MITRO207
Homework 5: Solutions

Problem 1: n-skeleton (Exercise 5.1)

Show that the colorless complex corresponding to independently assigning values from a set V^{in} to a set of $n+1$ processes is the n-skeleton of $\mathbf{s}^{|V^{in}|-1}$ (the complex of all the faces of the standard $(|V^{in}|-1)$-dimensional simplex).

The vertices of our colorless complex are all the values that could be used as inputs, i.e., V^{in}. Simplices of our complex are all sets of vertices of size at most $n+1$. Thus, every simplex in this complex is a face of $\mathbf{s}^{|V^{in}|-1}$, where vertices are in V^{in}.

A simplex in the complex is a set of vertices that can correspond to values that can appear in the same assignment. Since the values are assigned independently, and there are $n+1$ processes, every combination of at most n distinct values in V^{in} is a possible assignment.

Thus, the complex is the collection of all sets of vertices of size at most $n+1$, i.e., all the faces of the $(|V^{in}|-1)$-dimensional simplex of dimension at most n, or, put differently, the n-skeleton of $\mathbf{s}^{|V^{in}|-1}$.

Problem 2: solving set agreement with test-and-set

Recall the test-and-set objects from Problem 1 of Homework 4. Assuming that the input complex $\mathcal{I}$ is $\mathbf{s}^n$, give an $(n+1)$-process protocol for solving $\lceil \frac{n+1}{2} \rceil$-set agreement using test-and-set objects.

The idea is to split the processes in pairs and solve consensus within each pair.

For each $i = 0, \ldots, \lceil \frac{n+1}{2} - 1 \rceil$, we associate a distinct test-and-set object T_i with two processes p_{2i} and p_{2i+1} (the last pair may include only one process if n is an even number).

In the protocol, each p_i accesses $T_{\lceil i/2 \rceil}$: if it wins, it outputs its own identifier, otherwise it outputs the identifier of the other process associated with $T_{\lceil \frac{n+1}{2} \rceil}$.

It is easy to see that each two processes p_{2i} and p_{2i+1} output the same identifier and, thus, at most $\lceil \frac{n+1}{2} \rceil$ distinct values are output.

Moreover, only the identifiers of processes that invoked test-and-set instructions can be output. Hence, if a set of processes $\sigma \in \mathbf{s}^n$ participate, then only identifiers in σ are output, which gives a simplex in $\mathtt{skel}^{\lceil \frac{n+1}{2} \rceil}\sigma$.