MITRO207-2018
Homework 4

Problem 1: test-and-set

Consider the model in which $n+1$ processes $p_0, \dots, p_n$ communicate via a *test-and-set* (*T&S*) object. The object is initialized to 0 and exports one atomic operation *TAS*() which returns the value of the object and then sets the value to 1.

1. Draw the protocol complexes of the model for $n = 1$ (two processes) and $n = 2$ (three processes).
2. The task of *input-less consensus* is defined as a tuple $(\mathcal{I}, \mathcal{O}, \Delta)$, where the input complex $\mathcal{I}$ is the simplex $\{(0,0), \dots, (n,n)\}$ plus all its faces, the output complex $\mathcal{O}$ is a set of simplices $\{(0,0), \dots, (n,0)\}$, $\{(0,1), \dots, (n,1)\}$, $\dots$, $\{(0,n), \dots, (n,n)\}$ plus all their faces, and Δ maps each $\sigma = \{(i_1, i_1), \dots, (i_k, i_k)\}$ in $\mathcal{I}$ to the set of simplices of $\mathcal{O}$ in which vertices have the form (i,j) such that $i, j \in \{i_1, \dots, i_k\}$.

   Intuitively, every process has its identifier as an input, and the goal of the task is to agree on the identifier of one of the participating processes.

   Is the task of simplified consensus task solvable in the model above for $n = 1$? For $n = 2$? Explain why.

Problem 2

Let $\mathcal{K}$ be a complex and $\mathsf{Div}\,\mathcal{K}$ be a subdivision $\mathcal{K}$. For each $\tau \in \mathsf{Div}\,\mathcal{K}$, let $\mathsf{Car}(\tau)$ be the minimal simplex $\sigma \in \mathcal{K}$ (with all its faces), such that $\tau \in \mathsf{Div}\,\sigma$.

Show that $\mathsf{Car} : \mathsf{Div}\,\mathcal{K} \to 2^{\mathcal{K}}$ is a carrier map. Check whether the carrier map is strict.

Problem 3: commit-adopt task (Exercise 4.10 in the book)

The *commit-adopt* task is a variant of consensus, where each process is assigned an input value in some set V and it chooses, as an output, a pair $[\mathit{decision}, v]$, where *decision* is either *commit* (C) or *adopt* (A) and v is *one of the input values* in the execution. Moreover, (i) if a process decides $[C, v]$, then every other decision is $[., v]$, and (ii) if all inputs are the same, then every decision value must be $[C, .]$.

Define the task formally as a colorless task and show that it can be solved by a 2-layer colorless protocol, but not by a 1-layer colorless protocol.

Problem 4: simplicial approximation

Let $f : |\mathcal{A}| \to |\mathcal{B}|$ be a continuous map and $\phi : \mathcal{A} \to \mathcal{B}$ be a simplicial map. We say that ϕ is a *simplicial approximation* of f if:

$$\forall v \in V(\mathcal{A}) : \ f(\mathsf{St}^0(v)) \subseteq \mathsf{St}^0(\phi(v))$$

Show that the condition above is equivalent to:

$$\forall \sigma \in \mathcal{A} : \ f(\mathsf{Int}\,\sigma) \subseteq \mathsf{St}^0(\phi(\sigma))$$

Recall that

$$\mathsf{Int}\,\sigma = |\sigma| - \bigcup_{\tau \subsetneq \sigma} |\tau|$$

is the *interior* of $|\sigma|$ (the geometric realization of σ without the boundary), and

$$\mathsf{St}^0(\sigma) = \bigcup_{\tau \supseteq \sigma} \mathsf{Int}\,\tau$$

is the *open star* of σ (in a simplicial complex it belongs to).