MITRO207 2018
Exercise session

Problem 1

Let $\mathcal{A}$ and $\mathcal{B}$ be simplicial complexes and $\varphi : V(\mathcal{A}) \to V(\mathcal{B})$ be a simplicial map. Show that if $\mathcal{A}$ is connected, then the image of $\mathcal{A}$ under φ, denoted $\varphi(\mathcal{A})$ and defined as the set of vertices $\{\varphi(v) | v \in V(\mathcal{A})\}$ is *connected in* $\mathcal{B}$: every two vertices in $\varphi(\mathcal{A})$ are connected by a path in $\mathcal{B}$.

Show that it is not always the case that if $\varphi(\mathcal{A})$ is connected, then $\mathcal{A}$ is connected.

Problem 2

Figure 1 depicts $\mathcal{I}$, the input complex of the 1/5-agreement task, $\mathcal{P}$, the protocol complex of 1 layer of the read-write layered model, and $\mathcal{O}$, the output complex of the 1/5-agreement task.

Recall that Δ, the carrier map of the task, maps each of the endpoints of $\mathcal{I}$ to the corresponding endpoints of $\mathcal{O}$ and each of the edges of $\mathcal{I}$ to $\mathcal{O}$.

Prove that there does not exist a simplicial map δ from $\mathcal{P}$ to $\mathcal{O}$ carried by Δ.

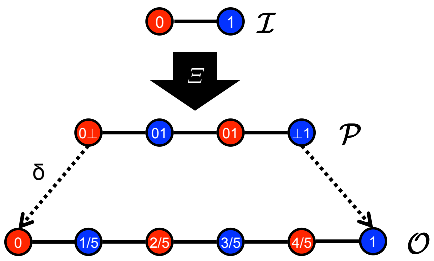

Figure 1: Complexes of the 1-layer read-write protocol and the 1/5-agreement task.

Problem 3

Consider the model in which $n+1$ processes $p_0, \ldots, p_n$ communicate via layered *participating-set* objects. The object stores a set of *values* (initially $\emptyset$), and exports one *atomic* operation *join*(V) that adds the set of values V to the object state and returns the state.

In the corresponding layered full-information protocol, we have a series of participating-set objects $M_1, M_2, \ldots$. Each process p_i starts with invoking $M_1.join(i)$, and then proceeds to the next layer, each time invoking M_i with the value returned by M_{i-1}.

1. Draw the protocol complexes of one layer of this model for $n = 1$ (two processes) and $n = 2$ (three processes).

2. In the colorless task of *set agreement*, every process proposes a value in a set V ($|V| = n + 1$) and outputs one of the proposed values, so that at most n different values are output.

   Formally, the input complex $\mathcal{I}$ is Δ^n (the standard n-dimensional simplex), the output complex $\mathcal{O}$ is $\texttt{skel}^{n-1}\Delta^n$ (the set of all faces of Δ^n of dimensions $n - 1$ or less), and the task carrier map is

$\texttt{skel}^{n-1}$ (mapping each simplex $\sigma \in \mathcal{I}$ to $\texttt{skel}^{n-1}\sigma$). In particular, for the n-dimensional simplex σ, $\Delta(\sigma)$ is the set of all proper faces of σ.

Show that set agreement can be solved by the 1-layered participating-set protocol (by presenting the corresponding simplicial decision map).

Hint: if a process sees n or less different values after accessing M_1, it can output any of these values. The only interesting case is when a process sees $n+1$ different values.

3. Show that for all $n \geq 2$ (three or more processes) and $L \geq 1$, the task of consensus *cannot* be solved by the L-layered participating-set protocol

   Hint: The complex of an L-layered protocol is the union of one-layer protocol complexes resulting from the simplices of the $(L-1)$-layered protocol complex.

   Check if the protocol complex remains connected, regardless of the number of layers .