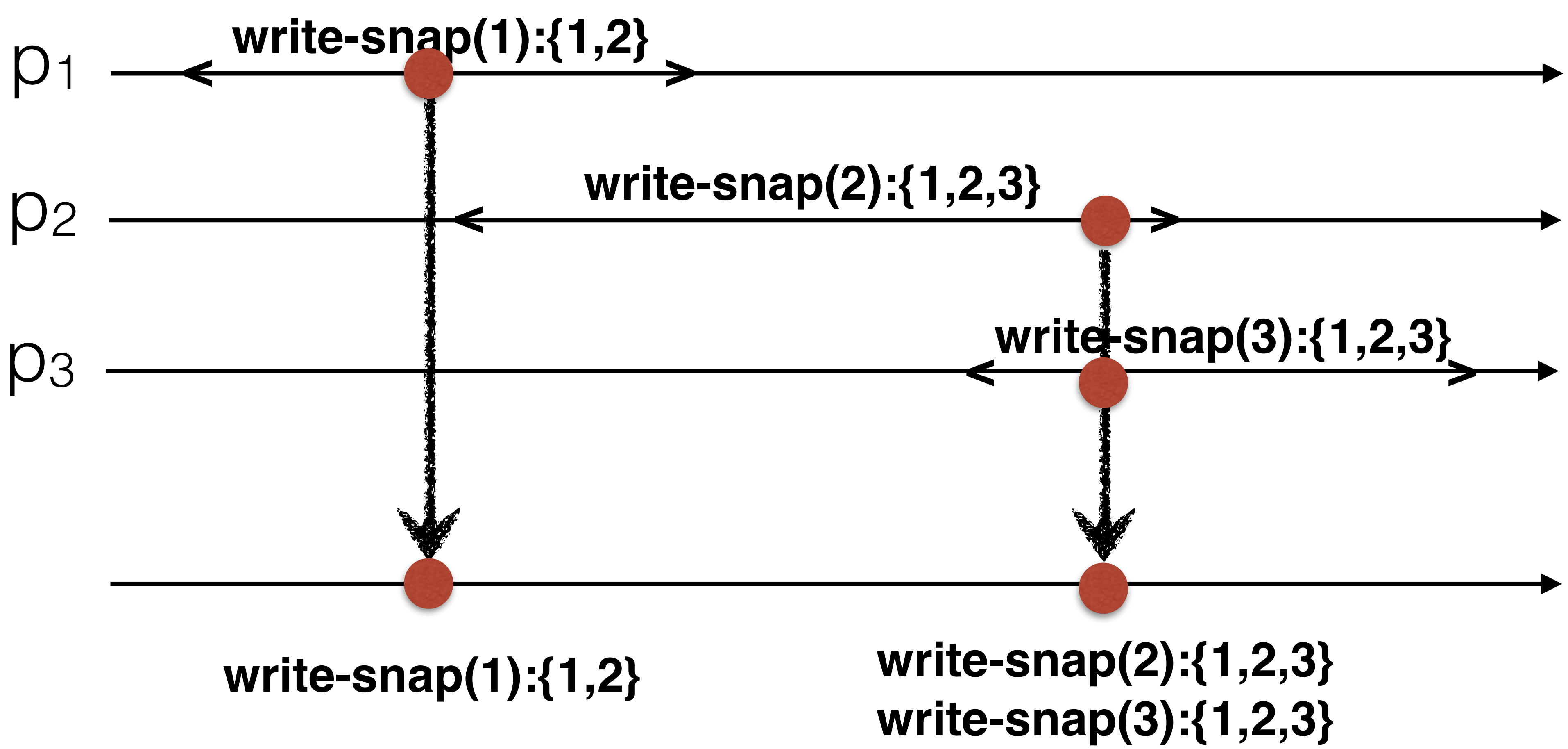

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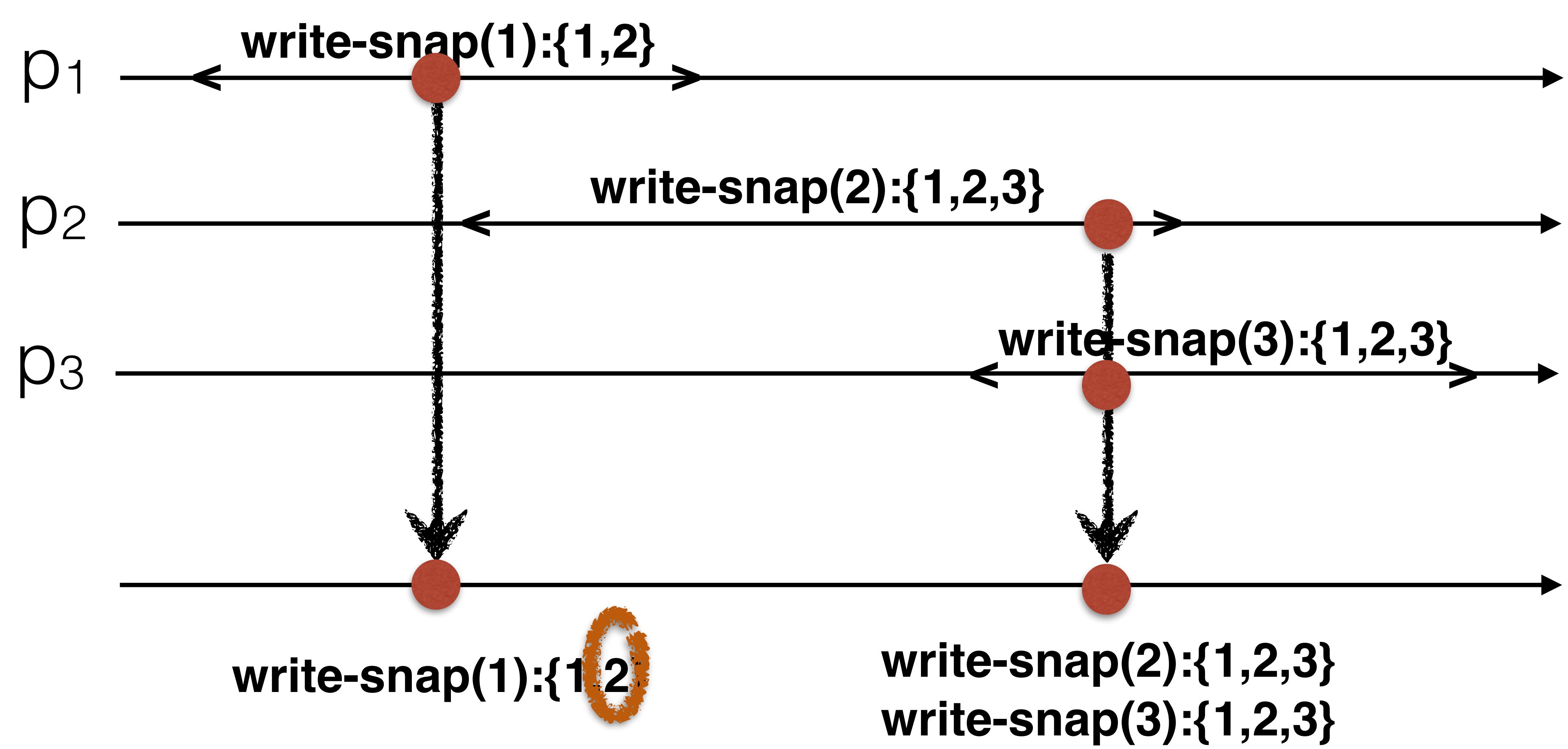

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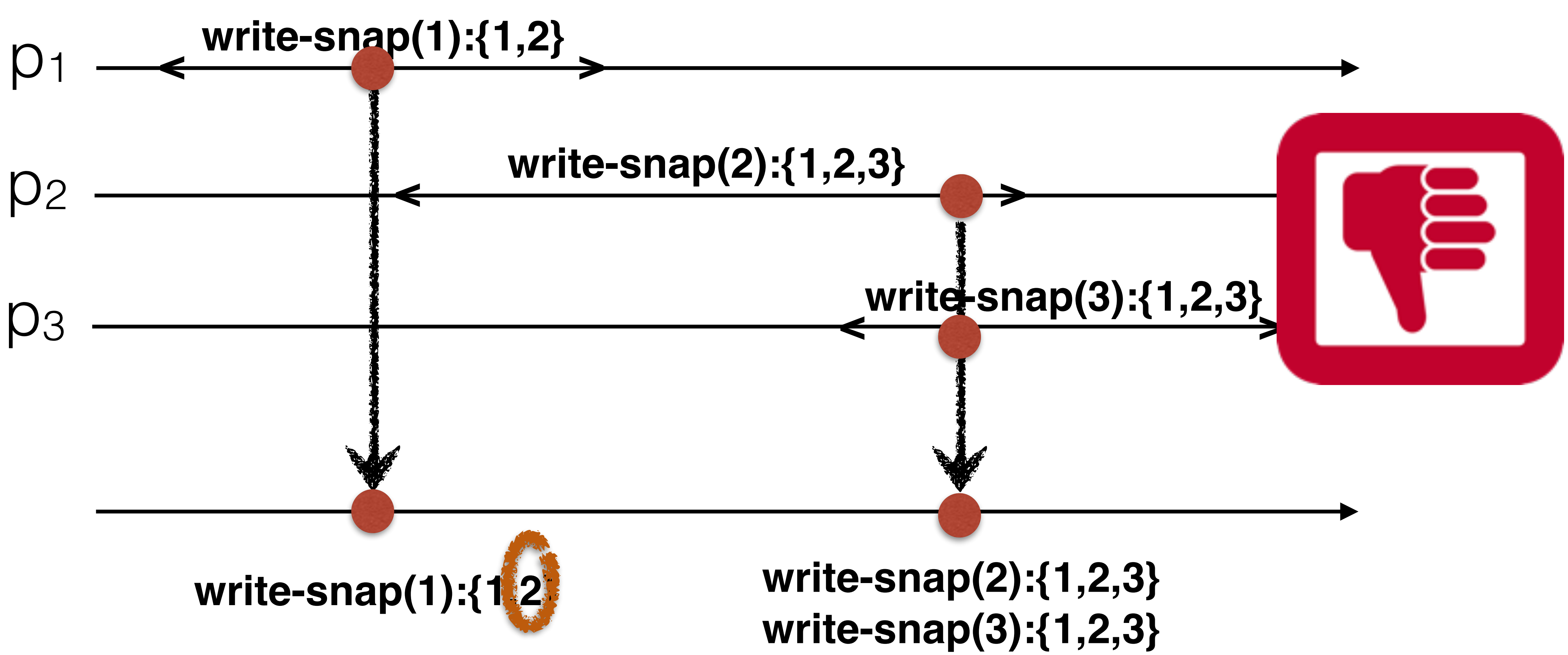

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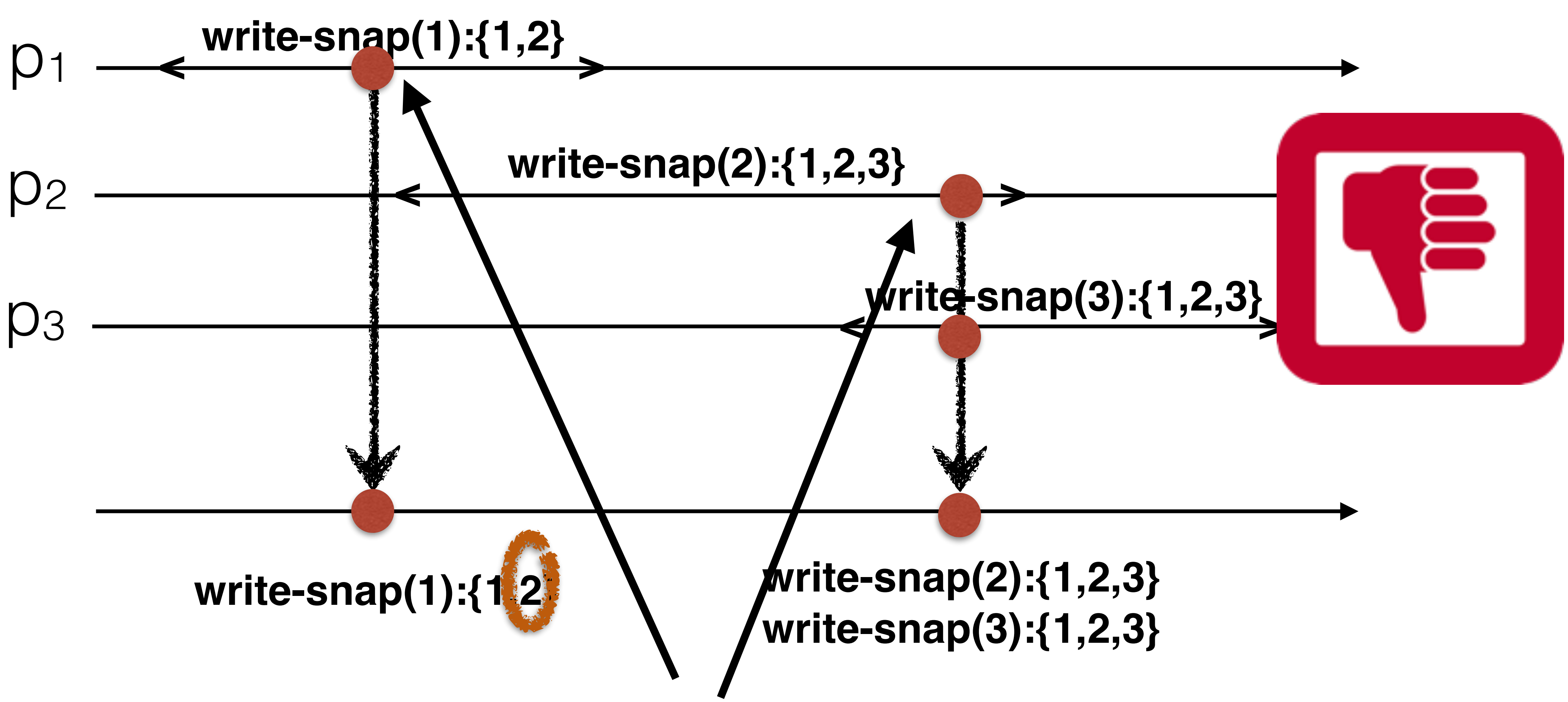
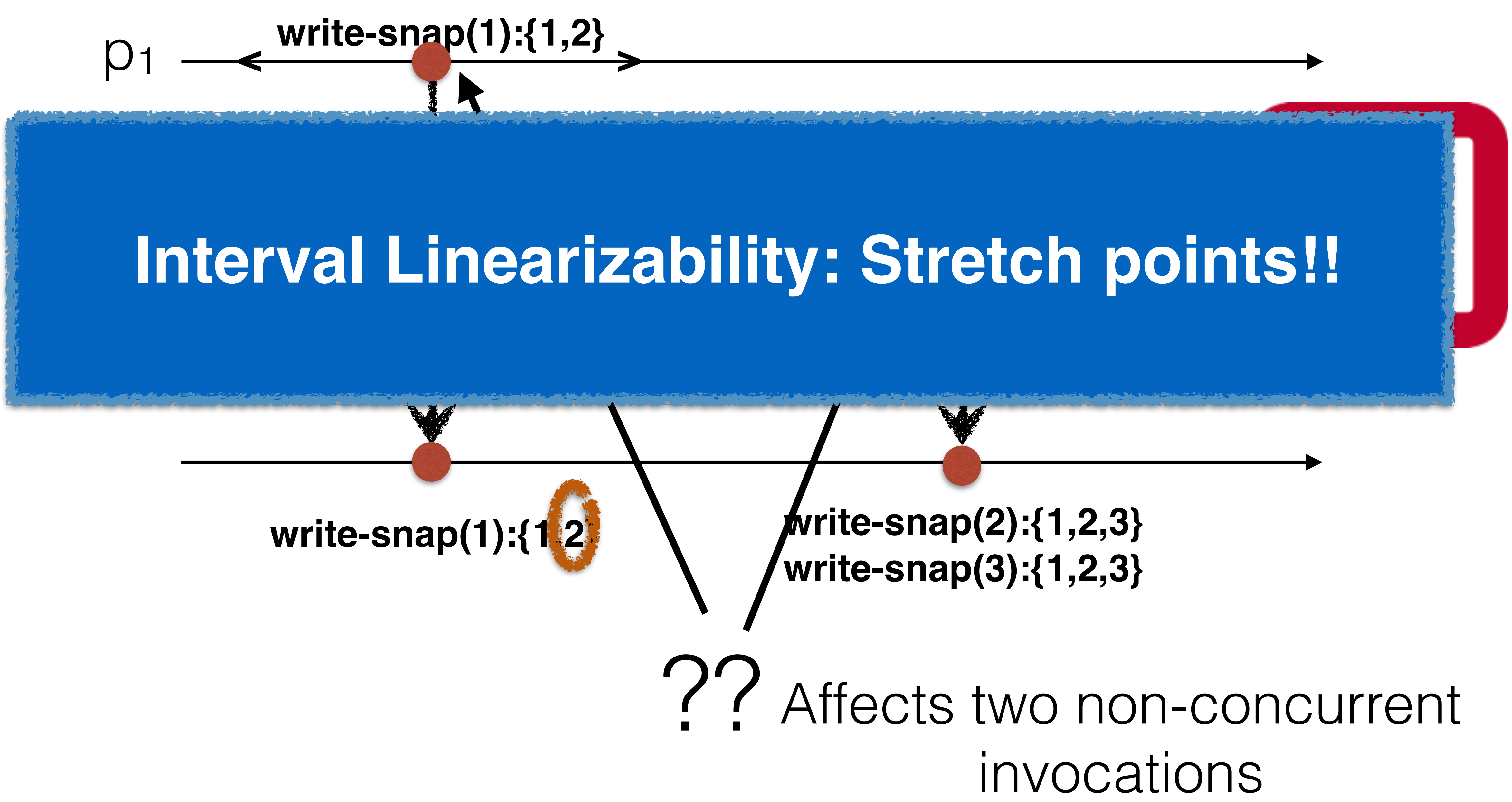


?? Affects two non-concurrent invocations

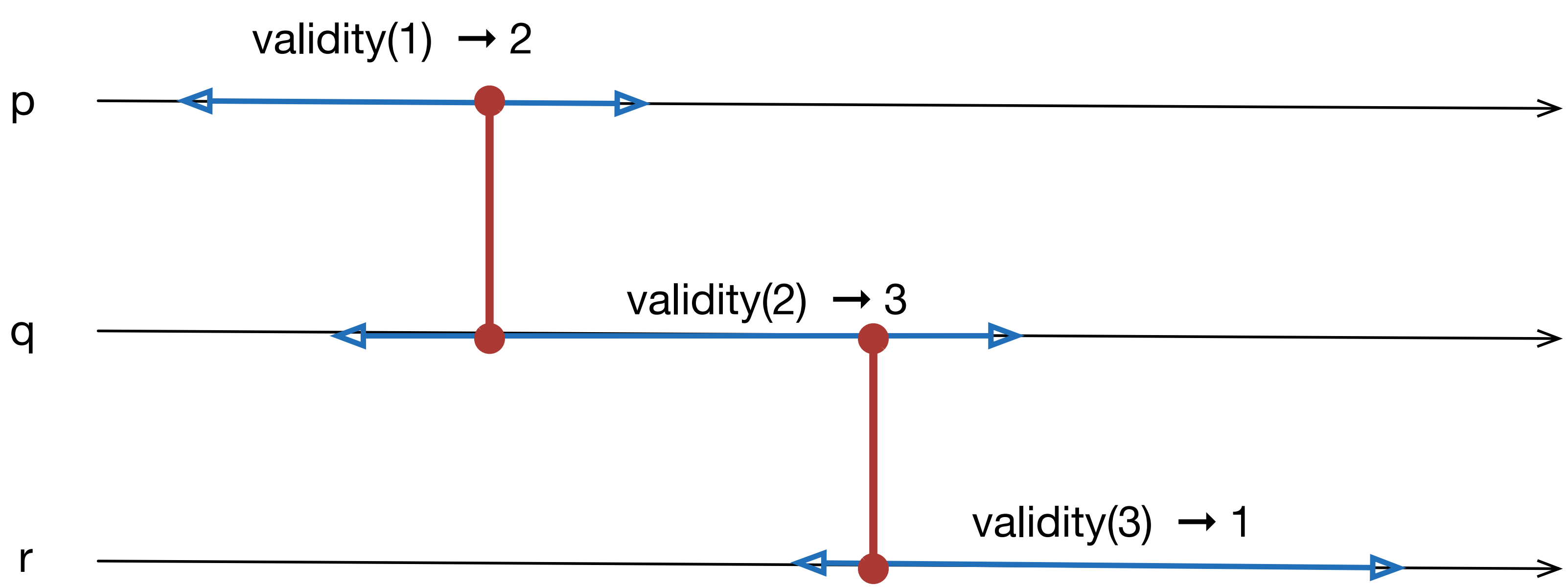
Limitations of Set Linearizability

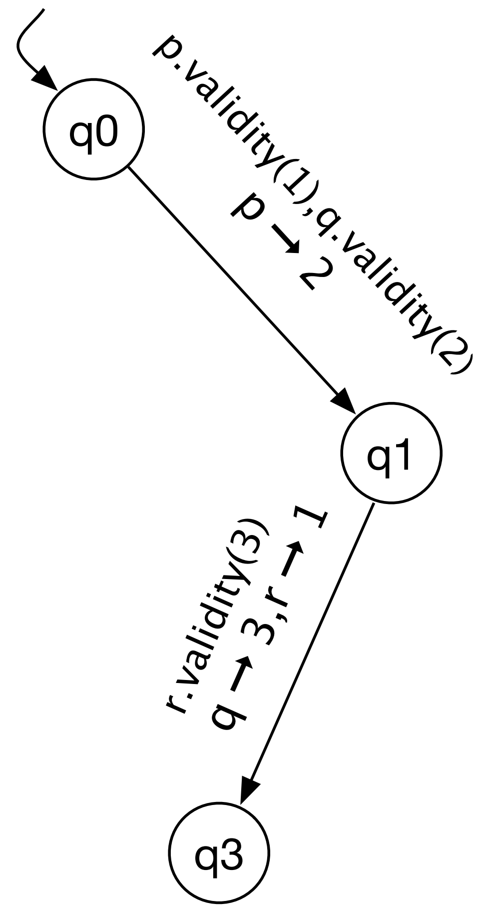


Interval-Sequential automata

- Mealy state machine
- If X is in state q and it receives as input a set of invocations I, then, if $(R,q') \in \delta(q,I)$, the meaning is that X may return the non-empty set of responses R and move to state q'.

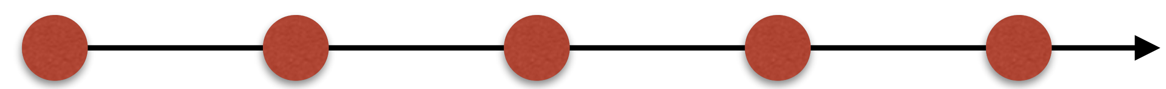
Interval-Sequential Validity Object



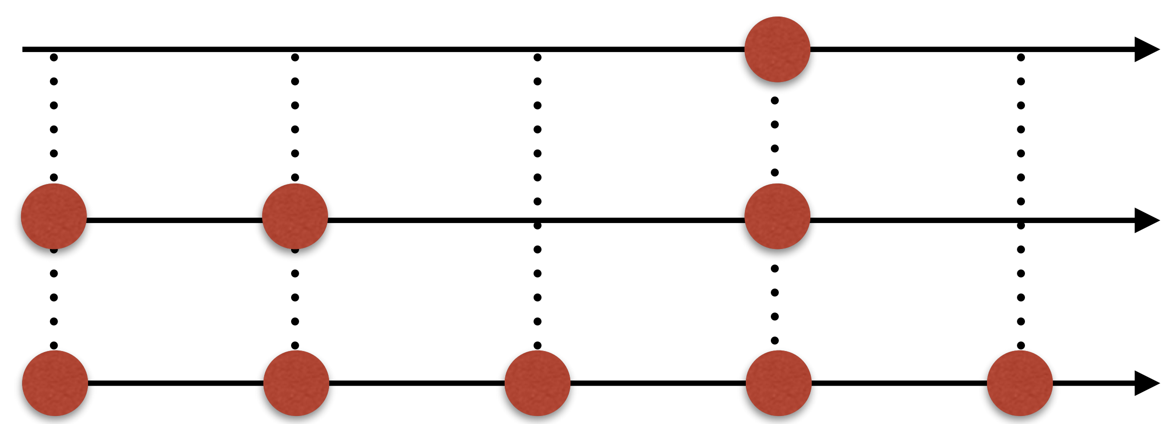


Interval Linearizability

Sequential



Set sequential



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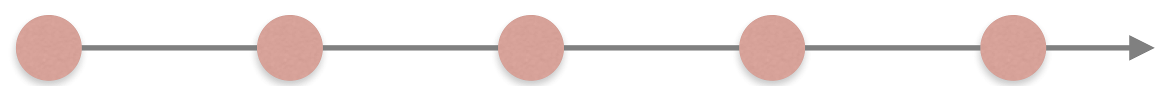

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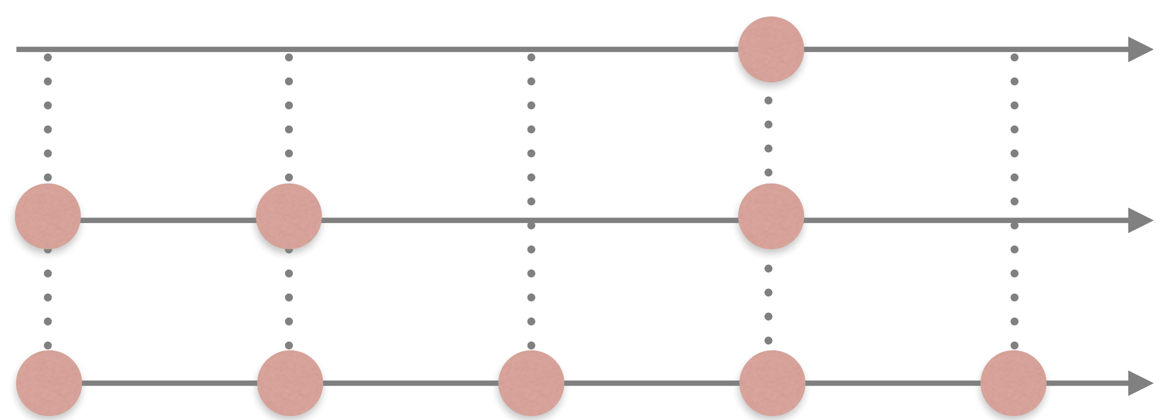

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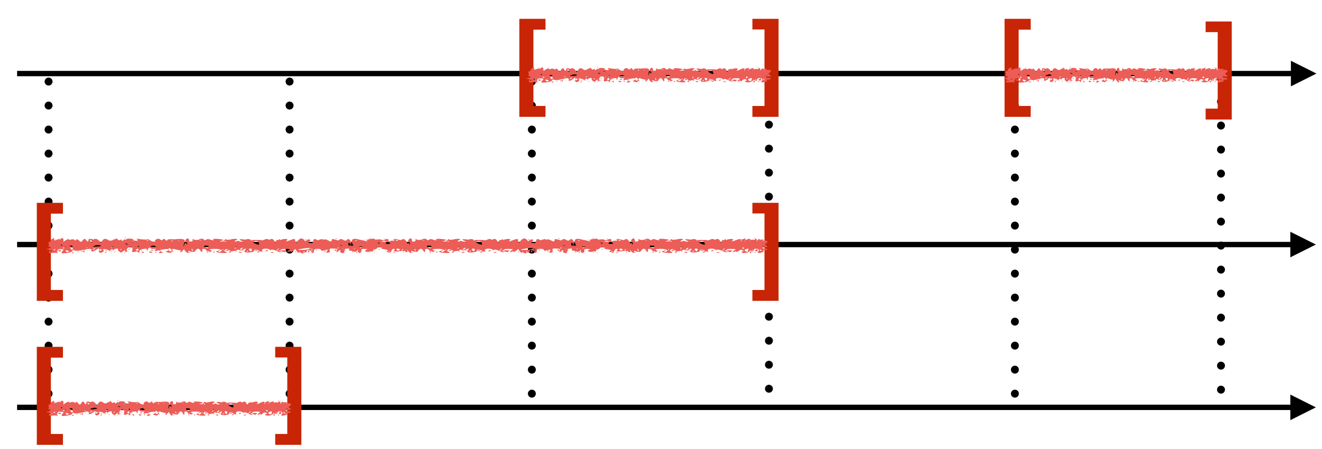


Interval Linearizability

- **Interval Sequential (IS) exec**: Grid with **'nicely'** ordered intervals

  First column: invocations.

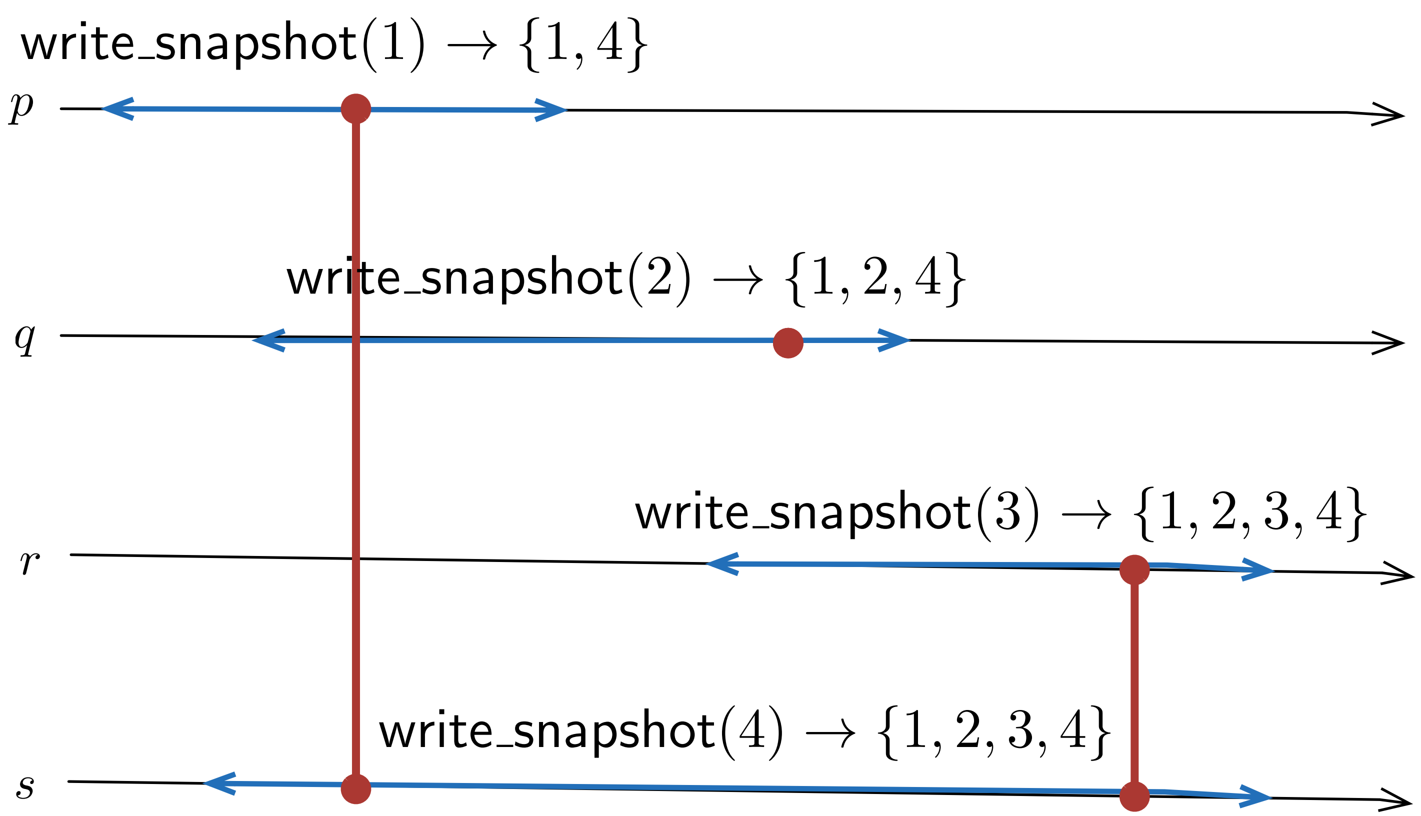
  Second column: responses to some invocations.

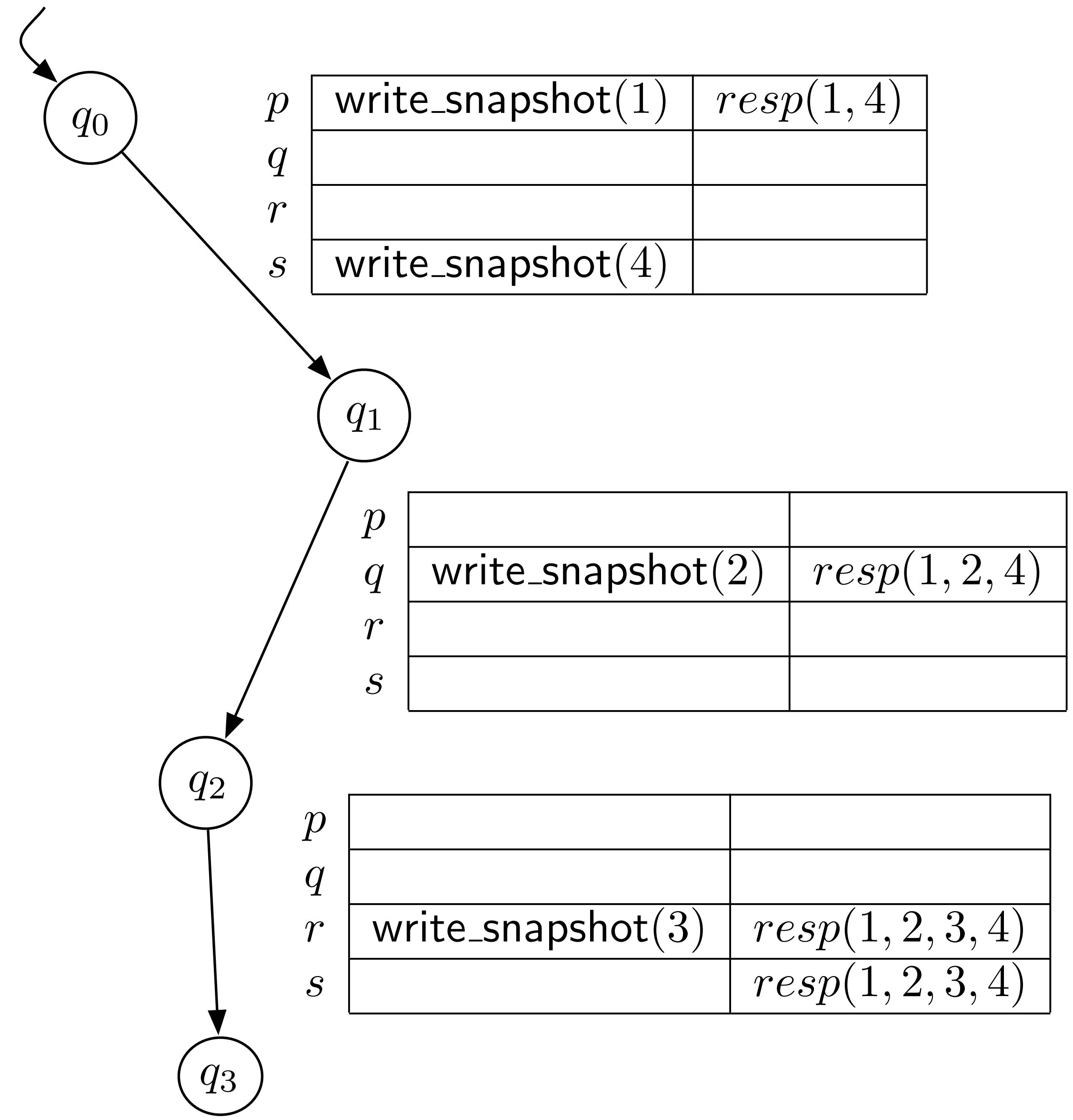
  Third column: new invocations.

  Fourth column: …

- **IS specification**: set with IS executions,
- alternatively IS automaton

Interval Linearizability and automaton





Interval Linearizability

- **Interval linearizable implementation**: each execution can be transformed into a IS execution, respecting real-time order (like in linearizability)

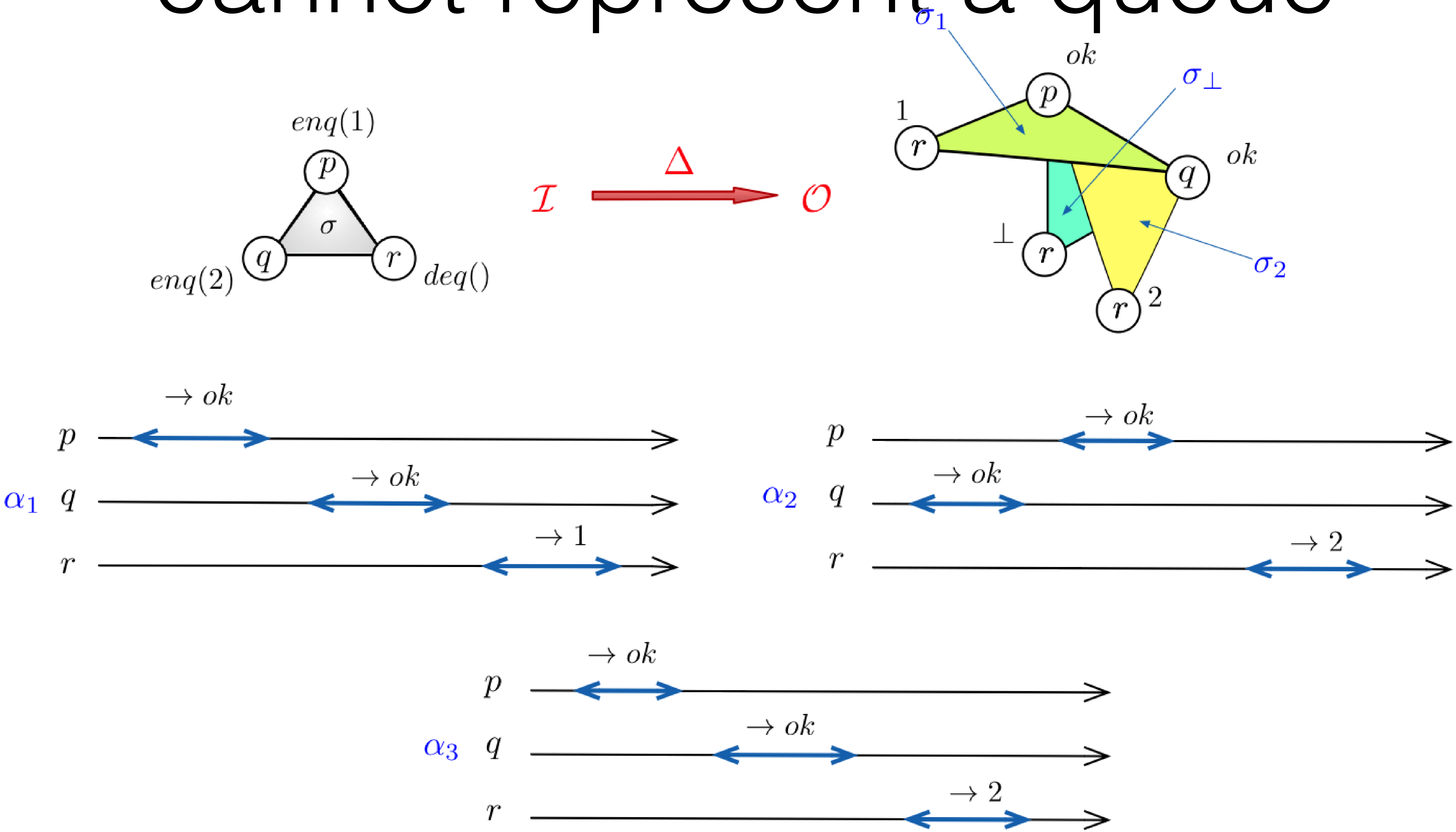
- **Not harder** to prove than linearizability. For each operation, two points (an interval) need to be found

- **Particular cases:** linearizability and set linearizability

Extended Tasks

- A **new value** on each vertex added in the **output complex** to **model memory**
- The **mapping** has the same definition but the **meaning** is **a bit different**
- **Particular case:** Tasks

Simple task interpretation cannot represent a queue



From Interval Linearizability to Extended Tasks

For every one-shot IS object X, there is an extended task equivalent to X

Idea of the proof: Every execution is represented with a simplex of appropriate dimension. New value model memory

By-product: Opens the possibility to apply topological techniques to sequential, set sequential and interval sequential objects.

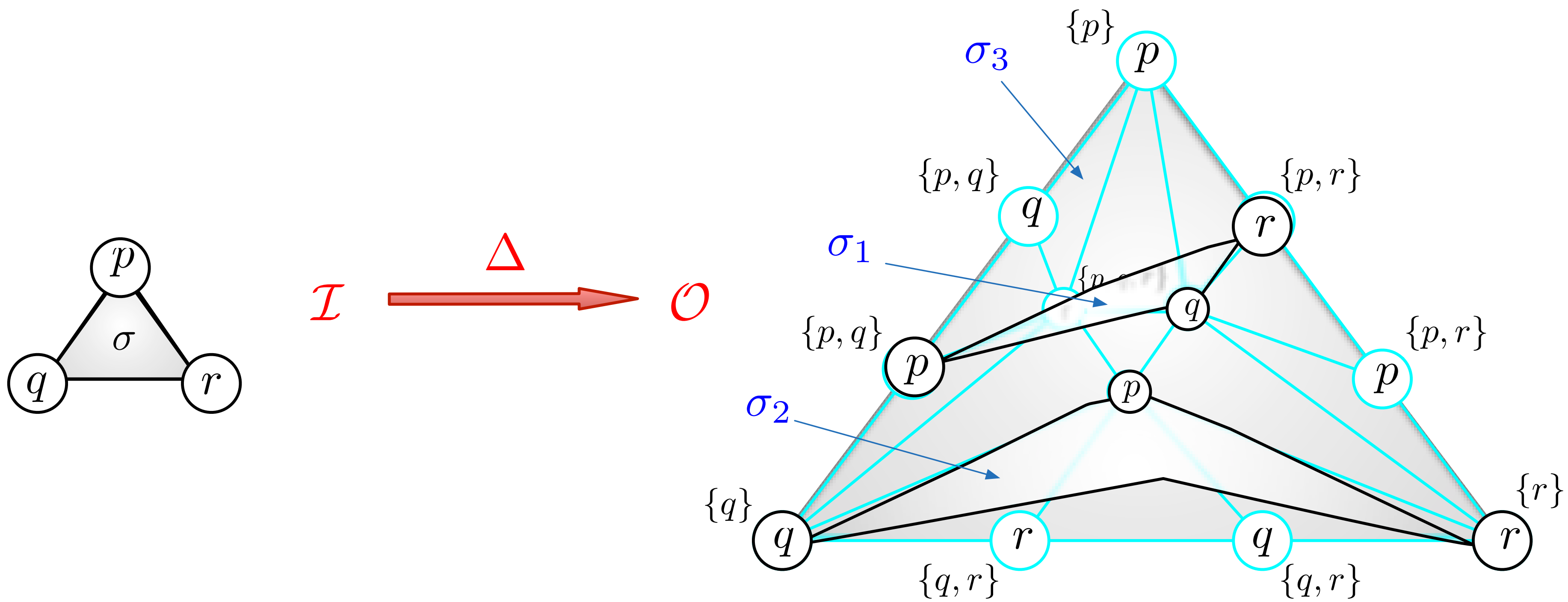
From Extended Tasks to Interval Linearizability

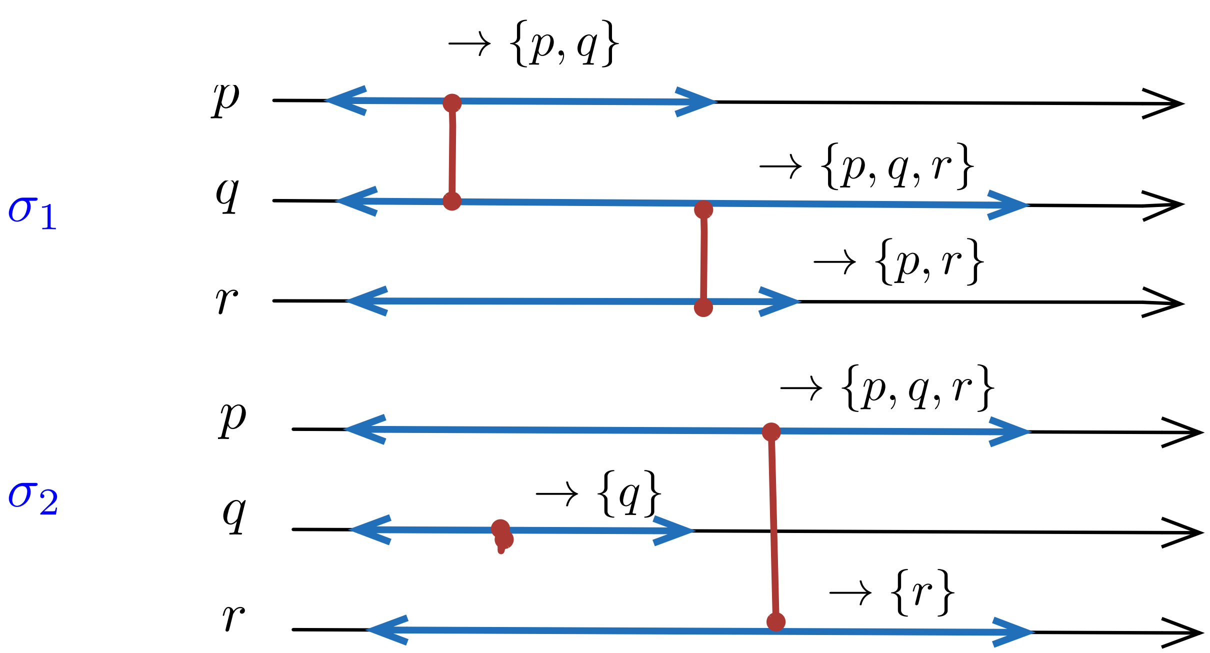
For every extended task T, there is a one-shot IS object equivalent to T

Idea of the proof: Model each output simplex as an IS execution. The interpretation of the mapping from input complex to output complex is not trivial, has to be done carefully.

By-product: Better understanding of the semantics of tasks.

From tasks to interval sequential automata





	init	*term*	*init*	*term*
p	prop(p)	$resp(p,q)$		
q	prop(q)			$resp(p,q,r)$
r			prop(r)	$resp(p,r)$

	init	*term*	*init*	*term*
p			prop(p)	$resp(p,q,r)$
q	prop(q)	$resp(q)$		
r			prop(r)	$resp(r)$

Interval Linearizability Properties

Interval Linearizability Properties

- Local property (like linearizability)

An execution E is interval linearizable if and only if each object X, $E|_X$ is interval linearizable

- Non-blocking property (like linearizability)

For every interval linearizable execution E, there is an interval linearization with all ops in E completed

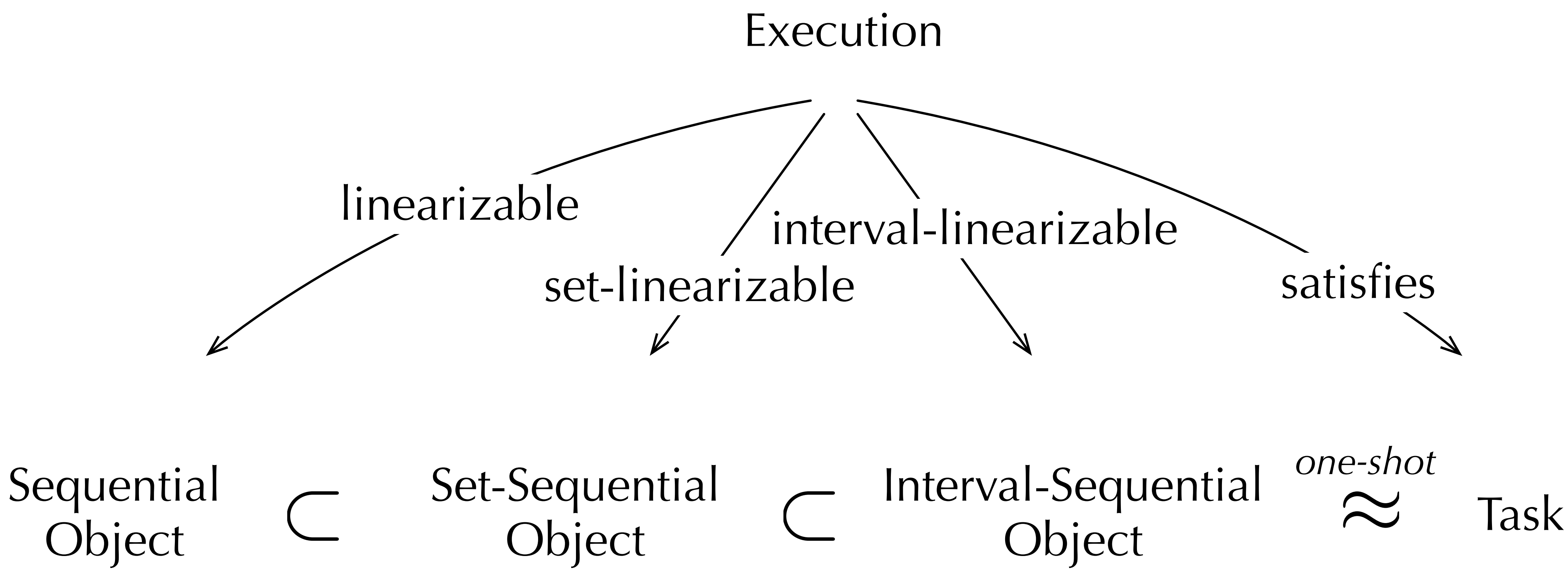
Completness Result

A general definition: Prefix-closed set of executions (with no restrictions, not necessarily one-shot)

Most general definition one can imagine?

For every prefix-closed set of executions, there is a IS object that model the set

Conclusion



- Set-based spec = multi-shot tasks = IS linearizability
- We are working on extend task definition further, to model multi-shot objects
- and on applying topological techniques to objects

Thanks!!